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Introduction to the Method of Finite Elements by a balance Sheet Problem: A Simplification for an Initial understanding of the Method

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Abstract**—** *The Finite Element method is one of the most widely used methods by Engineers in the various areas of activity, especially Mechanical Engineering, to design or solve problems. However, the understanding of the method is not always easy to perform, since in the literature, when explaining the method, the examples are generic or presented quickly. Thus, this paper presents the solution of a problem involving a rocking beam (set), which is solved analytically and later by the finite element method. The comparison of the solutions found is established as reflection analysis. Elasticity theory, Ordinary Differential Equations and Finite Element Method are used to approximate the reader of the Finite Element Method, in a concise and objective, easy-to-understand reading performed with a reduced explanation. Comparing the method by means of a problem.*

Keywords— Finite Element Method, Beams, Differential Equations.

I. INTRODUCTION

Mathematical models are increasingly present in the area of Engineering. In Elasticity Theory, for example, there are several models that work with tensions, deformations and displacements of structures. Be applied to one-dimensional, two-dimensional or three-dimensional problems. In the search for these models, [1] says that several problems fall into the Differential Equations, whether these are ordinary, when the treated model is one-dimensional or partial where the model can be two-dimensional or three-dimensional. Therefore, a problem of obtaining a model for the understanding of a certain phenomenon can become a problem of solving the model found, that is, the problem of physical phenomenon can be modeled by a Differential Equation, and however, it needs a method resolution.

In [2], the resolution methods can be analytical and / or numerical. Where, among the various numerical methods, for the described situation, the Finite Element Method (MEF) stands out. This method was developed by the need to solve complex problems of Engineer, and can advance, nowadays, with the advent of Technology. Today it is widely used for the resolution and reflection of various problems in the Engineering area. The purpose of this method is to approach the searched solution in a discrete way.

Due to its potentialities, for the Engineer interested, it becomes interesting to understand such a method.

II. OBJECTIVE

To present succinctly, by the resolution of a problem of balance beam (set), the Finite Element method;

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III. METHODOLOGY

A problem is presented, on beam, to deduce the solution via analytical mode and via numerical mode. Being the second resolution based on the MEF, in order to lead the reader, to a first contact with the method. The problem proposed is:

A set beam of homogeneous steel, whose length is 2 meters, has a cross section in I format and W250x17,9 specification. Such a beam will be subjected to a Q (concentrated) charge of intensity 10 kN at its free end.

The sketch of the problem, using the presented conditions, with reference axes adopted, can be worked in a one-dimensional way.

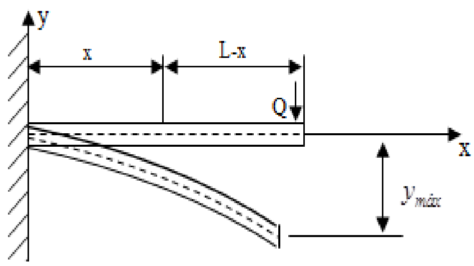

Fig. 1: Diagram elaborated for Resolution

To determine the bending moment, using the right side, it is established:

$$M(x) = -Q(L-x) \qquad (1)$$

Using the differential equation whose solution is the deflection curve, we have

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$$\int \frac{d^2y}{dx^2}dx = \int \frac{-Q(L-x)}{EI}dx \Rightarrow \frac{dy}{dx} = \frac{1}{EI}\left(\frac{Qx^2}{2} - QLx\right) + C \quad (2)$$

From the proposed problem statement, initial conditions can be extracted.

At the point where the beam is embedded, that is, in the adopted frame, y (0) = 0 and y '(0) = 0. Thus, by replacing in (2), the value of C = 0 is found. Then, as the analytic resolution continues:

$$\int \frac{dy}{dx} = \int \frac{1}{EI}\left(\frac{Qx^2}{2} - QLx\right)dx \Rightarrow y(x) = \frac{Qx}{2EI}\left(\frac{x^3}{3} - Lx^2\right) + C \quad (3)$$

Soon

$$y(x) = \frac{Qx}{2EI}\left(\frac{x^3}{3} - Lx^2\right) \quad (4)$$

The function (4) provides the generalized elastic line for all problems of homogeneous beams (with spheres) with concentrated load at the free end, worked in a one-dimensional manner, that is, the extension with respect to one axis is relatively larger than when compared to others. For the problem in question we have: L = 2 m, Q = 10 kN = 10000 N, E = Pa and I =, the values of E and I, for the steel used can be obtained in the table of NBR-8800 2008, obtained from the [3] website. Thus, the elastic curve is:

$$y(x) = \frac{10000x}{2.200.10^9.2291.10^{-8}}\left(\frac{x^3}{3} - 2x^2\right) \Rightarrow y(x) = \frac{5x}{4582}\left(\frac{x^3}{3} - 2x^2\right) \quad (5)$$

Making a table and analyzing values of the deflections in the points: x = 0, x = 1 and x = 2, we obtain:

Table.1: Analysis by the analytical method of some deflections

X	Deflexão (y) - mm
0	0
1	-1,8187108977157
2	-11,6397497453805

It is found that the largest deformation occurs at x = 2 m, as was expected at the free end of the beam, where the load Q is acting, making this point move approximately m or 11.64 mm.

To prepare the numerical response via MEF, we use [4] and [5]. Initially, the Elasticity Theory equations and the discretization of the worked object. Discretization can be thought of here as dividing the beam into other "pieces". For the problem in question, the beam was divided into two elements of the same length (T and R), that is, L = 1 m for each element, with such discretization we obtain three nodes (1, 2, 3), according to the figure, which also contains the adopted framework.

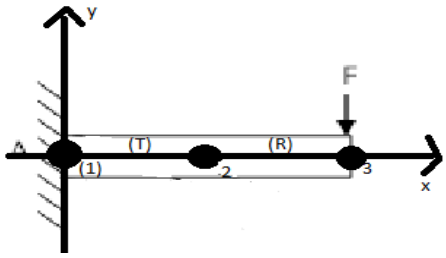

Fig. 2: Beam being discretized by elements

With this discretization configuration, at the end of the process, the method will provide the displacements relative to the established nodes (1, 2, and 3), that is, relative displacements ax = 0, x = 1 ex = 2, according to the established initial constructions and the adopted reference.

Once the discretization is established, a stiffness matrix is assembled for each of the elements. This rigidity matrix is established by the theorems of the Variational Calculus, Elasticity Theory and Physics, worked in a discrete way. For the configuration adopted, each element (T and R) will have its stiffness matrix.

For the problem in question each element will have a stiffness matrix in the format presented:

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad (6)$$

Where L is the length of the element and A is the area of the profile adopted (in this case, it is the steel beam W250x17,9), to obtain the value of A, a query to Gerdau's website, and then we have that A =. Then, replacing the values of A, E, I and L in (6) and overlapping the matrices we have

$$\begin{pmatrix} 4.6210^8 & 0 & 0 & -4.6210^8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.498410^7 & 2.749210^7 & 0 & -5.498410^7 & 2.749210^7 & 0 & 0 & 0 \\ 0 & 2.749210^7 & 1.832810^7 & 0 & -2.749210^7 & 9164000.0 & 0 & 0 & 0 \\ -4.6210^8 & 0 & 0 & 9.2410^8 & 0 & 0 & -4.6210^8 & 0 & 0 \\ 0 & -5.498410^7 & -2.749210^7 & 0 & 1.0996810^8 & 0.0 & 0 & -5.498410^7 & 2.749210^7 \\ 0 & 2.749210^7 & 9164000.0 & 0 & 0.0 & 3.665610^7 & 0 & -2.749210^7 & 9164000.0 \\ 0 & 0 & 0 & -4.6210^8 & 0 & 0 & 4.6210^8 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5.498410^7 & -2.749210^7 & 0 & 5.498410^7 & -2.749210^7 \\ 0 & 0 & 0 & 0 & 2.749210^7 & 9164000.0 & 0 & -2.749210^7 & 1.832810^7 \end{pmatrix}$$

(7)

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From the theory of elasticity it is known that: [K] [D] = [F], the product of the global stiffness matrix [K], by the displacement matrix [D], is equal to the force matrix acting on the system [F].

The force matrix and the displacement matrix are assembled from the forces acting on the problem and from the degrees of freedom of each node of the problem, respectively.

$$D = \begin{pmatrix} u_1 \\ v_1 \\ r_1 \\ u_2 \\ v_2 \\ r_2 \\ u_3 \\ v_3 \\ r_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_2 \\ r_2 \\ 0 \\ v_3 \\ r_3 \end{pmatrix} \quad F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -10000.0 \\ 0 \end{pmatrix} \quad (8)$$

Note that the force matrix [F], there is only one force acting in the opposite direction to the established reference frame, with respect to the y-axis, thus occupying the indicated position with the negative signal and the displacement matrix [D], was established from the following (2) and (3) will have two degrees of freedom - displacement with respect to the axis and rotation with respect to the xy-plane - so we will have no degree of freedom for the problem in question. in the displayed mode, ie, where there are no displacements, it is assumed to be zero.

To solve this linear system you can use mathematical software such as Visual Calculus Numerical (VCN), Maxima, Scilab and / or Maple or even Microsoft Excel. When solving the linear system, we find the mentioned displacements.

$$D = \begin{pmatrix} u_1 \\ v_1 \\ r_1 \\ u_2 \\ v_2 \\ r_2 \\ u_3 \\ v_3 \\ r_3 \end{pmatrix} = (9) \quad \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ -0.0018187108977157699 \\ -0.0032736796158882258 \\ 0.0 \\ -0.0058198748726902377 \\ -0.0043649061545176788 \end{pmatrix}$$

It is interesting here to compare the displacements suffered by nodes (1), (2) and (3), in relation to the y axis, that is, those obtained in matrix D. Interestingly, in node (1), as already was expected, there is no displacement, either horizontal, vertical or rotation, $u_1 = v_1 = r_1 = 0$ this is due to the fact that node (1) has no degree of freedom in the proposed problem. Thus, by comparing the analytical method with the numerical method and presenting it in a table, we have:

Table 2: Numerical method comparison and analytical method

X	Deflexão (Em relação a y) – Método Numérico - mm	Deflexão (Em relação a y) – Método Analítico - mm
0	0	0
1	-1,8187108977	-1,818710897157
2	-5,8198748726	-11,6397497453805

When comparing the methods, it is possible to verify that the first two approximations are satisfactory, that is, referring to node (1), which occurs at x = 0 and to node (2), which occurs at x = 1. However, 3, which occurs at x = 2, a very large error can be verified by comparing the analytical method with the numerical method.

An attempt to control this error is to work with more elements in the discretization, but the global linear system will have more unknowns to consider and the stiffness matrix will be larger. What is not configured at the present time as a major problem, since the technology supports linear systems such as those mentioned.

Therefore, with this small demonstration of the method, it is possible to verify that the approximations found for the solution of the proposed problem are satisfactory to present the MEF as a numerical method for solving structural problems. If the approximations in another problem are not enough or to the liking of the engineer it is possible to refine the solution found, as much as one wants.

IV. CONCLUSION

In order to employ a reductionist character, the Theory, by MEF, which is rigorous and elegant, was aimed here, to offer a reader interested north to the understanding of this numerical method and its theory. In short, the solutions of the same problem, by the analytical method and by the numerical method, succinctly, the general objective of the article is reached, together with the specific ones: an introduction by the resolution of a problem on MEF beams

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Thus, it is possible to analyze the complement of analytical and numerical solutions, expressing a problem that offers the intersection of these solutions. However, it is necessary for the future engineer, the engineer and the interested reader to verify that there are problems that the analytical solution is not suitable, so to resort to the numerical solution becomes a complement, and this occurs with the MEF.

Allowing those interested in using the method, a first contact or even glimpsing an application of the method is a way of contributing to the academic milieu. This leaves the possibility for the reader to broaden his knowledge about the MEF, understanding its full potential and theory.

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