

# Application of Cubic Spline Interpolation to Fit the Stress-Strain Curve to SAE 1020 Steel

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**Abstract** —This article presents the methodology to use cubic splines interpolation method to fit the stress-strain curve, in the field of permanent deformation, based on experimental data obtained on performed tests on a standardized specimen of SAE 1020 steel hot rolled flat. The third-order polynomial for each interval between knots was used to fit the stress-strain curve.

**Keywords**— Cubic Spline, Permanent Deformation, Curve, Interpolation, Steel.

## I. INTRODUCTION

According to Vicente Chiaverini (1986), the materials tend to deform when subjected to mechanical stresses, and, depending on the nature of each material, their behavior during deformation may vary. Metals can undergo considerable permanent deformation before to rupture. The application of cubic splines for the construction of the strain-strain curve, in the field of permanent deformation, through the mathematical manipulation of data obtained in experimental tests, allows the identification of third-order interpolator polynomials to obtain the value of the acting axial tension for a certain measured deformation.

The benefits are the possibility of obtaining values of the axial tension for a given deformation, point of constraint and better approximation of the stress-strain curve.

## II. MATERIAL AND METHODOLOGY OF THE TEST

The experimental data were obtained by tensile tests performed on samples of specimens made of SAE 1020 hot rolled material. The tests were performed on a vertical traction machine.

### 2.1 SPECIMEN USED ON EXPERIMENTAL TESTS

The specimens used in the tensile tests were prepared according to the recommendations of ASTM E8 / E8M. The material used was SAE 1020 hot rolled steel.

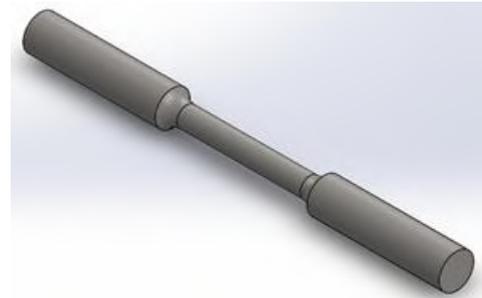


Fig. 1: Specimen Used on Experimental Tests

### 2.2 METHODOLOGY OF THE EXPERIMENTAL TESTS

The tests were carried out on four identical samples of the test specimen, in order to compare the values obtained in the tests and to guarantee the responses of the equipment used.

The specimens were fixed to the vertical traction machine by means of wedge-shaped jaws, suitable to reduce slippage during load application, which reduces errors in data collection.

The axial tensile load was applied to the test pieces gradually, causing the deformation of the same. The axial tensile load was raised until fracture of each of the test specimens used in the tests.

### 2.3 RESULTS OBTAINED IN THE EXPERIMENTAL TESTS

The results obtained in the tensile tests of each of the four test specimens are described in the table below. Seven measuring points were taken, all within the permanent deformation field, where the values of the axial tensile load and its respective value of permanent deformation of the specimen were collected.

The value of the deformation collected in the first measurement covers the field of elastic deformation, which was linear for all the specimens tested. As this work aims to work only with the data of the curve of permanent deformation, which is not linear, the data referring to the field of elastic deformation were suppressed.

Table. 1: Results of Experimental Tests

Sample	Measurement	Valor	Valor	Valor	Valor	Valor	Valor	
		1 <sup>a</sup>	2 <sup>a</sup>	3 <sup>a</sup>	4 <sup>a</sup>	5 <sup>a</sup>	6 <sup>a</sup>	7 <sup>a</sup>
#01	Load (kgf)	2501	2989	3294	3447	3443	3447	3386
	Stress (kgf/mm <sup>2</sup> )	38,20	45,66	50,31	52,65	52,59	52,65	51,72
	Strain (mm)	7,44	9,30	11,16	13,02	14,88	16,74	18,60
#02	Load (kgf)	2562	2987	3233	3386	3507	3509	3385
	Stress (kgf/mm <sup>2</sup> )	39,13	45,63	49,38	51,72	53,57	53,60	51,70
	Strain (mm)	7,44	9,30	11,16	13,02	14,88	16,74	18,60
#03	Load (kgf)	2555	2980	3244	3380	3450	3452	3383
	Stress (kgf/mm <sup>2</sup> )	39,03	45,52	49,55	51,63	52,70	52,73	51,67
	Strain (mm)	7,44	9,30	11,16	13,02	14,88	16,74	18,60
#04	Load (kgf)	2531	2981	3270	3375	3448	3451	3380
	Stress (kgf/mm <sup>2</sup> )	38,66	45,53	49,95	51,55	52,67	52,71	51,63
	Strain (mm)	7,44	9,30	11,16	13,02	14,88	16,74	18,60

### III. MATHEMATICAL ANALYSIS OF RESULTS OF EXPERIMENTAL TESTS

#### 3.1 MATHEMATICAL TREATMENT OF TEST RESULTS

Based on the results obtained in the experimental tests, a new the table was elaborated with the results that will be taken as basis for the application of the interpolating polynomials of the cubic splines. "Table 2" was constructed by taking as reference the mean values obtained in the experimental tests of the four specimens of the specimen.

Table. 2: Mean values obtained in the experimental tests.

Measurement	Value						
	1 <sup>a</sup>	2 <sup>a</sup>	3 <sup>a</sup>	4 <sup>a</sup>	5 <sup>a</sup>	6 <sup>a</sup>	7 <sup>a</sup>
Load (kgf)	2537,25	2984,25	3260,25	3397,00	3462,00	3464,75	3383,50
Stress (kgf/mm <sup>2</sup> )	38,76	45,56	49,80	51,89	52,88	52,92	51,68
Strain (mm)	7,44	9,30	11,16	13,02	14,88	16,74	18,60

The data in Table 2 will be used to develop the mathematical equations that will be demonstrated in the next topic.

#### 3.2 MATHEMATICAL TREATMENT OF TEST RESULTS

According to Steven C. Charpa and Raymond P. Canale (2011), the spline concept originated from a drawing technique in which a thin, flexible strip (called a spline) was used to draw a smooth curve through a set of points . A smooth cubic curve results from interspersing the strip between the pins. Thus, the name "cubic splines" was adopted for such polynomials.

Cubic splines will be applied to determine a third-order polynomial, for each of the intervals, of the seven experimental measurements, in order to approximate the strain x strain curve of the experimental results.

First, we will write the third-order polynomials for each of the experimental data ranges.

$$S(x) = a_1x^3 + b_1x^2 + c_1x + d_1 . x \in [7,44;9,30] \quad (1)$$

$$S(x) = a_2x^3 + b_2x^2 + c_2x + d_2 . x \in [9,30;11,16] \quad (2)$$

$$S(x) = a_3x^3 + b_3x^2 + c_3x + d_3 . x \in [11,16;13,02] \quad (3)$$

$$S(x) = a_4x^3 + b_4x^2 + c_4x + d_4 . x \in [13,02;14,88] \quad (4)$$

$$S(x) = a_5x^3 + b_5x^2 + c_5x + d_5 . x \in [14,88;16,74] \quad (5)$$

$$S(x) = a_6x^3 + b_6x^2 + c_6x + d_6 . x \in [16,74;18,60] \quad (6)$$

Applying the condition that the function values must be equal at the interior knots, and the first and last functions must pass through the end points, we have the following equations:

$$S(7,44)=411,83.a_1+55,35.b_1+7,44.c_1+d_1=38,76 \quad (7)$$

$$S(9,30)=804,36.a_1+86,49.b_1+9,30.c_1+d_1=45,56 \quad (8)$$

$$S(9,30)=804,36.a_2+86,49.b_2+9,30.c_2+d_2=45,56 \quad (9)$$

$$S(11,16)=1389,93.a_2+124,55.b_2+11,16.c_2+d_2=49,80 \quad (10)$$

$$S(11,16)=1389,93.a_3+124,55.b_3+11,16.c_3+d_3=49,80 \quad (11)$$

$$S(13,02)=2207,16.a_3+169,52.b_3+13,02.c_3+d_3=51,89 \quad (12)$$

$$S(13,02)=2207,16.a_4+169,52.b_4+13,02.c_4+d_4=51,89 \quad (13)$$

$$S(14,88)=3294,65.a_4+221,41.b_4+14,88.c_4+d_4=52,88 \quad (14)$$

$$S(14,88)=3294,65.a_5+221,41.b_5+14,88.c_5+d_5=52,88 \quad (15)$$

$$S(16,74)=4691,01.a_5+280,23.b_5+16,74.c_5+d_5=52,92 \quad (16)$$

$$S(16,74)=4691,01.a_6+280,23.b_6+16,74.c_6+d_6=52,92 \quad (17)$$

$$S(18,60)=6434,86.a_6+345,96.b_6+18,60.c_6+d_6=51,68 \quad (18)$$

Applying the condition that the first derivatives at the interior knots must be equal, we have the following equations:

$$259,47.a_1+18,60.b_1+c_1=259,47.a_2+18,60.b_2+c_2 \quad (19)$$

$$373,64.a_2+22,32.b_2+c_2=373,64.a_3+22,32.b_3+c_3 \quad (20)$$

$$508,56.a_3+26,04.b_3+c_3=508,56.a_4+26,04.b_4+c_4 \quad (21)$$

$$664,24.a_4+29,76.b_4+c_4=664,24.a_5+29,76.b_5+c_5 \quad (22)$$

$$840,68.a_5+33,48.b_5+c_5=840,68.a_6+33,48.b_6+c_6 \quad (23)$$

Applying the condition that the second derivatives at the interior knots must be equal, we have the following equations:

$$55,80.a_1+2.b_1=55,80.a_2+2.b_2 \quad (24)$$

$$66,96.a_2+2.b_2=66,96.a_3+2.b_3 \quad (25)$$

$$78,12.a_3+2.b_3=78,12.a_4+2.b_4 \quad (26)$$

$$89,28.a_4+2.b_4=89,28.a_5+2.b_5 \quad (27)$$

$$100,44.a_5+2.b_5=100,44.a_6+2.b_6 \quad (28)$$

Applying the condition that the second derivatives at the end knots are zero, we have the following equations:

$$44,64.a_1+2.b_1=0 \quad (29)$$

$$111,60.a_6+2.b_6=0 \quad (30)$$

Solving the linear system, from equation 7 to equation 30, with their respective twenty-four unknowns, the following values were obtained.

$$\begin{aligned} a_1 &= -0,05497, b_1 = 1,22701, c_1 = -5,28285 \text{ e } d_1 = 32,78479; \\ a_2 &= -0,03342, b_2 = 0,62565, c_2 = 0,30979 \text{ e } d_2 = 15,44759; \\ a_3 &= 0,07327, b_3 = -2,94646, c_3 = 40,17451 \text{ e } d_3 = -133,42537; \\ a_4 &= -0,00696, b_4 = 0,18739, c_4 = -0,62827 \text{ e } d_4 = 43,65871; \\ a_5 &= -0,02213, b_5 = 0,86487, c_5 = -10,70909 \text{ e } d_5 = 93,65957; \\ a_6 &= 0,04421, b_6 = -2,46692, c_6 = 45,06517 \text{ e } d_6 = -217,5607. \end{aligned}$$

In this way, we can write the polynomials that approximate the value of the acting axial stress as a function of the deformation measured in the experimental tests, for each of the intervals between the measurements.

$$\begin{aligned} S(x) &= -0,05497.x^3 + 1,22701.x^2 - 5,28285.x + 32,78479; \\ &\text{if } x \in [7,44;9,30] \\ S(x) &= -0,03342.x^3 + 0,62565.x^2 + 0,30979.x + 15,44759; \\ &\text{if } x \in [9,30;11,16] \\ S(x) &= 0,07327.x^3 - 2,94646.x^2 + 40,17451.x - 133,42537; \\ &\text{if } x \in [11,16;13,02] \\ S(x) &= -0,00696.x^3 + 0,18739.x^2 - 0,62827.x + 43,65871; \\ &\text{if } x \in [13,02;14,88] \\ S(x) &= -0,02213.x^3 + 0,86487.x^2 - 10,70909.x + 93,65957; \\ &\text{if } x \in [14,88;16,74] \\ S(x) &= 0,04421.x^3 - 2,46692.x^2 + 45,06517.x - 217,56079; \\ &\text{if } x \in [16,74;18,60] \end{aligned}$$

### 3.3 APPROXIMATION OF THE STRESS-STRAIN CURVE WITH THE USE OF THE CUBIC SPLINES

The cubic splines that approximate the stress values for the deformations evidenced in the experimental tests, obtained through the mathematical methods demonstrated in topic 2.2, were used to construct the strain-strain curve, as can be seen in Fig. 2.

The curve was constructed with the application of the cubic splines in the numerical calculation software VCN.

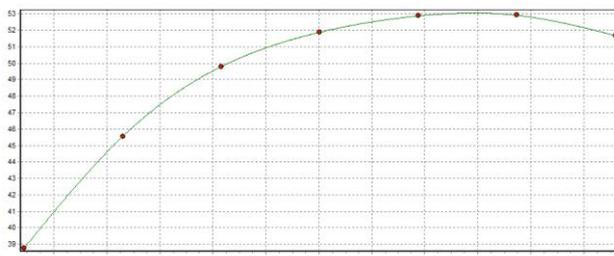


Fig. 2: Stress x strain curve obtained through the cubic spline polynomials

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