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Investigation of Multi Linear Regression Methods on Estimation of Free Vibration Analysis of Laminated Composite Shallow Shells

Ali Dogan, Omer Faruk Cansiz, Kevser Unsalan, Nurullah Karaca*

Department of Civil Engineering, İskenderun Technical University, TURKEY
***C**orresponding Author: nurullah.karaca@iste.edu.tr (N.Karaca)

***Abstract*— *This paper presents regression method's in estimating the free vibration analysis and compared with SDSST method. In this study, the free vibration analysis of the cross-ply laminated composite cylindrical shallow shells has been studied using shear deformation shallow shell theory (SDSST). First, the kinematic relations of strains and deformation are given. Then, using Hamilton's principle, governing differential equations are developed for a general curved shell. Finally, the stress-strain relation for the laminated, cross-ply composite shells are obtained. By using some simplifications and assuming Fourier series as a displacement field, the governed differential equations are solved by the matrix algebra for shallow shells. Employing the computer algebra system called MATHEMATICA; a computer program has been prepared for the solution [1]. The results obtained by this solution are compared with the results obtained by (ANSYS) programs. In this article, regression method's and SDSST method's abilities in estimating the free vibration with the laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables, are compared with different and similar aspects. In comparing with linear, interaction, quadratic and pure quadratic models, which are constructed with multiple linear regression approach, the quadratic model provides better results.*

Keywords*— *Anisotropy, Finite Element Method (FEM), Multi Linear Regression, Shell Theory, Structural Composites, Free Vibration.

I. INTRODUCTION

A structural composite material consists of two or more constituents combined on a macroscopic scale to form a useful material. Different materials must be put together in a three dimensional body. The goal of this three dimensional composition is to obtain a property which none of the constituents possesses. In other words, the target is to produce a material that possesses higher performance properties than its constituent parts for a particular purpose. Some of these properties are mechanical strength, corrosion resistance, high temperature resistance, heat conductibility, stiffness, lightness and appearance. In accordance with this definition, the following conditions must be satisfied by the composite material. It must be manmade and unnatural. It must comprise of at least two different materials with different chemical components separated by distinct interfaces. It must possess properties, which none of the constituents possesses alone and that must be the aim of its production. The material must behave as a whole, i.e. the fiber and the matrix material (material surrounding the fibers) must be perfectly bonded. Structures composed of composite materials offer lower weight and higher strength and stiffness than those composed of most metallic materials [2].

Shells are common structural elements in many engineering structures, including concrete roofs, exteriors of rockets, ship hulls, automobile tires, containers of liquids, oil tanks, pipes, aerospace etc. A shell can be defined as a curved, thin-walled structure. It can be made from a single layer or multilayer of isotropic or anisotropic materials. Shells can be classified according to their curvatures. Shallow shells are defined as shells that have rise of not more than one fifth of the smallest planform dimension of the shell [2]. Shells are three-dimensional (3D) bodies bounded by two relatively close, curved surfaces. Since the 3D equations of elasticity are complicated, all shell theories (thin, thick, shallow and deep, etc.) reduce the 3D elasticity problem into a 2D one. This is done usually by Classical Lamination Theory-CLT and Kirchhoff hypothesis. A number of theories exist for layered shells. Many of these theories were developed originally for thin shells and based on the Kirchhoff–Love kinematic hypothesis that straight lines normal to the undeformed mid-surface remain straight and normal to the middle surface after deformation. Many studies have been performed on characteristics of shallow shells. Qatu [2] uses energy functional to develop equation of motion. Reddy [3] are presented effect of shear deformation for laminated composite plates and shells. Dogan [4] studied the effects of anisotropy and curvature on free vibration characteristics of laminated composite cylindrical shallow shells. Also Dogan [5] are presented the mode-shape analysis of the cross-ply laminated composite cylindrical shallow shells using SDSST and finite element method.

There has been a lot of number of studies on the regression analysis of composites. Dong etc [6] studied dimension variation prediction for composites with finite element analysis and regression modeling. The studied provides information to develop practical and forward-looking dimension control techniques for composite material. Lee etc. [7] were studied on regression of the response surface of laminated composite structures. In this study, the response surface of composite laminated structures was calculated using regression analysis. Ply angles of composite structure was investigated using regression analysis method. Satapathy etc. [8] studied targeted material design of fly ash filled composites for friction braking application by non-linear regression technique. The study demonstrated non-linear regression technique successfully analyzed materials design of heterogeneous composites. Ziari etc. [9] studied predicting rutting performance of carbon nano tube asphalt binders using regression models and neural networks. Prediction models were obtained using regression and artificial neural network of rutting performance of carbon nano tube reinforced asphalt binders and performances of these models were compared. Artificial neural network model performance is better than regression model results. Oladipo etc. [10] studied optimization and modelling using non- linear regression technique to enhanced removal of crystal violet by low cost alginate and acid activated bentonite composite beads. In this work, operational parameters investigated and for this used non-linear regression as method of analysis. Yadollahi etc. [11] studied application of adaptive network-based fuzzy inference system (ANFIS) and regression models to predict the compressive strength of geopolymer composites. Materials properties of geopolymer composites specified very difficult for this reason geopolymer are widely complex materials. ANFIS, linear and non-linear regression methods were studied to determine best approach of composite materials.

II. THEORY

A lamina is produced with the isotropic homogenous fibers and matrix material. Any point on a fiber, and/or on matrix and/or on matrix-fiber interface has crucial effect on the stiffness of the lamina. Due to the big variation on the properties of lamina from point to point, macro-mechanical properties of lamina are determined based on the statistical approach. According to FSDT, the transverse normal do not cease perpendicular to the mid-surface after deformation. It will be assumed that the deformation of the plates is completely determined by the displacement of its middle surface.

The theory of shallow shells can be obtained by making the following additional assumptions to thin and thick (or shear deformation) shell theories. It will be assumed that the deformation of the shells is completely determined by the displacement of its middle surface. The derivation of equations of motion is based on two assumptions. The first assumption is that the shallow shell has small deflections. The second assumption is that the shallow shell thickness is small compared to its radii of curvature. Also, the radii of curvature are very large compared to the in-plane displacement. Curvature changes caused by the tangential displacement component u and v are very small in a shallow shell, in comparison with changes caused by the normal component w.

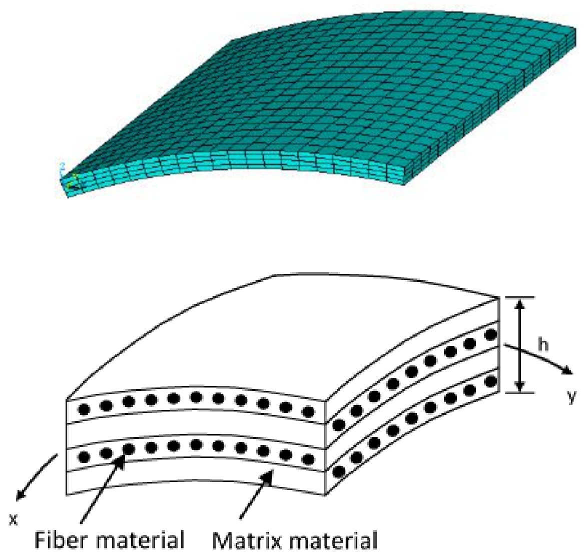

Fig. 1: Fiber and matrix materials in laminated composite shell

Using the given equation below (Eq.1) nth layer lamina plate stress-strain relationship can be defined in lamina coordinates,

$$\begin{bmatrix}\sigma_\alpha\\ \sigma_\beta\\ \sigma_z\\ \tau_{\beta z}\\ \tau_{\alpha z}\\ \tau_{\alpha\beta}\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26}\\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36}\\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0\\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{36} & 0 & 0 & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_\alpha\\ \varepsilon_\beta\\ \varepsilon_z\\ \gamma_{\beta z}\\ \gamma_{\alpha z}\\ \gamma_{\alpha\beta}\end{bmatrix} \quad (1)$$

The displacement based on plate theory can be written as

$$u(\alpha,\beta,z)=u_0(\alpha,\beta)+z\,\varphi_x(\alpha,\beta)$$

$$v(\alpha,\beta,z)=v_0(\alpha,\beta)+z\varphi_y(\alpha,\beta) \quad (2)$$

$$w(\alpha,\beta,z)=w_0(\alpha,\beta)$$

where u, v, w, φ_x and φ_y are displacements and rotations in x, y, z direction, orderly. u_o, v_o and wo are mid-plane displacements.

$$\begin{aligned}
\varepsilon_\alpha &= \frac{1}{(1+z/R_\alpha)}(\varepsilon_{0\propto}+z\kappa_\alpha)\\
\varepsilon_\beta &= \frac{1}{(1+z/R_\beta)}(\varepsilon_{0\beta}+z\kappa_\beta)\\
\varepsilon_{\alpha\beta} &= \frac{1}{(1+z/R_\alpha)}(\varepsilon_{0\propto\beta}+z\kappa_{\alpha\beta})\\
\varepsilon_{\beta\alpha} &= \frac{1}{(1+z/R_\beta)}(\varepsilon_{0\beta\alpha}+z\kappa_{\beta\alpha})\\
\gamma_{\alpha z} &= \frac{1}{(1+z/R_\alpha)}(\gamma_{0\propto z}+z(\psi_\alpha / R_\alpha))\\
\gamma_{\beta z} &= \frac{1}{(1+z/R_\beta)}(\gamma_{0\beta z}+z(\psi_\beta / R_\beta))
\end{aligned} \tag{3}$$

$$\begin{aligned}
\varepsilon_{0\alpha} &= \frac{1}{A}\frac{\partial u_0}{\partial\alpha}+\frac{v_0}{AB}\frac{\partial A}{\partial\beta}+\frac{w_0}{R_\alpha}\\
\varepsilon_{0\beta} &= \frac{1}{B}\frac{\partial v_0}{\partial\beta}+\frac{u_0}{AB}\frac{\partial B}{\partial\alpha}+\frac{w_0}{R_\beta}\\
\varepsilon_{0\alpha\beta} &= \frac{1}{A}\frac{\partial v_0}{\partial\alpha}-\frac{u_0}{AB}\frac{\partial A}{\partial\beta}+\frac{w_0}{R_{\alpha\beta}}\\
\varepsilon_{0\beta\alpha} &= \frac{1}{B}\frac{\partial u_0}{\partial\beta}-\frac{v_0}{AB}\frac{\partial B}{\partial\alpha}+\frac{w_0}{R_{\alpha\beta}}\\
\gamma_{0\alpha z} &= \frac{1}{A}\frac{\partial w_0}{\partial\alpha}-\frac{u_0}{R_\alpha}-\frac{v_0}{R_{\alpha\beta}}+\psi_\alpha\\
\gamma_{0\beta z} &= \frac{1}{B}\frac{\partial w_0}{\partial\beta}-\frac{v_0}{R_\beta}-\frac{u_0}{R_{\alpha\beta}}+\psi_\beta\\
\kappa_\alpha &= \frac{1}{A}\frac{\partial\psi_\alpha}{\partial\alpha}+\frac{\psi_\beta}{AB}\frac{\partial A}{\partial\beta}\\
\kappa_\beta &= \frac{1}{B}\frac{\partial\psi_\beta}{\partial\beta}+\frac{\psi_\alpha}{AB}\frac{\partial B}{\partial\alpha}\\
\kappa_{\alpha\beta} &= \frac{1}{A}\frac{\partial\psi_\beta}{\partial\alpha}-\frac{\psi_\alpha}{AB}\frac{\partial A}{\partial\beta}\\
\kappa_{\beta\alpha} &= \frac{1}{B}\frac{\partial\psi_\alpha}{\partial\beta}-\frac{\psi_\beta}{AB}\frac{\partial B}{\partial\alpha}
\end{aligned} \tag{4}$$

Equation of motion for plate structures can be derived by Hamilton's principle

$$\delta\int_{t_1}^{t_2}(T+W-(U+U_F)\,dt=0 \tag{5}$$

where T is the kinetic energy of the structure

$$T=\frac{\rho}{2}\int\left\{\frac{\partial u}{\partial t}\right\}^2+\left\{\frac{\partial v}{\partial t}\right\}^2+\left\{\frac{\partial w}{\partial t}\right\}^2 d\alpha d\beta dz \tag{6}$$

W is the work of the external forces

$$W=\int_x\int_y(q_\alpha u_0+q_\beta v_0+q_n w_0+m_\alpha\psi_\alpha + m_\beta\psi_\beta)\,ABd\alpha d\beta \tag{7}$$

in which q_x, q_y, q_z, m_x,m_y are the external forces and moments, respectively. U is the strain energy defined as,

$$U=\frac{1}{2}\int_V\{\sigma_\alpha\varepsilon_\alpha+\sigma_\beta\varepsilon_\beta+\sigma_z\varepsilon_z+\sigma_{\alpha\beta}\gamma_{\alpha\beta}+\sigma_{\alpha z}\gamma_{\alpha z} + \sigma_{\beta z}\gamma_{\beta z}\}dV \tag{8}$$

Solving equation 5 gives set of equations called equations of motion for plate structures. This gives equation 9 in simplified form as,

$$\begin{aligned}
&\frac{\partial}{\partial\alpha}(BN_\alpha)+\frac{\partial}{\partial\beta}(AN_{\beta\alpha})+\frac{\partial A}{\partial\beta}N_{\alpha\beta}-\frac{\partial B}{\partial\alpha}N_\beta + \frac{AB}{R_\alpha}Q_\alpha+\frac{AB}{R_{\alpha\beta}}Q_\beta+ABq_\alpha = AB(\bar{I}_1\ddot{u}^2+\bar{I}_1\ddot{\Psi}_\alpha{}^2)\\
&\frac{\partial}{\partial x}(AN_y)+\frac{\partial}{\partial x}(BN_{xy})+\frac{\partial B}{\partial x}N_{yx}-\frac{\partial A}{\partial y}N_x + \frac{AB}{R_y}Q_y+\frac{AB}{R_{xy}}Q_x+ABq_y = AB(\bar{I}_1\ddot{v}^2+\bar{I}_2\ddot{\Psi}_y{}^2)\\
&-AB\left(\frac{N_x}{R_x}+\frac{N_y}{R_y}+\frac{N_{xy+}N_{yx}}{R_{xy}}\right)+\frac{\partial}{\partial x}(BQ_x) + \frac{\partial}{\partial y}(AQ_y)+ABq_z+ = AB(\bar{I}_1\ddot{w}^2)\\
&\frac{\partial}{\partial x}(BM_x)+\frac{\partial}{\partial y}(AM_{yx})+\frac{\partial A}{\partial y}M_{xy}-\frac{\partial B}{\partial x}M_y - ABQ_x+\frac{AB}{R_x}P_x+ABm_x = AB(\bar{I}_2\ddot{u}^2+\bar{I}_3\ddot{\Psi}_x{}^2)\\
&\frac{\partial}{\partial y}(AM_y)+\frac{\partial}{\partial x}(BM_{xy})+\frac{\partial B}{\partial x}M_{yx}-\frac{\partial A}{\partial y}M_x - ABQ_y+\frac{AB}{R_y}P_y+ABm_y = AB(\bar{I}_2\ddot{v}^2+\bar{I}_3\ddot{\Psi}_y{}^2)
\end{aligned} \tag{9}$$

$$[I_1,I_2,I_3,I_4,I_5]=A_{ij}=\sum_{k=1}^{N}\int_{h_{k-1}}^{h_k}\rho^{(k)}\,[1,z,z^2,z^3,z^4]dz \tag{10}$$

$$\bar{I_i}=\left(I_i+I_{i+1}\left(\frac{1}{R_\alpha}-\frac{1}{R_\beta}\right)+\frac{I_{i+2}}{R_\alpha R_\beta}\right) \tag{11}$$

When the shell has small curvature it is referred to as a shallow shell. Shallow shells are defined as shells that have a rise of not more than 1/5th the smallest planform dimension of the shell [2]. It has been widely accepted that shallow shell equations should not be used for maximum span to minimum radius ratio of 0.5 or more. For shallow shells, Lame parameters are assumed to equal to one (A=B=1).

The Navier type solution might be implemented to thick and thin plates. This type solution assumes that the displacement section of the plates can be denoted as sine and cosine trigonometric functions.

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Assume a shell with shear diaphragm boundaries on all edges. For simply supported thick shells, boundary conditions can be arranged as follows:

$$N_\alpha = w_0 = v_0 = M_\alpha = \psi_\beta = 0 \qquad \alpha = 0, a$$
$$N_\beta = w_0 = u_0 = M_\beta = \psi_\alpha = 0 \qquad \alpha = 0, b \tag{12}$$

The displacement functions of satisfied the boundary conditions apply

$$u_0(\alpha,\beta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} U_{mn}\cos(x_m x)\sin(y_n y)\sin(\omega_{mn}t)$$
$$v_0(\alpha,\beta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} V_{mn}\sin(x_m x)\cos(y_n y)\sin(\omega_{mn}t)$$
$$w_0(\alpha,\beta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_{mn}\sin(x_m x)\sin(y_n y)\sin(\omega_{mn}t) \tag{13}$$
$$\psi_\alpha(\alpha,\beta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_{mn}\cos(x_m x)\sin(y_n y)\sin(\omega_{mn}t$$
$$\psi_\beta(\alpha,\beta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_{mn}\sin(x_m x)\cos(y_n y)\sin(\omega_{mn}t$$

where $x_m = m\pi/a$, $y_n = n\pi/b$.

Substituting the above equations into the equation of motion in matrix form,

$$\begin{bmatrix} M_{11} & 0 & 0 & M_{14} & 0 \\ 0 & M_{22} & 0 & 0 & M_{25} \\ 0 & 0 & M_{33} & 0 & 0 \\ M_{41} & 0 & 0 & M_{44} & 0 \\ 0 & M_{52} & 0 & 0 & M_{55} \end{bmatrix} \begin{bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \\ \ddot{\psi}_{\alpha mn} \\ \ddot{\psi}_{\beta mn} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \psi_{\alpha mn} \\ \psi_{\beta mn} \end{bmatrix} = \begin{bmatrix} -P_\alpha \\ -P_\beta \\ -P_n \\ m_\alpha \\ m_\beta \end{bmatrix} \tag{14}$$

Following equation can be used directly to find the natural frequencies of free vibrations. The number of terms that taken into account in the m and n cycle is one (i.e. m=1 and n=1). Following equation can be used directly to find the natural frequencies of free vibrations.

$$[K]\{\Delta\} + (\omega_{mn})^2[M]\{\Delta\} = 0 \tag{15}$$

III. NUMERICAL EXAMPLES

As an example, a simply supported cylindrical shell which has a ratio of radius of curvature (ratio of shell width/shell radius) equals from 0 to 0.1 (0, 0.025, 0.05, 0.1) in one plane and infinite radius of curvature in other plane, has been considered (Fig. 3.a). The shell, in hand, has a quadrangle planform where the ratio of plan-form dimensions varies from 1 to 4 (a/b=1, 2, 4). As a material, a laminated composite has been used as [0°/90°/0°], [0°/90°/90°/0°], [0°/90°/0°/0°/90°/0°], symmetrical cross-ply stacking sequence (Fig. 3.b). Ratio of modulus of elasticity (E_1/E_2) which is the ratio of modulus of elasticity in fiber direction to matrix direction, has been taken from 1 to 50 (1, 2, 5, 10, 15, 25, 50). Effect of shell thickness ratio that ratio of shell width to shell thickness, a/h=100, 50, 20, 10 and 5, has been examined.

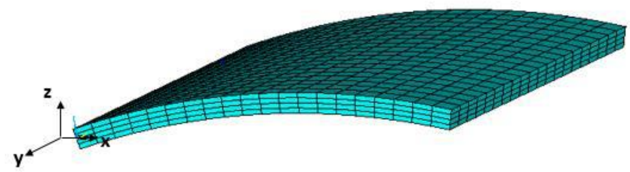

Fig. 2.a: Cylindrical shallow shell [12]

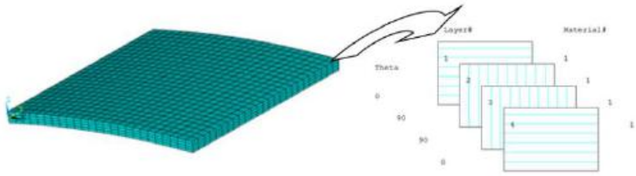

Fig. 2.b: Layered sequence for cylindrical shallow shell [12]

For each case, the shell has been solved with two theories. First theory used in the solution of composite laminated shallow shell is shear deformation shallow shell theory (SDSST). The second theory is the Finite element model (FEM). Entire structure is meshed by finite elements in this theory. Then assuming a suitable displacement fields for each meshing element, the behavior of the structure has been obtained. In this paper, finite element package programs, ANSYS [12] have been used. The structure is meshed by 25x25 elements in ANSYS model. A 8-noded quadratic element is considered as a meshing element named as SHELL99. The element has 100 layers to model the composite materials used in the structure. For each layer geometric and material properties is entered to program. Furthermore, thicknesses of each layer, fiber orientations and stacking sequence must be entered carefully. Orthotropic and lamination properties of the problem could be modeled by using this element. Regardless of the point used, programs provided geometric shape as seen in Fig. 3.

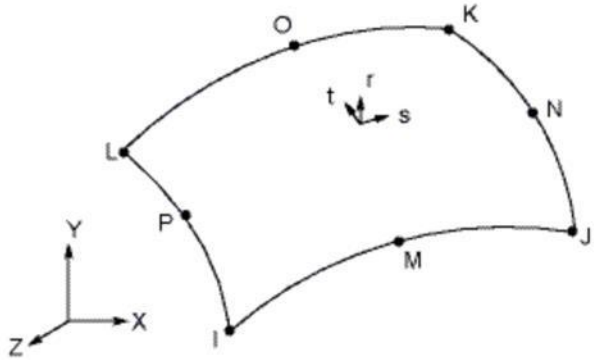

Fig. 3: 8-point quadratic elements for ANSYS

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IV. MULTIPLE LINEAR REGRESSION METHODS FOR ESTIMATION OF FREE VIBRATION

The regression equations are the formulization of the relationship between dependent and independent variables. These equations aims in this section to estimate the dependent variable which is the free vibration using the independent variables which are the laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables at the base of multiple linear regressions. Multiple linear regression models are built from a potentially large number of predictive terms according to independent variables. Each model has different number of coefficients. Coefficients of multiple linear regressions models are determined according to 1260 different Ansys models data for values of free vibration in this study. The terms produced from the variables are highly effective in increasing the accuracy of estimates. For example, the number of interaction terms increases exponentially with the number of predictor variables. If there is no theoretical basis for choosing the form of a model, and no assessment of correlations among terms, it is possible to include unnecessary terms in a model that confuse the identification of significant effects. Multiple linear regression models often take the form of something like in (Eq.16)

$$y = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + \beta_3 * x_3 + \beta_4 * x_4 + \beta_5 * x_5 + \beta_6 * x_1 * x_2 + \beta_7 * x_1 * x_3 + \beta_8 * x_1 * x_4 + \beta_9 * x_1 * x_5 + \beta_{10} * x_2 * x_3 + \beta_{11} * x_2 * x_4 + \beta_{12} * x_2 * x_5 + \beta_{13} * x_3 * x_4 + \beta_{14} * x_3 * x_5 + \beta_{15} * x_4 * x_5 + \beta_{16} * x_1^2 + \beta_{17} * x_2^2 + \beta_{18} * x_3^2 + \beta_{19} * x_4^2 + \beta_{20} * x_5^2 + \varepsilon \quad (16)$$

In this formula, x_1, x_2, x_3, x_4, and x_5 represent independent variables, y represents dependent variable, β_j (j=1,…,20) represent coefficients of regression, ε represents error. A response variable y is modeled as a combination of constant, linear, interaction, and quadratic terms formed from five predictor variables x_1, x_2, x_3, x_4, and x_5. Uncontrolled factors and experimental errors are modeled by ε. The regression estimates model coefficients (β_j) given on x_1, x_2, x_3, x_4, x_5 and y.

With the multiple linear regression approach, linear additive model, pairwise interaction model, quadratic model and pure quadratic model are formed in MATLAB program [13] by the data set. The data set consisting of 1260 results of Ansys models is used to determine the coefficients of the regressions formulas. The formula of linear additive model based on multiple linear regression is shown in (Eq.17).

$$W = -35{,}82584 - 2{,}66296 * LN + 16{,}98051 * AR + 0{,}17772 * TR + 30{,}76336 * CR + 0{,}53356 * OR \quad (17)$$

In this formula, W, LN, AR, TR, CR and OR letters stand for free vibration, laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio respectively. Pairwise interaction model based on multiple linear regression has a formula which is shown in (Eq.18).

$$W = 20{,}03485 - 5{,}31571 * LN + 1{,}51{,}42 * AR - 0{,}336 * TR + 1{,}39285 * CR - 0{,}81184 * OR + 2{,}12392 * LN * AR + 0{,}03255 * LN * TR - 1{,}16235 * LN * CR + 0{,}12116 * LN * OR + 0{,}09175 * AR * TR - 11{,}44856 * AR * CR + 0{,}21859 * AR * OR + 1{,}15932 * TR * CR + 0{,}00699 * TR * OR + 1{,}18130 * CR * OR \quad (18)$$

In this formula, W, LN, AR, TR, CR and OR letters stand for free vibration, laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio respectively. Quadratic model based on multiple linear regression has a lot of terms about five independent variables. These terms take part in (Eq.19).

$$W = 2051247 - 0{,}21684 * LN - 16{,}83334 * AR + 0{,}05440 * TR - 23{,}52538 * CR - 0{,}43582 * OR + 2{,}12392 * LN * AR + 0{,}03255 * LN * TR - 1{,}16235 * LN * CR + 0{,}12116 * LN * OR + 0{,}09175 * AR * TR - 11{,}44856 * AR * CR + 0{,}21859 * AR * OR + 1{,}15932 * TR * CR + 0{,}00699 * TR * OR + 1{,}1813 * CR * OR - 0{,}55769 * LN^2 + 3{,}56684 * AR^2 - 0{,}0037 * TR^2 + 240{,}58983 * CR^2 - 0{,}00746 * OR^2 \quad (19)$$

In this formula, W, LN, AR, TR, CR and OR letters stand for free vibration, laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio respectively. The formula of pure quadratic model based on multiple linear regression is shown in (Eq.20).

$$W = -35{,}34822 + 7{,}76183 * LN - 1{,}36325 * AR + 0{,}56813 * TR + 5{,}84513 * CR + 0{,}90957 * OR - 0{,}55769 * LN^2 + 3{,}56684 * AR^2 - 0{,}0037 * TR^2 + 240{,}58983 * CR^2 - 0{,}00746 * OR^2 \quad (20)$$

In this formula, W, LN, AR, TR, CR and OR letters stand for free vibration, laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio respectively.

After the regression analyses, the models are compared with the performance criteria which are correlation coefficient, mean percent error and mean square error used in this study. These methods on error comparison are very useful for finding the best models. The correlation coefficient is used to understand the correlation between the results of the models and the Ansys values. The mean percent error criterion is calculated to take into consideration the error ratio according to each result of Ansys which has small or big value of free vibration. Another criterion to find the best model in this study is

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the mean square error (MSE) which was used to evaluate the performances of the models.

It is seen that the quadratic model is the best model to estimate compressive strength when the multiple linear regression methods are compared (Table 1). Performance criteria for the estimation of the results of quadratic model are as follows: correlation coefficient (R) 0.916, mean square error 62,2995 mean percent error 24,23%.

Table 1: Comparison of Multiple Linear Regression Methods for Estimation of W

	R	MSE	MPE
Linear model	0,784	159,1663	40,65
İnteraction model	0,883	86,4958	27,54
Quadratic model	0,916	62,2995	24,23
Pure Quadratic model	0,817	135,4546	38,10

V. COMPARISON OF SHEAR DEFORMATION SHALLOW SHELL THEORY AND REGRESSION WITH MODELS

When is the SDSST model compared with the quadratic model based on multiple linear regression approach, on the whole data set. MSE of the SDSST is less than Regression Model (Table 2). Likewise, while the Regression model has 0.916 R value, the SDSST model has 0.989. When we look at the MPE value, the SDSST model again has a better value of 3,75% compared to 24,23% in the Regression model. Figure 5 shows the results of the Ansys and SDSST models in the graphical environment, while Figure 6 compares the results of the Ansys and Regression results. While the results of the SDSST model only show large deviations in moderate values, the regression results show large deviations in almost all values.

Table 2: Comparison of R, MSE and MPE of SDSST and Regression Model Results

	R	MSE	MPE (%)
Regression	0,91632	62,2995	24,23
SDSST	0,98845	9,2358	3,75

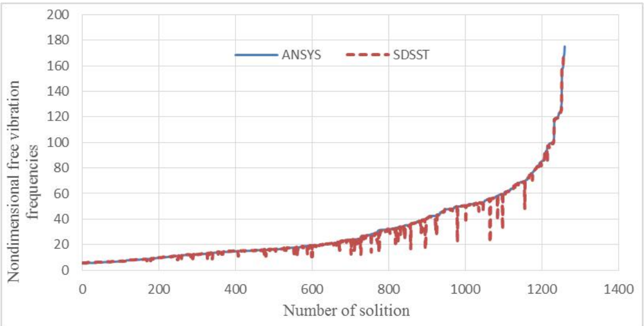

Fig. 4: Comparison of Ansys and SDSST Model Results for each of all data set

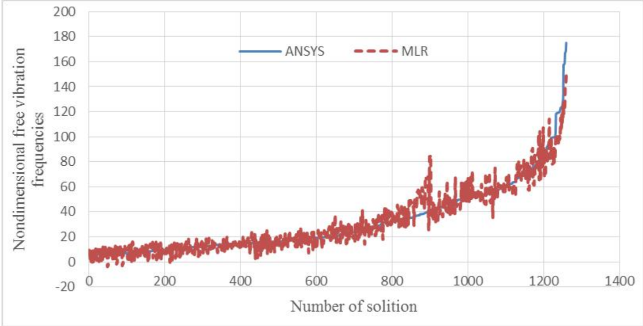

Fig. 5: Comparison of Ansys and MLR Model Results for each of all data set

VI. RESULTS OF THE MODELS

In this article, regression methods and SDSST method's abilities in estimating the free vibration with the laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables, are compared with different and similar aspects. In comparing with linear, interaction, quadratic and pure quadratic models, which are constructed with multiple linear regression approach, the quadratic model provides better results. With developing quadratic regression model, MPE declined to 24,23%, R rose to 0,916. The MSE value, which is a very important criterion for comparing model results became less than 62,2995 and positive developments are seen in the whole evaluation criteria. Similarly, from the models that are based on the SDSST theory provides the best result. With the SDSST model, the R rose to 0,989, the MPE declined to 3,7%, while the important criterion of MSE, which shows high performance in model comparisons declined to 9,24. When comparing the quadratic model which is the best among the regression based models and the SDSST, the results of the SDSST based model are superior. In estimating the free vibration, SDSST methods show better performance than the multi linear regression methods.

VII. CONCLUSION

According to outcomes, the under mentioned conclusions can be drawn:

The quadratic model based on multiple linear regression contains laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables and the quadratic model was fitted all of the data set. The quadratic model has 0,916, 62,2995, 24,23% R, MSE and MPE values, respectively.

The results of improved SDSST model compare with quadratic model, it is seen that the SDSST model has better results than the quadratic model with 0.989 R , 9,24 MSE and 3,75 % MPE values.

The fluctuations of data set of the free vibration were very well reflected using SDSST models constituted laminate

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number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables. Furthermore, the SDSST models gave better reflection than the multiple linear regression models.

It has an incentive effect for future studies to know that both of the methods, multiple linear regression with quadratic terms and SDSST, produce better results to estimate free vibration using laminate number, aspect ratio, thickness ratio, curvature ratio and orthotropic ratio variables.

The results show that the MLR method cannot achieve better results than the SDSST method. To achieve better results, different statistical methods or artificial intelligence techniques should be used. Models created using artificial intelligence techniques in particular can achieve better results than the SDSST model.

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