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Hydraulic jump on smooth and uneven bottom

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Abstract— *The mechanism of absorption of excess power of the flow within hydraulic jump has been studied in the article based on theoretical manner. Mathematic model of hydraulic jump has been investigated by taking additional water body (mass) in hydraulic jump zone as basic. Theoretical research has shown that main part of excess power is discharge for rotation of additional water mass and a formula has been obtained to make calculation thereof. The article also has provided a formula for calculating the portion of flow energy needed for overcoming friction resistance emerged in bed bottom. Because of conducted studies, formulas have been suggested for calculating hydraulic jump length occurred in flat and uneven beds. Obtained formulas have been mutually analyzed with results found by other researchers.*

Keywords— ***hydraulic jump, variable mass, motion quantity, additional discharge, critical section, jump length.***

I. INTRODUCTION

It is obvious that process of absorption of excess flow energy in tailraces of hydro-technical facilities happen by hydraulic jump. One of the main parameters when designing tailraces of hydro-technical facilities is accurate and proper calculation of length of emerged hydraulic jump. Study of energy losses in the jump zone bears great importance when analyzing flow structure. Number of studies have been devoted to the given problem
[1,2,3,4,5,6,7,8,9,10,11,14,15,16,17,18,19,20,21,22,23].
Results of these studies indicate that intensive turbulent agitation takes place in the area of hydraulic jumping, which causes penetration of large vortex structures in the form of additional discrete liquid masses from the turbulent (stormy) zone into transit (tranquil) one.

An analysis of the existing assignments on this issue suggests that most of them are devoted to a flow with a constant mass.

Currently, several empirical formulas are used in practice to determine the length of hydraulic jump [1, 5].

a) The formula of N.N.Pavlovsky:

$l=2{,}5\,(1{,}9\,h_2-h_1)$

b) The formula of M.O. Chertausov:

$$l = 10{,}3 h_1 (\sqrt{Fr} - 1)^{0{,}81}$$

c) The formula of Safrenech:

$L = 4{,}5\, h_2$

r) The formula of Bakhmetov-Matchick:

$L= 5\,(h_2-h_1)$

Great majority of these formulas have been proposed considering analysis of the results of studies conducted in various laboratories globally that carry out hydraulic investigations. Results derived from the calculation formulas sometimes vary from each other up to 50-80%. Proper design of water stilling wells constructed in tailraces of hydro-technical facilities depends on accurate calculation of hydraulic jump length. Studies regarding hydromechanical analysis of energy absorption within hydraulic jump have not been conducted in known formulas. All calculation formulas have been empirically suggested based on the results of laboratory tests performed within a given range.

Theoretical research.

Unlike existing tasks and works, we consider a hydraulic jump in which the motion of a liquid occurs with a variable mass with decreasing number of motions. With a sudden transition of the flow from a turbulent state to a calm one between sections I-I and II-II, a hydraulic jump is generated within which highly complex hydrodynamic process takes place (Fig. 1).

Both connection to the main flow (between sections I-I and K-K) and separation of the additional flow (between sections K-K and II-II) from it happens within the limits of hydraulic jump. In this case, specific water discharge in the range of I-I will be q_0, it will increase in the range of K-K and become (q_0+q_d), where q_d is the specific flow rate of the connected flow. Separation of connected discharge $-q_d$ from main stream takes place in the area between the ranges of K-K and II-II.

As a result, specific discharges of main stream in sections of I-I and II-II are the same and equal to - q_0.

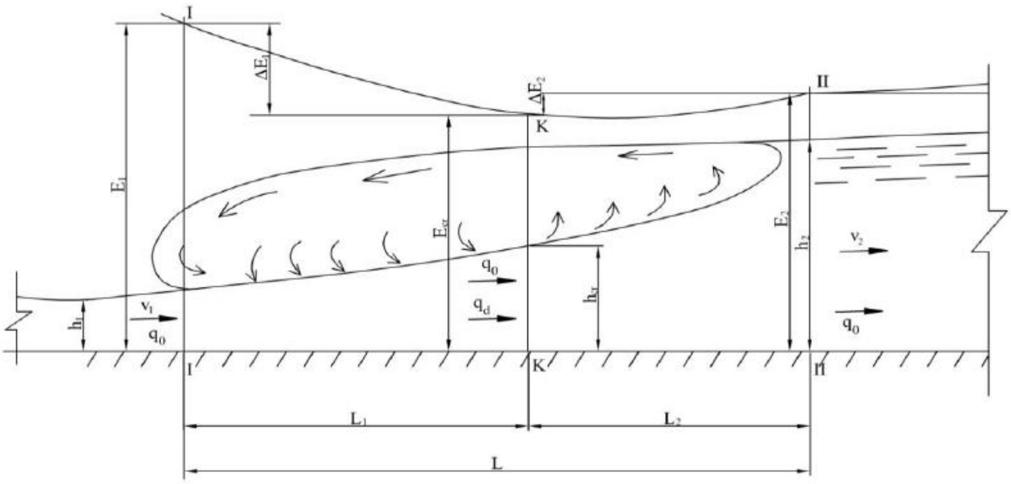

Fig.1: Calculation diagram

It has been established according to available studies that energy of the stream within the hydraulic jump is getting decreased. However, according to results of the studies we have conducted it becomes obvious that the nature of the change in energy of the stream between sections I-I and II-II differs to some extent from existing similar studies. When processing the data of numerous studies it is confirmed that specific energy of the stream decreases to a minimum - E_{cr} in section K-K along the length of the hydraulic jump, and then, it increases somewhat due to restoration of the pressure and equals to E_2 in section II-II.

We reviewed G.A. Petrov equation in the following form □10,11,12,13,14□, to study pressure loss in hydraulic jump:

$$d\left(\frac{\alpha_0 \upsilon^2}{2g}\right) + \frac{dP}{\gamma} + dz + i_f dx + \frac{\alpha_0(\upsilon - \theta)}{g} \cdot \frac{\upsilon dQ}{Q} = 0 \quad (1)$$

where: Q - is discharge along the flow; □-average flow velocity along the flow; P-hydrodynamic pressure; i_f - hydraulic slopeof the friction; □-projection of velocities of attached particles on the direction of main stream velocity.

Accepting bed bottom as horizontal and integrating equation (1) between the sections I-I and II-II along the flow length and after not complicated transformation, we get:

$$\frac{\alpha_1 \upsilon_1^2}{2g} + h_1 = \frac{\alpha_2 \upsilon_2^2}{2g} + h_2 + \int_1^2 i_f dx + \int_1^2 \frac{\alpha_0(\upsilon - \theta)}{g} \cdot \frac{\upsilon}{Q} \cdot dQ \quad (2)$$

Supposing that energy loss during the jump is equal to difference in energy of E_1 and E_2 in sections I-I and II-II, instead of (2) we find out:

$$E_1 - E_2 = \left(\frac{\alpha_1 \upsilon_1^2}{2g} + h_1\right) - \left(\frac{\alpha_2 \upsilon_2^2}{2g} + h_2\right) = \int_1^2 i_f dx + \frac{\alpha_0}{g}\int_1^2 \frac{\upsilon \cdot (\upsilon - \theta)}{Q} dQ \quad (3)$$

Equation (3) reminds the Bernoulli equation, but with new term on the right side. During flow movement with variable discharge along the path, we determine the pressure loss on the friction against the bottom and the side walls of the channel bed by the formula of the same kind as with constant discharge, i.e.:

$$h_f = \int_1^2 i_f dx = \frac{\upsilon_{av}^2}{C_{av}^2 R_{av}} \quad (4)$$

where C_{med}- is average value of Chezy coefficient between the sections; R_{med}- average value of hydraulic radius between the sections.

Another integral in (3) expresses mainly the pressure loss caused by the variability of the flow discharge:

$$h_{mix.} = \frac{\alpha_0}{g}\int_1^2 \frac{\upsilon \cdot (\upsilon - \theta)}{Q} dQ \quad (5)$$

where: □-means the ratio of the projection of the velocity of the attached flow to the main one.

We will assume in future studies that □1=□2=1 and □=0 (meeting of two streams happens at an angle of 90 °).

Taking into account assumptions and considering that b = 1.0 m, $dQ = q_x dx$, we determine the value of the pressure loss by formula (5) as follows:

$$h_{mix} = \frac{\alpha}{g}\int_1^2 \frac{\upsilon \cdot (\upsilon - \theta)}{Q} dQ = \frac{1}{g}\int_1^2 \frac{\upsilon^2 q_x}{bh\upsilon} dx = \frac{1}{g}\int_1^2 \frac{\tilde{\upsilon}_x q_x}{\tilde{h}_x} dx \quad (6)$$

$\tilde{\upsilon}_x$ and $\tilde{h}_x$ in expression (6) are variable values and depend on the length of the jump.

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We determine the value of q_x, for running value of separable or connected value from the expression.

$$q_x = \frac{q_d}{l_x} \tag{7}$$

Where: l_x-is the length of connection and separation section; q_d – is additional discharge.

By integrating he dependence (6) within the boundaries of I-I and K-K sections, we determine pressure loss for mixing additional discharge with main discharge:

$$\Delta h_{mix} = \frac{1}{g}\int_1^K \frac{\widetilde{\upsilon}_{x_1}}{\widetilde{h}_{x_1}} \cdot \frac{q_d}{l_1} dx \tag{8}$$

We apply change of υ_x and h_x in these sections according to straightforward principle:

$$\widetilde{\upsilon}_{x_1} = \upsilon_1 - (\upsilon_1 - \upsilon_{cr})\frac{x}{l_1}, \tag{9}$$

$$\widetilde{h}_{x_1} = h_1 + (h_{cr} - h_1)\frac{x}{l_1} \tag{10}$$

Considering (9) and (10), equation (8) obtains the following form:

$$\Delta h_{mix} = \int_0^{l_1} \frac{\upsilon_1 - (\upsilon_1 - \upsilon_{cr})\frac{x}{l_1}}{h_1 + (h_{cr} - h_1)\frac{x}{l_1}} \cdot \left(\frac{q_d}{g l_1}\right) dx \tag{11}$$

For hydraulic jump area between I-I and K-K sections with water depth h_1 and h_{cr}, expression (11) can be presented in the following form after minor transformations:

$$dh_{mix} = \frac{\alpha q_d}{gl}\int_0^l \frac{\upsilon_1 - \frac{\upsilon_1 - \upsilon_{cr}}{l}x}{h_1 + \frac{h_{cr} - h_1}{l}x} \cdot dx \tag{12}$$

By integrating and transforming expression (12), we find out dependence of pressure loss on the mixing of the additional discharge with the primary one during the jump in the form below:

$$\Delta h_{mix} = \frac{\alpha q_d}{g(h_{cr} - h_1)}\left[\frac{(\upsilon_1 h_{cr} - \upsilon_{cr} h_1)}{h_{cr} - h_1}\ln\frac{h_{cr}}{h_1} + (\upsilon_{cr} - \upsilon_1)\right] \tag{13}$$

By integrating the dependence (8) within the limits of K-K – II-II sections, we determine pressure loss on separation of the additional discharge from the primary one:

$$\Delta h_{sec} = \frac{1}{g}\int_K^2 \frac{\widetilde{\upsilon}_{x_2}}{\widetilde{h}_{x_2}} \cdot \frac{q_d}{l_{x_2}} dx, \tag{14}$$

We accept the change of υ_x and $\widetilde{h}_x$ according to straightforward principle in the following manner:

$$\widetilde{\upsilon}_{x_2} = \upsilon_{cr} - (\upsilon_{cr} - \upsilon_2)\frac{x}{l_2} \tag{15}$$

$$\widetilde{h}_{x_2} = h_{cr} + (h_2 - h_{cr})\frac{x}{l_2}. \tag{16}$$

Considering (15) and (16), equation (14) obtains the following form:

$$\Delta h_{sec} = \int_0^{l_2} \frac{\upsilon_{cr} - (\upsilon_{cr} - \upsilon_2)\frac{x}{l_2}}{h_{cr} + (h_2 - h_{cr})\frac{x}{l_2}} \cdot \left(\frac{q_d}{g l_2}\right) dx, \tag{17}$$

Expanding the integral in expression (17) and conducting certain transformations, we get the formula for pressure loss on separation of additional discharge during the jump:

$$\Delta h_{sec} = \frac{\alpha q_d}{g(h_2 - h_{cr})}\left[\frac{(\upsilon_{cr} h_2 - \upsilon_2 h_{cr})}{h_2 - h_{cr}}\ln\frac{h_2}{h_{cr}} + (\upsilon_2 - \upsilon_{cr})\right]. \tag{18}$$

Thus, two expressions (13) and (18) were obtained for determining pressure loss during hydraulic jump on connection and separation of additional discharge.

It should be mentioned that, according to the adopted scheme, the energy of the stream gets decreased before the critical section. Arriving at minimum value in the critical section, and further due to separation of additional discharge, the flow partially recovers its energy. This condition indicates that pressure restoration happens in the section K-K – II-II. Pressure restoration value is determined by the expression (18).

The values q_d. and h_{cr}. can be determined from the following expression [13] with known magnitudes of hydraulic parameters of the jump:

$$h_{cr} = \sqrt{\frac{1}{3}\left[h_2(h_1 + h_2) + h_1^2\right]}, \tag{19}$$

$$q_d = \sqrt{g}\left[\sqrt{\frac{h_{cr}^3}{\alpha_0}} - \sqrt{\frac{h_1 h_2}{2\alpha_0}(h_1 + h_2)}\right]. \tag{20}$$

Being aware of parameters on hydraulic jump elements, it is possible to determine pressure loss due to the connection and separation of the additional discharge along the length of the jump according to formulas (13) and (18) with great accuracy.

Proposed dependencies (13) and (18) enables for determining energy loss in hydraulic jumping. These energy losses are formed under the influence of a surface roller. On the other hand, part of the flow energy is spent

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on overcoming the resistance of the bed bottom. Specific energies E_1, E_2 and $E_{c.}$ in sections I-I, II-II and K-K of the hydraulic jump are identified by using hydraulic flow parameters. At the same time, energy losses between the sections I-I and K-K constitute $\Delta E_1=E_1-E_{cr}$, and between the sections of K-K and II-II they become $\Delta E_2=E_{cr}-E_2$. In all cases, terms of E_{cr}□E_2, and at the same time ΔE_2□0 are satisfied. Hence it is obvious that during flow movement within the boundaries between the sections K-K and II-II, the specific flow energy increases additionally from the minimum (section K-K) to E_2 (section II-II) by an amount of ΔE_2. While calculating for (13) and (18) of values Δh_1 and Δh_2 you can determine energy loss to overcome resistance along a segment of length L_1 from the expression $\Delta H_1=\Delta E_2-\Delta h_1$ and along the length of section L_2 from the expression $\Delta H_2=\Delta h_2+\Delta E_2$. It becomes obvious from the presented material that, in hydraulic jump, energy loss necessary for overcoming bottom resistance of the bed will be equal to $\Delta H_1+\Delta H_2=\Delta E$.

It should be mentioned that in order to determine hydraulic parameters of the stream and to find the magnitude of value of the pressure loss in the hydraulic jump according to (13), (18) and other expressions we recommended, the data of laboratory studies by D.A. Akhutin [2] were processed, the results of which are presented in table 1. It is obvious from this table that experiments were carried out at the values of the conjugate depths h_1=1,30…7,35 cm and h_2=10,60…32,80 cm, and the length of the hydraulic jump fluctuated within the range of 35 ... 185 cm.

Pressure losses in the hydraulic jump calculated by (13) and (18) are respectively Δh_1=1,01…36,0 cm and Δh_2=0,85…3,87 cm. The total pressure loss along the length of the hydraulic jump varied from 0.15 to 11.81 cm. Furthermore, according to the data in the table, it is obvious that for all the experiments ΔE_2 is negative and the conditions of $\Delta E_1>\Delta h_1$ and ΔE_2□Δh_2 are satisfied. This statement fully confirms the recommendations we made on the assessment of hydraulic jumps.

To determine the magnitude of pressure loss for overcoming the bottom resistance of the channel, bypassing formulas (13) and (18), after processing numerous data, a dependence was obtained in the form:

$$\Delta E = E_1\left[0{,}78 - 0{,}89\left(\frac{h_1}{h_2}\right)^{0{,}15}\right], \qquad (21)$$

where h_1, h_2 – are first and second conjugate depths in hydraulic jumping; E_1 – is specific flow energy within initial section of hydraulic jump.

Elements of hydraulic jump and energy loss in it

Table 1.

Water depth in section I-I, h_1, sm	Water depth in section II-II, h_2, sm	Length of hydraulic jump, L_j, sm	Froude number	Flow velocity in section I-I, υ_1, sm/s	Flow velocity in section II-II, υ_2, sm/s	Flow energy in section I-I, E_1, sm	Flow energy in section II-II, E_2, sm	Critical depth, $h_{cr.}$, sm	Additional water discharge, $q_{ad.}$, sm²/s	Critical velocity, $\upsilon_{cr.}$, sm/s	Critical flow energy, $E_{cr.}$, sm	$\Delta E_1=E_1-E_{cr.}$	$\Delta E_2=E_{cr.}-E_2$	Δh_1 for (4.27), sm	Δh_2 for (4.32), m	$\Delta H_1=\Delta E_1-\Delta h_1$, sm	$\Delta H_2=\Delta E_2+\Delta h_2$, sm	$\Delta E=\Delta H_1+\Delta H_2$, sm
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1,3	10,9	55	39,3	224	26,7	26,9	11,26	6,7	251,99	81,07	10,05	16,82	-1,21	13,36	1,3	3,47	0,08	3,55
1,3	16,0	82	81,9	323,2	26,3	54,5	16,35	9,63	516,58	97,22	14,45	40,08	-1,90	31,08	2,07	9,0	0,17	9,17
1,9	8,80	35	13,0	155,9	33,7	14,3	9,38	5,71	130,99	74,84	8,56	5,73	-0,81	4,73	0,85	1,0	0,03	1,03
1,9	14,7	72	33,8	251,0	32,4	34,0	15,24	9,09	380,84	94,41	13,63	20,38	-1,61	16,26	1,71	4,12	0,10	4,22

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1,9	21,0	107	66,6	352,3	31,9	65,2	21,52	12,71	749,50	111,66	19,06	46,11	-2,46	36,00	2,67	10,1	0,21	10,31
2,52	6,20	15	4,30	102,6	41,7	7,9	7,090	4,49	39,240	66,35	6,730	1,15	-0,35	1,01	0,36	0,14	0,00	0,15
2,52	13,7	63	17,5	208,0	38,3	24,6	14,45	8,73	283,61	92,54	13,09	11,47	-1,35	9,37	1,42	2,11	0,06	2,17
2,52	19,1	104	32,5	283,5	37,4	43,5	19,81	11,82	558,72	107,69	17,73	25,75	-2,08	20,57	2,21	5,18	0,13	5,31
2,52	24,4	127	51,7	357,6	36,9	67,7	25,10	14,87	894,61	120,77	22,30	45,38	-2,79	35,72	3,01	9,66	0,22	9,88
3,11	12,6	48	10,2	176,7	43,6	19,0	13,57	8,32	202,02	90,34	12,48	6,54	-1,09	5,46	1,13	1,08	0,03	1,11
3,08	18,5	90	21,0	252,1	42,0	35,5	19,4	11,7	472,40	107,0	17,5	18,0	-1,89	14,58	1,99	3,40	0,10	3,50
3,06	23,0	120	32,0	310,0	41,2	52,0	23,9	14,2	735,40	118,2	21,4	30,7	-2,50	24,51	2,66	6,15	0,16	6,31
3,07	29,5	160	51,0	391,8	40,8	81,3	30,3	18,0	1185,6	132,8	27,0	54,3	-3,37	42,79	3,63	11,6	0,26	11,86
3,72	16,0	65	11,4	204,0	47,4	24,9	17,1	10,5	303,50	101,4	15,7	9,2	-1,43	7,65	1,48	1,56	0,05	1,61
3,68	23,1	123	22,8	287,1	45,7	45,7	24,2	14,5	675,6	119,3	21,8	23,9	-2,39	19,35	2,52	4,58	0,13	4,71
3,70	26,7	140	29,6	328,0	45,5	58,5	27,8	16,6	902,1	127,6	24,9	33,7	-2,87	26,98	3,05	6,69	0,18	6,86
3,72	32,2	170	41,8	390,5	45,1	81,5	33,2	19,8	1296,8	139,2	29,6	51,8	-3,61	41,06	3,87	10,76	0,26	11,02
4,42	15,4	48	7,8	184,0	52,8	21,7	16,8	10,4	237,7	101,0	15,6	6,1	-1,22	5,14	1,25	0,93	0,03	0,96
4,40	20,1	95	12,7	234,3	51,3	32,4	21,4	13,1	447,6	113,2	19,6	12,8	-1,85	10,58	1,92	2,21	0,07	2,28
4,40	24,2	128	17,9	277,8	50,5	43,7	25,5	15,4	670,7	122,9	23,1	20,6	-2,40	16,83	2,51	3,80	0,11	3,91
4,42	30,5	160	27,3	343,8	49,8	64,7	31,8	19,0	1077,3	136,6	28,5	36,1	-3,24	29,04	3,43	7,10	0,19	7,29
4,40	33,7	185	33,2	378,3	49,4	77,4	34,9	20,8	1315,8	143,0	31,3	46,1	-3,68	36,80	3,92	9,29	0,24	9,53
4,93	12,8	30	4,7	150,3	57,9	16,4	14,5	9,2	126,3	94,8	13,7	2,7	-0,78	2,36	0,79	0,35	0,01	0,36
4,93	18,0	70	8,5	202,6	55,5	25,9	19,6	12,1	314,3	108,8	18,1	7,8	-1,47	6,54	1,51	1,22	0,04	1,26
4,94	23,2	110	13,4	254,6	54,2	38,0	24,7	15,0	566,4	121,4	22,5	15,4	-2,16	12,74	2,24	2,70	0,08	2,78
4,97	27,4	135	18,0	295,9	53,7	49,6	28,9	17,4	809,2	130,8	26,1	23,4	-2,72	19,12	2,85	4,32	0,13	4,45

4,97	31,6	160	23,4	337,7	53,1	63,1	33,0	19,8	1088,5	139,5	29,8	33,3	-3,29	26,94	3,46	6,41	0,18	6,59
5,62	16,5	65	5,8	178,5	60,8	21,9	18,4	11,5	218,0	106,2	17,2	4,6	-1,14	3,97	1,16	0,64	0,02	0,66
5,67	21,5	100	9,1	224,8	59,3	31,4	23,3	14,3	425,0	118,6	21,5	9,9	-1,79	8,34	1,84	1,59	0,05	1,64
5,65	25,8	130	12,7	265,4	58,1	41,6	27,5	16,8	650,7	128,2	25,1	16,4	-2,37	13,57	2,46	2,84	0,09	2,93
5,65	31,2	190	18,0	315,9	57,2	56,5	32,9	19,8	984,2	139,5	29,8	26,8	-3,10	21,82	3,24	4,93	0,15	5,08
6,27	15,0	35	4,1	158,0	66,0	19,0	17,2	10,9	141,1	103,5	16,4	2,6	-0,83	2,29	0,84	0,31	0,01	0,32
6,27	19,8	80	6,6	201,0	63,6	26,9	21,9	13,6	312,2	115,5	20,4	6,4	-1,45	5,50	1,48	0,94	0,03	0,97
6,27	23,9	115	9,2	237,5	62,3	35,0	25,9	15,9	500,4	125,0	23,9	11,1	-2,00	9,35	2,06	1,79	0,06	1,85
6,27	28,9	150	12,9	282,0	61,2	46,8	30,8	18,8	776,8	135,7	28,1	18,7	-2,67	15,42	2,77	3,24	0,10	3,34
6,25	32,6	185	16,2	315,3	60,4	56,9	34,5	20,9	1013,9	143,1	31,3	25,6	-3,17	20,98	3,31	4,64	0,14	4,78
7,35	16,5	30	3,6	162,1	72,2	20,7	19,2	12,2	145,9	109,5	18,3	2,4	-0,83	2,14	0,84	0,27	0,01	0,28
7,25	21,5	70	5,9	204,5	69,0	28,6	23,9	15,0	328,2	121,1	22,4	6,1	-1,50	5,28	1,52	0,86	0,03	0,89

Hence, energy loss in in hydraulic jumps will be:

$$E_1 - E_2 = \Delta h_f + \Delta h_{mix} + \Delta h_{\sec}, \qquad (22)$$

It is obvious from the equation (22) that for hydraulic jumps, following conditions must be necessarily fulfilled:

$$\Delta E = E_1 - E_2 - (\Delta h_{mix} - \Delta h_{\sec}). \qquad (23)$$

We use Darcy-Weisbach formula for critical section form to determine length of hydraulic jump:

$$\Delta E = (\lambda_p + \lambda_f)\frac{L_{jmp}}{h_{cr}} \cdot \frac{\upsilon_{cr}^2}{2g}, \qquad (24)$$

where: ΔE- is the energy for overcoming frictional resistance;

λ_p - coefficient of hydraulic friction from the slopeof the gradient of pressure [9];

λ_f- coefficient of hydraulic friction from the slopeof the friction [2,9,15] ;

$L_{jmp.}$- length of jump; $\upsilon_{cr.}$- critical flow velocity; $h_{cr.}$- critical flow depth.

We define the following from (24) for length of hydraulic jump:

$$L_{jmp} = \frac{2g}{\upsilon_{cr}^2} \cdot \frac{\Delta E h_{cr}}{\lambda_p + \lambda_f} \qquad (25)$$

For determining λ_p and λ_f we processed data of laboratory studies by several authors [2,3,6,8,9,14,15,16,17,20,21,22,23]. Based on results of these studies an expression was found for determining λ_p in the following form:

$$\lambda_p = 0{,}0021 Fr^{0{,}84} \qquad (26)$$

In this case, the value of λ_f is determined both for smooth and uneven bed separately. Coefficient of hydraulic friction λ_f for smooth bed is determined by the formula:

$$\lambda_{f.smo.} = 0{,}08 \cdot \lambda_0^{0{,}42} \left(\frac{h_{cr}}{h_1}\right)^{1{,}6} \qquad (27)$$

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λ_0 is determined by formula [15] in dependency (27).

$$\lambda_0 = \frac{0{,}035}{\mathrm{Re}^{0,25}},$$

where: Re- is Reynolds number and determined by the formula:

$$\mathrm{Re} = \frac{\upsilon_1 \cdot h_1}{\nu} \quad (28)$$

For uneven bed, hydraulic friction coefficient is determined from the expression:

$$\lambda_{f(fric.)} = 0{,}033\lambda_f^{0,34}\left(\frac{h_{cr}}{h_1}\right)^{1,6} \quad (29)$$

In dependency (29) the magnitude of λ_f value is determined from (30).

$$\lambda_f = \frac{1}{\left(3{,}9\lg\frac{h}{k} + 4\right)^2} \quad (30)$$

where: K- is determined by A.P.Zegjda methodology [1,5,15].

It should be mentioned that comparison of these dependencies was carried out with the data of experimental studies by M.D. Chertausov (table 2). By using presented data, the length of the hydraulic jump is calculated both with the dependences we proposed, and according to the recommendations of N.N. Pavlovsky and M.D. Chertausov, which are given in table 2. Comparison of the obtained dependences with the results of M.D.Chertausov's experimental data indicated their satisfactory convergence [16]. At that time discrepancy between the experimental and calculated lengths of the hydraulic jump changed: ±0,4…22% according to the dependencies recommended by us, ±1,0…36,0% by N.N.Pavlovsky formula, ±2,1…29% according to M.D.Chertausov expression.

Results of the comparison of length of the hydraulic jump according to the experimental data of M.D. Chertausov

Table 2.

h_1-first conjugate depth, cm	h_2-second conjugate depth, cm	q_0-specific discharge, cm^2/c on 1 p.sm	Experimental length of hydraulic jump, L_0, cm	Jump length according to the author, L_a, cm	*Difference between L_0 and* L_a, %	Jump length according to N.N.Pavlovsky, L_n, cm	*Difference between L_0 and* L_n, %	Jump length according to M.D.Chertausov, L_r, cm	*Difference between L_0 and* L_r, %
1	2	3	4	5	6	7	8	9	10
0,24	2,92	33,0	11,5	11,6	-0,4	13	-15	13	-16,9
3,75	33,8	1527,9	180,0	178,9	0,6	151	16	161	10,4
2,20	15,5	544,1	70,0	74,7	-6,7	68	3	77	-10,1
1,95	15,02	493,8	60,0	73,2	-22,0	66	-11	73	-21,0
1,96	14,93	492,4	67,0	72,6	-8,2	66	1	73	-9,0
2,25	17,64	622,3	87,0	87,7	-0,9	78	10	85	2,1
1,94	15,85	518,0	70,0	78,1	-11,5	70	-1	76	-8,9
0,34	2,980	40,6	13,0	12,2	6,1	13	-2	14	-10,1
1,01	9,22	216,2	41,5	42,6	-2,6	41	1	44	-6,5
0,24	2,54	28,8	10,5	10,1	3,8	11	-9	12	-13,1
0,22	2,67	28,9	11,5	10,5	9,1	12	-6	12	-7,0
0,24	2,92	33,0	11,5	11,6	-0,4	13	-15	13	-16,9

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0,17	2,18	20,7	10,0	8,3	17,1	10	1	10	0,8
0,35	4,5	61,2	19,0	18,4	3,0	21	-8	21	-9,0
0,21	2,72	28,7	12,0	10,5	12,2	12	-3	12	-3,9
0,25	3,55	40,7	14,0	13,8	1,1	16	-16	16	-15,9
0,21	3,11	32,6	12,0	11,9	1,1	14	-19	14	-17,3
0,23	3,79	41,5	17,0	14,6	14,4	17	-3	17	2,0
0,18	3,34	32,2	13,0	12,1	7,1	15	-19	15	-14,4
0,11	2,45	18,4	10,0	8,3	17,0	11	-14	10	-2,9
0,19	4,16	41,1	19,0	14,9	21,6	19	-2	18	6,4
0,18	4,10	39,40	14,0	14,2	-1,3	19	-36	18	-29,0
0,10	2,63	18,80	9,0	8,5	5,8	12	-36	11	-22,0
3,35	13,64	617,1	61,5	54,9	10,8	56	8	65	-5,6
3,23	13,88	613,4	59,0	56,9	3,5	58	2	69	-16,6
3,29	14,42	642,0	62,0	60,3	2,7	60	3	71	-14,5
3,14	15,27	658,0	79,5	68,7	13,5	65	19	73	7,9
3,03	14,60	618,5	65,0	64,2	1,3	62	5	72	-10,9
2,68	15,5	608,6	65,0	72,4	-11,4	67	-3	78	-19,4
2,68	15,89	622,8	78,0	75,1	3,7	69	12	79	-1,3
2,16	14,35	501,0	67,0	69,4	-3,6	63	6	69	-3,5
2,50	16,62	624,2	98,5	81,3	17,5	73	26	81	17,9
0,40	2,750	41,20	14,0	11,2	20,2	12	14	13	5,3
2,10	14,51	498,3	66,0	70,4	-6,6	64	4	70	-6,6
0,51	3,68	62,1	16,5	15,5	5,9	16	2	18	-7,0

Extensive research activities have been carried out under the leadership of academician M. Vyzgo regarding the impact of bed roughness on hydraulic jump length [4]. The length of hydraulic jump has been studied under laboratory conditions within same hydraulic parameters in smooth and uneven beds. Results of carried studies are summarized and presented in Figure 2. As it is obvious from this graph, obtained results are subject to parabolic functioning appropriateness.

The results of the calculation done by formulas (25), (29), (29) and (30) that we obtained through theoretical method for the length of hydraulic jump occurring in uneven beds have been compared to research results by academician M. Vyzgo (Figure 2).

When comparing the parameters of the hydraulic jump according to the recommendations developed by us for the uneven bottom, the data of M.S. Vyzgo and Y.A. Kuzminova were used [4], results of which are presented in Figure 2. According to data of this figure, hydraulic jump parameters determined according to our recommendations, as well as by M.S. Vyzgo expression $L_{.frik.}=L_0$ almost coincide, which is confirmed by graphics

$$\frac{L_{frik}}{L_0} = f\left(\frac{\Delta}{h_2}\right).$$

Furthermore, according to recommendations made by us, a schedule of changes was developed $\frac{L}{(h_2 - h_1)} = f\left(\frac{\upsilon_1}{\sqrt{gh_1}}\right)$ for hydraulic jump and (figure 3) where it where it was given together with similar graphs of US Bureau of Reclamation, N.N. Pavlovsky and M.D. Chertausov [14]. The analysis of materials of figure 3 indicate that schedule of change for length of hydraulic jump developed according to the methodology suggested by us ic satisfactorily compliant with the graph of US Bureau of Reclamation [18,19] and well confirmed by experimental data of A.N.Akhutin and

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A.N.Rakhmanov. When determining the length of hydraulic jump according to the schedules of the US Bureau of Reclamation, the discrepancy does not exceed 9,5…15,6%. However, in case of $\frac{\upsilon_1}{\sqrt{gh_1}} < 4$ these graphs significantly differ from the graphs of N.N. Pavlovsky and M.D. Chertausov. Significant deviations of all these graphs descend from the experimental data of Pistrovsky and Bradley. It should be noted that at a value of $\frac{\upsilon_1}{\sqrt{gh_1}} > 4$ the convergence of the lengths of the hydraulic jump determined by different, including dependencies recommended by us are satisfactory.

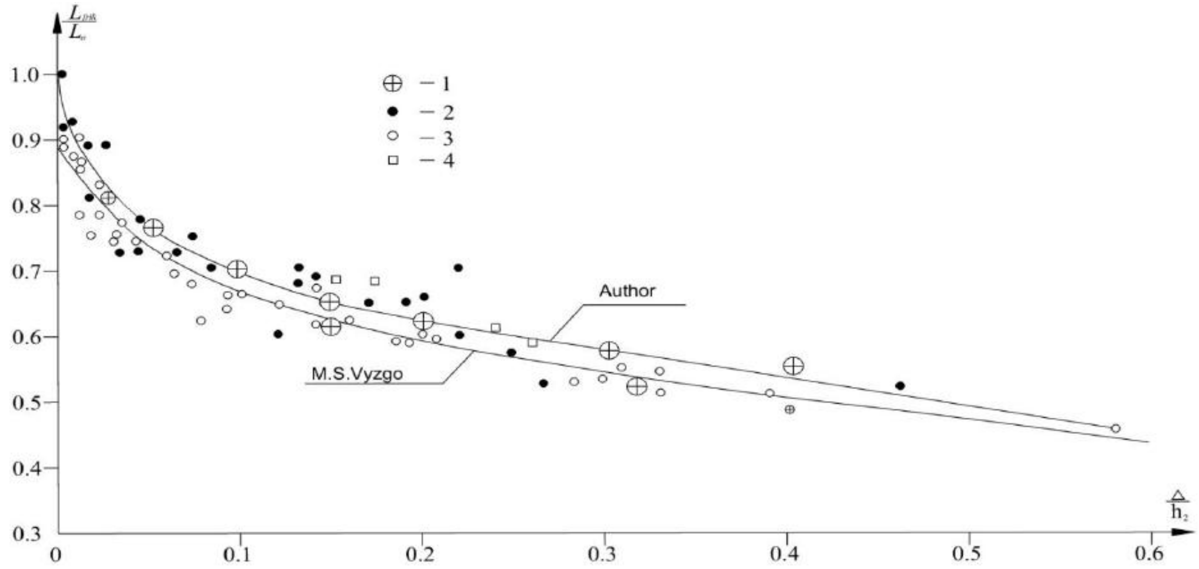

Fig.2: Diagram $\frac{L_{frik}}{L_0} = f\left(\frac{\Delta}{h_2}\right)$.

1- K.A. Suleymanova; 2; 3; 4 – Y.M.Kuzminova

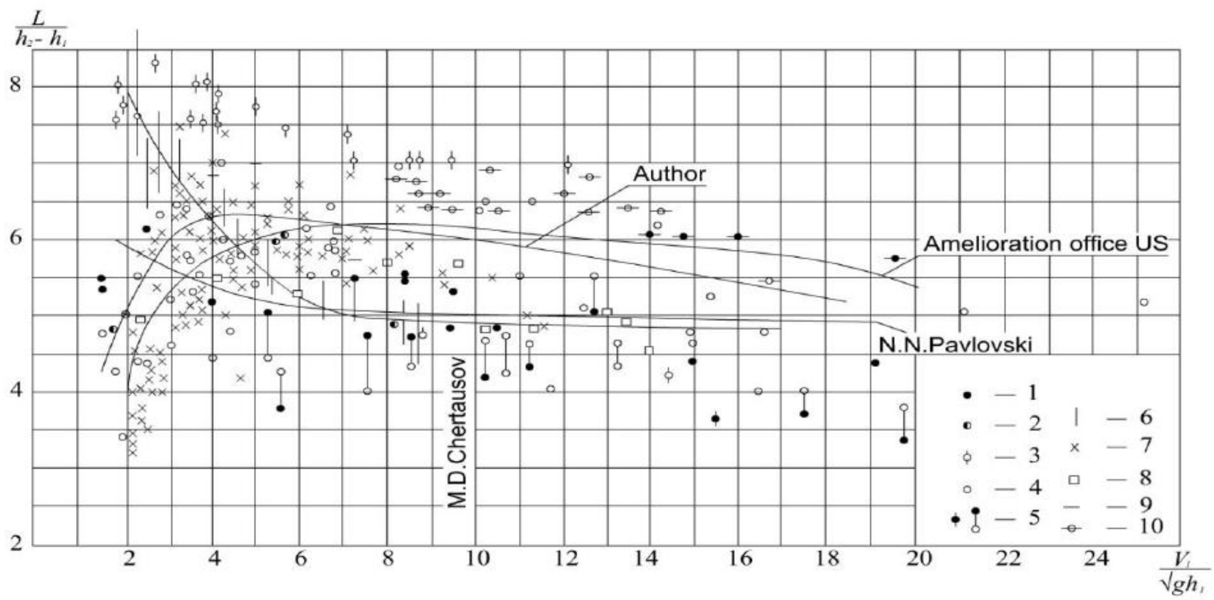

Fig.3: Jump length: 1- according to experiments of Safranch; 2 - according to experiments of Einwachter; 3 - according to experiments of Voychiska; 4 - according to experiments of Smetana; 5 - according to experiments of Pistrovsky; 6 - according to experiments of Bakhmetyev and Matchsko; 7 - according to experiments of Akhutin; 8 - according to experiments of Rakhmanov; 9 - according to experiments of Mur; 10 - according to experiments of Bradley and Peterky.

Main outcomes

1. The mechanism of flow energy absorption within hydraulic jump zone has been explained by the model for the rotation of q_d - additional water body on the mainstream and overcoming the resistance emerged in bed bottom against the flow. According to this selected mathematical model, the maximum flow energy - E_1 within I-I section in the hydraulic jump zone falls to minimum - $E_{cr.}$ value while decreasing to the critical depth located in K-K cross section and then increases to - E_2 value within section of II-II.
2. Within the hydraulic jump zone, q_d - additional water body is connected to mainstream between the section of I-I and K-K and the process of energy loss occurs, the formula (13) is developed to calculate the same. Between the cross sections K-K and II-II the process of separation of q_d - additional water body from the mainstream happens; the formula f to omputed. Among the K-K and II-II fragments, the main stream is the qd - separation of the water mass, which is composed of the formula (18) has been developed to calculate the additional energy generated at that time.
3. A proposal has been made for using the formula (19) to calculate the critical depth in the hydraulic jump zone and using the formula (20) to calculate the discharge of q_d - additional water body.
4. Formulas (21) and (23) have been developed to calculate the energy used by the flow bed for overcoming the bed bottomresistance.
5. It is recommended to use formulas of (25), (26), (27), (28), (29) and (30) obtained by theoretical method for calculating the length of hydraulic jump formed in smooth and rough beds.

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