https://dx.doi.org/10.22161/ijaers.5.5.28 ISSN: 2349-6495(P) | 2456-1908(O)

A Novel Constraint Narrowing Technique for MIMO Unstable System

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***Abstract*— *Frequency response data collection can be a boon for modeling of MIMO uncertain plant. System stability can be assessed either by transfer function or by state-space method. Both will arrive at matrix transformation and further decision approach. Both can be considered for diagonalization of matrix. It is a proven fact that when the matrix is diagonalized the elements of the principle diagonal are the Eigen values and these Eigen values are closed loop poles from which stability can be assessed. The feature of such a diagonal matrix is that its principle diagonal elements contain gains of all the feedback paths. Singular value decomposition is used here for diagonalization. Singular value decomposition technique has been demonstrated by many authors but, application of PCA with Euclidian norm has not been paid attention so far. The systems numerical array is fed to a digital processing tool such as Mat lab and SVD-PCA (Singular Value Decomposition- Principal Component Analysis) is applied to determine the reduction of disturbance or noise and to provide minimum sensitivity and error correction. There are Hull, Box and KB consistency narrowing techniques used previously and the idea is extended further and an SVD-PCA-Norm technique which is now referred as LA criteria has been demonstrated here.*

Keywords*— *Constraint Narrowing, Degree of Freedom, Hull consistency, ICST, MIMO, Pre-filter, QFT.

I. INTRODUCTION

Good performance of control system is the result of combination of feed-forward and feedback control systems. Stability is the constraint applicable to feedback control due to the uncertainty in tracking and measured noise filtering, whereas sensor availability and modeling errors limit the performance of feed-forward system.

Generally, a 2-DOF is selected for demonstration in which the output of the plant and reference are available to the control system. The number of degree of freedom is defined as the number of closed loop transfer function that can be designed independently. In 2-DOF closed loop systems, there are transfer functions from disturbance to output and reference to output can be designed independently.

Many control requirements are assessed in frequency domain. The ability of the control system to reject the disturbances whose frequency components are concentrated on a certain band determines its performance. It is a proven fact that the effective control band is the one whose worst-case sensitivity is below 6 dB which indicates a minimum attenuation of 50% of output disturbance. In PCA actually very few components are selected which is as good as rejecting frequency components in a particular band and thus amounts to 50% of disturbance rejection. For disturbance rejection a comparison of the worst case open loop response and the closed loop response will determine how effective control design has been. In other words, there are finite set of constraints which specify which value combination from given variable domains are admitted and the value combination satisfying all constraints, that means rounding off errors and this has been done by PCA-Euclidian Norm.

II. DESIGN CONSTRINATS AND SATISFACTION

Quantitative Feedback Theory (QFT) is for robust stability, tracking and disturbance rejection. The constraints applied over certain intervals are sensitivity S (jw), Complementary Sensitivity T(jw), Gain Margin, Phase Margin, Resonant Peak, and Bandwidth. These constraints over the interval are satisfied in order to get good stability and disturbance rejection by having:

- High Gain at Low Frequency
- Low Gain at High Frequency
- Sensitivity must lie between 1 - 1.5
- Complementary Sensitivity must lie between 1.2 - 2.0
- Gain Margin should be in the range of 1.7-4.0
- Phase Margin should be in the range of 30^0- 45^0
- Damping Ratio ζ =0.64 for maximum response speed

https://dx.doi.org/10.22161/ijaers.5.5.28 ISSN: 2349-6495(P) | 2456-1908(O)

- Peak Resonant Frequency $\omega_p = \omega_n \sqrt{1-2\zeta^2}$
- Bandwidth BW= $\omega_n = \sqrt{1-2\zeta^2 + \sqrt{2-4\zeta^2+4\zeta^4}}$

Referring to above inferences a conclusion is reached whereby High gain, low sensitivity and larger bandwidth provides stability. In this paper a LA criterion is demonstrated which suggest application of SVD-PCA Norm technique to reduce noise and disturbance and uncontrolled variables elimination by pairing.

III. INFERENCES AND VALIDATIONS

This section brings out the proven inferences and its validation with respect to LA criteria.

A. Sensitivity must be minimized to get good disturbance rejection

The plant, the controller and the pre-filter are the components of a control system which are governed by noted equations $Y(s) = \frac{P(s)C(s)}{1+P(s)C(s)} F(s)C(s) + \frac{P(s)}{1+P(s)C(s)} D(s) - \frac{P(s)C(s)}{1+P(s)C(s)} N(s) --------- (1)$

If N(s) = 0, then desired output is achieved as Y(s) = F(s)R(s); the output follows the reference input.

Let us consider $C(s) = -\frac{3(1-2s)}{s+1}$ and $P(s) = \frac{0.5}{1-2s}$

The characteristics equation is given by 1+P(s) C(s) = 0 which becomes $1 + \frac{3(1-2s)}{s+1}\frac{0.5}{1-2s} = 0$

That implies s+2.5 =0; a single root by which the system is said to be stable.

For N(s) = F(s) R(s) = 0 then

$$\mathrm{Y(s)} = \frac{\mathrm{P(s)}}{1+\mathrm{P(s)C(s)}}\,\mathrm{D(s)}$$
$$= \frac{-0.5(s+1)}{(1-2s)(s+2.5)}\mathrm{D(s)} ----------- \quad (2)$$

A pole at s= +0.5 implies that output response to a disturbance is unstable; this is because the characteristic equation does not include pole-zero cancellation.

If |P(s) C(s)| » 1 then $\frac{1}{1+P(s)C(s)} \approx 0$ *and* $\frac{P(s)C(s)}{1+P(s)C(s)} \approx 1$

If $\frac{Y}{D} \approx 0$, then output response to disturbance is good and $\frac{Y}{F(s)R(s)} \approx 1$ implies that set point tracking occurs.

The constraint on sensitivity and complementary sensitivity is that S(s) + T(s) = 1. High loop gain at low frequency and low gain at high frequency are some of the inferences related to sensitivity and complementary sensitivity which requires that:

- For tracking of reference signal and good rejection of disturbance it is required that S(s) ≈0; T(s) ≈ 1, which can be met by having loop gain say |L(s)| ≥ 1
- To prevent propagation of measurement noise to the error and output signals it is required that T(s) ≈0 => S(s) ≈ 1which is met by having loop gain say |L(s)| ≤ 1
- In general, |L(s)| ≥ 1 s required at low frequency and |L(s)| ≤ 1 is required at high frequency.

B. Maximum Gain corresponds to the Eigen Vector associated with maximum Eigen Value.

In the above equation (1) to make N(s) = 0 the system gain must be maximum and maximum gain corresponds to the eigenvector associated to the maximum Eigen value.

Let us consider a 2x2 matrix to assess this concept: A = $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

The characteristics equation is given by | A- λI | = 0, arranging the equation we get,

$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ - λ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = 0 solution of which gives λ= 6 and λ = 1as Eigen values and the largest Eigen value is 6.

The Eigenvectors can be found by $|A - \lambda I| \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Solving for the above equation we get

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad \rightarrow \quad \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Putting λ= 6 and λ = 1in the above equation we get

$$\frac{x}{4} = \frac{y}{1}$$

And hence the Eigen vectors are (4, 1).

The maximum Eigen vector corresponding to the maximum Eigen value is 4 and hence the maximum gain is 4.

The maximum gain implies minimum sensitivity which is required for stability of a system.

C. Rejection of frequency components in a particular band amounts to minimum 50% of attenuation of Noise and Disturbance.

To assess, consider the following matrix

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$$K=\begin{bmatrix} 0.48 & 0.90 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{bmatrix}$$

It's SVD which gives:

$$U=\begin{bmatrix} 0.5714 & 0.3766 & 0.7292 \\ 0.6035 & 0.4093 & -0.6843 \\ -0.5561 & 0.8311 & 0.0066 \end{bmatrix}$$

$$\Sigma=\begin{bmatrix} 1.618 & 0 & 0 \\ 0 & 1.143 & 0 \\ 0 & 0 & 0.0097 \end{bmatrix}$$

$$V=\begin{bmatrix} 0.0541 & 0.9984 & 0.0151 \\ 0.9985 & -0.0540 & -0.0068 \\ -0.0060 & 0.0154 & -0.9999 \end{bmatrix}$$

And Its PCA gives

$$PCA=\begin{bmatrix} 0.2082 & -0.9386 & 0.2749 \\ -0.9780 & -0.2026 & 0.0489 \\ 0.0089 & -0.2791 & -0.9602 \end{bmatrix}$$

As can be seen the SVD- PCA gives out compressed data array amounting to minimum 50% reduction and hence a comprehensive reduction of noise disturbance. It is a proven fact that the effective control band is the one whose worst-case sensitivity is below 6 db which indicates a minimum attenuation of 50% of output disturbance. In PCA actually very few components are selected which is as good as rejecting frequency components in a particular band and thus amounts to 50% of disturbance rejection. SVD matrix provides three matrices U, the row matrix, Σ, the diagonal matrix V, the column matrix maximum, minimum gain and its ratio the condition number N. The first column of matrix V from controller output transfer matrix is the combination of manipulation with highest effect on the control objective and the first column of matrix U from disturbance output matrix points out better measure of controlled variables. Minimum condition number N (close to 1) must be achieved to have gain and sensitivity stability. The condition number with K is 197.8571, the condition number with V is 0.9999 and it has been referred to as V can be fed back to attain stability and its CN number is near unity, which can contribute to stability and the condition number with PCA is 1. The SVD-PCA are helping to bring down the highest condition number from 197.8571 to near unity. An SVD gives an idea of system matrix acting upon an input at particular frequency and PCA treated as Euclidian Distance which can be used to pairing and deleting the uncontrollable values.

The Algorithm

Input: Array Gain Matrix

- Get SVD of the gain matrix SVD = $U\Sigma V^T$
- Get condition number CN = $\frac{\overline{\delta}}{\underline{\delta}}$
- Get lowest singular value $\lambda_{\min}$
- Assign columns of U (V) with most weighted output (input) vector.
- Compare different sets of input/output (pairing) which is achieved by norm.
- Another inference is that the larger the condition number of the diagonal matrix the more unstable is the system. In the above example Σ is the diagonal matrix and its condition number N≈ 198which is quite large and the system oscillates and hence it should be kept as low as possible to attain stability.

D. **One Transfer Function is enough to assess the stability**

The stability can be drawn at by Nyquist plot and in that if Eigen value locus does not encircle the point (-1, 0) the MIMO system is closed loop stable.

The state space representation of a system in standard format is:

$$X = Ax + Bu$$
$$Y = Cx + Du$$

The equations above gives rise to four matrices namely; A – State Matrix; B – Control Matrix; C – Output Matrix and D – Transmission Matrix. For closed loop stability only one transfer matrix must be checked instead of four.

Consider an example

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

From the Transfer Matrix Y(s) = G(s) U(s) and taking the Laplace transform of state equations

sX(s) – x (0) = AX(s)) + BU(s) -- (3a)

Y(s) = CX(s) + DU(s) - (3b)

Putting x (0) = 0 and simplifying we get a generalized equation

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$Y(s) = (Si-A)^{-1}BU(s)$- - - - - - - - - - - - -- - - - - - - - - - - - - - - - - - - (4)

Substituting equation (4) in equation (3b)

$Y(s) = [C\ (SI - A^{-1})\ B + D]\ U(s)$ and from the transfer matrix

$G(s) = C\ (SI - A^{-1})\ B + D$ which becomes as D = 0

$G(s) = C\ (SI - A^{-1})\ B$ and from the given example this equation can be put as:

$$G(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s+1 & 1 \\ -6.5 & s \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{s^2+s+6.5}$$

$$\begin{bmatrix} s & -1 \\ 6.5 & s+1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+s+6.5} \begin{bmatrix} s-1 & s \\ s+7.5 & 6.5 \end{bmatrix} \rightarrow \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{s-1}{s^2+s+6.5} & \frac{s}{s^2+s+6.5} \\ \frac{s+7.5}{s^2+s+6.5} & \frac{6.5}{s^2+s+6.5} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

This 2x2 system gives rise to four transfer functions and it is enough to work on one transfer function to assess stability.

$$\left[\frac{Y_1(s)}{U_1(s)}\right] = \frac{s-1}{s^2+s+6.5}$$

$$\left[\frac{Y_1(s)}{U_2(s)}\right] = \frac{s}{s^2+s+6.5}$$

$$\left[\frac{Y_2(s)}{U_1(s)}\right] = \frac{s+7.5}{s^2+s+6.5}$$

$$\left[\frac{Y_2(s)}{U_2(s)}\right] = \frac{6.5}{s^2+s+6.5}$$

If we plot and analyze Nyquist plot for all the transfer functions which are as shown below and it can be observed that the plots do not encircle (-1, 0) point and hence the systems is said to be closed loop stable and the point that only one transfer function is enough to conclude the stability of the system can be satisfied. The transfer functions show negative real values of pole (positive real values for unstable system) and when tested for stability they all show value 1 (0 for unstable), which are arrived at by using Matlab.

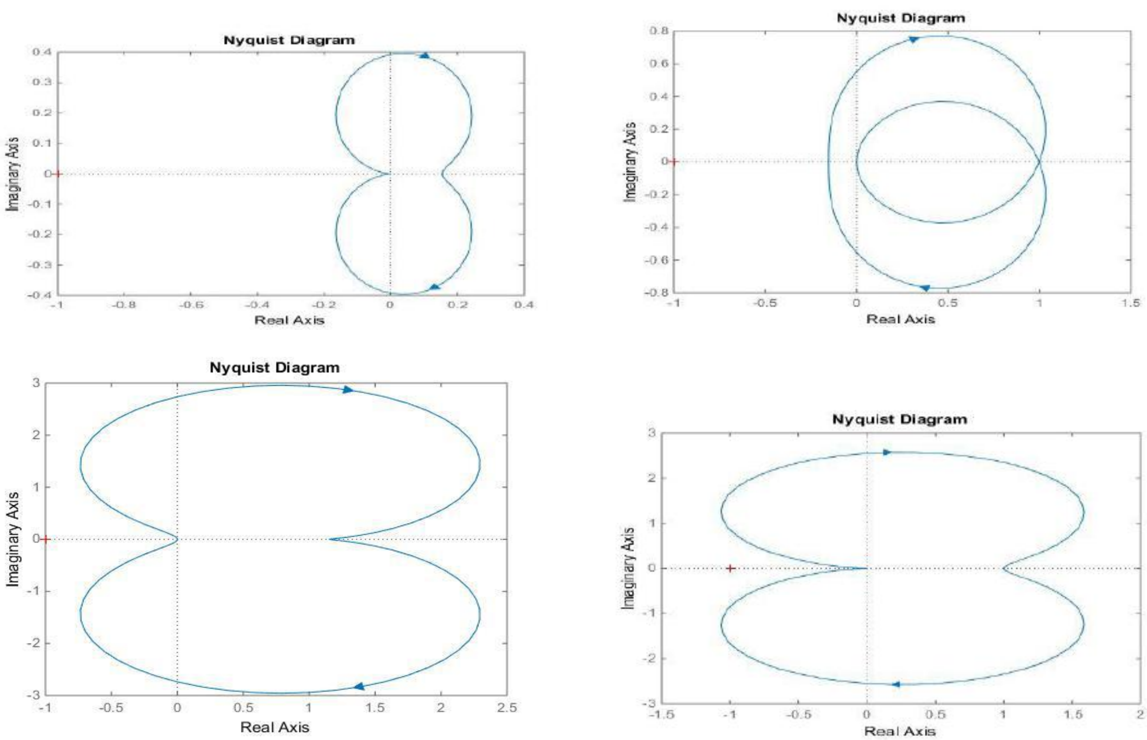

Fig.1-4: Nyquist plots for Transfer functions

IV. POINTS OF DISCUSSON

Feedback control compensates for disturbance and modeling errors. Set points always change with disturbances. Satisfactory reference tracking can be achieved with a high gain feedback. Stable controller design can be had by maintaining a high gain feedback system and the tracking of reference signal i.e., pre-filter design can be done by proper loop shaping in which the closed loop stability is determined by closed loop polynomial determinant and its characteristics loci which can be plotted by using Nyquist plots. The characteristics loci are the Eigen-values of the transfer matrix and in that if Eigen value locus does not encircle the point (-1, 0) the MIMO system is said to be closed loop stable.

Zero steady state error is obtained for a constant reference signal or disturbance signal by having low frequency slope of loop gain |L(s)| at -20db/dec and for a linearly increasing reference signal or disturbance signal a low frequency slope of -40 db/dec is required. Adequate phase margin must be provided which can be achieved by having the slope of the magnitude curve at the gain crossover frequency at -20db/dec.

Bandwidth indicates the frequency range for which satisfactory set point tracking occurs and it should be large enough for speedy response. The performance specifications of closed loop control system are robust stability for which high gain and low sensitivity must be maintained and robust tracking for which bandwidth must be high.

V. CONCLUSION

The paper demonstrates how LA criteria propose to solve closed loop stability at constrained intervals. The constraints may represent various bounds of control system stability. Uncontrolled systems behavior can be fed in the shape of numerical array to the processing tool such as Matlab. The SVD-PCA-NORM (LA criteria) is applied to the array until satisfactory compressibility is observed and the ultimate result is reduction in measurement noise and disturbance. The challenge of this criterion now lies in extending the idea to unknown source of disturbance and measurement noise, tracking and sensitivity minimization along with gain maximization when calculations goes unpredictable. Our first paper laid the foundation for this research work, whereas this paper illustrates the conceptual ground work and the upcoming paper will demonstrates this LA criterion for a real time dynamic system.

REFERENCES

[1] Prof. Masayuki Fujita, Robust Control, Spring, 2017
[2] Robust Control Toolbox User’s Guide R2017a, MathWorks, 2017.
[3] Chhaya Sunil Khandelwal a, Ranjan Maheshewarib ,U.B.Shinde, Review paper on applications of principal component analysis in multimodal biometrics system, 2nd International Conference on Intelligent Computing, Communication & Convergence (ICCC-2016), Science Direct 2016.
[4] Prof. Guy Beale, course notes on multivariable and robust control.
[5] Reed Tillotson, A Review of a Singularly Valuable Decomposition: The SVD of a Matrix, June 6, 2013.
[6] BurakDemirel, State-Space Representations of Transfer Function Systems, Referral notes,February 2, 2013
[7] Rambabu Kalla, P. S. V. Natraj, Synthesis of fractional –order QFT controller using interval constraint satisfaction technique, System & Control Engineering, IIT Mumbai, India.
[8] Chandrima Roy et al, Quantitative Feedback Theory based Controller Design of an Unstable System, International Journal of Computer Application, International Conference on Commuication, Circuits and Systems, pp 11-14, Ic3s-2012.
[9] Ivan Dokmanic, Juri Ranieri, and Martin Vetterli, Relax and Unfold: Microphone Localization with Euclidean Distance Matrices, 23rd European Signal Processing Conference (EUSIPCO)
[10] Okko H. Bosgra, HuibertKwakernaak, GjerritMeinsma, Control Systems, Notes for a course of the Dutch Institute of Systems and Control Winter term 2007.
[11] S. Skogestad and I. Postlethwaite, Multivariable Feedback Control; Analysis and Design, Second edition, Wiley, 2005.
[12] Nael H. El-Farra, Panagiotis D. Christodes, Bounded robust control of constrained multivariable nonlinear processes, Chemical Engineering Science 58 (2003) 3025 – 3047.
[13] Thomas McAvoy, Richard D. Braatz, Controllability of Processes with Large Singular Values, Ind. Eng. Chem. Res. 2003, 42, 6155-6165.
[14] E. Boje, “Pre-filter design for tracking error specifications in QFT”, Int. J. Robust and Nonlinear Control, 13637- 642, 2003.
[15] Derek Rowell, Analysis and Design of Feedback Control Systems, State-Space Representation of LTI Systems, October 2002.
[16] Katsuhiko Ogata, Modern Control Engineering, 4th Ed., Prentice Hall Inc., New Jersey, 2002.
[17] E. Boje, “Multivariable Quantitative feedback design for tracking error specifications”, Automatica, 38, 131-138, 2002.
[18] Jonathan How, Feedback Control, 2001.
[19] C.-C. Cheng, Y.-K. Liao and T.-S. Wang, “Quantitative feedback design of uncertain multivariable control systems”, Int. J. Control, 65(3), 537-553, 1996.

https://dx.doi.org/10.22161/ijaers.5.5.28 *ISSN: 2349-6495(P) | 2456-1908(O)*

[20] Winnie H. Y. Ng, L. F. Yeung,Robust Sequential Design Procedurefor Multivariable control systems with parameters bounded by Intervals, IFAC Large Scale Systems. London. UK. 1995

[21] C. Borghesani, Y. Chait, and O. Yaniv, Quantitative Feedback Theory Toolbox User's Guide, The MathWorks Inc. 1994.