Towards a Music Algebra: Fundamental Harmonic Substitutions in Jazz

Carmine Cataldo

Independent Researcher, PhD in Mechanical Engineering, Jazz Pianist and Composer, Battipaglia (SA), Italy
Email: catcataldo@hotmail.it

***Abstract*—*In this paper the most common harmonic substitutions, at least as far as jazz music is concerned, are unconventionally addressed. The novelty consists in introducing a new method finalized to formally defining and logically applying all the fundamental harmonic substitutions, by exploiting an unusually rigorous notation. After defining the substitutions and discussing their applicability, we resort to them in order to modify some simple harmonic progressions substantially based upon a banal major turnaround. As explicitly suggested by the title, the modifications are carried out by following an extremely formal line of reasoning: all the logic passages are accurately described by resorting to a notation so similar to the one commonly employed in mathematics and physics, that the harmonic analysis of a song turns out to be de facto comparable to the demonstration of a theorem.*

Keywords*—*Music Algebra, Unconventional Notation, Harmonic Substitutions, Ionian Scale, Turnaround.

I. INTRODUCTION

In this paper, all the fundamental harmonic substitutions [1] [2] are discussed by resorting to a notation, generally employed in fields such as mathematics and physics, extremely formal and rigorous. We herein exclusively refer to a single harmonization, the *Ionian* one: consequently, for the sake of clarity, we reveal in advance that the so-called *"Modal Interchange"* is not addressed in this article. Obviously, the line of reasoning we exploit can be equally followed starting from other scales, such as the *Natural Minor Scale* (that can be obtained from the *Ionian Scale* by means of a banal translation), the *Ipoionian Scale* (*Bach Minor Scale*), the *Harmonic Scales* (Major and Minor).

II. DIATONIC SUBSTITUTIONS

Two chords that arise from the harmonization of the same scale are interchangeable if the distance between them (between the roots) is equal to a diatonic third (both ascending and descending). [1] [2] Exclusively referring to the *Ionian Harmonization*, the *Diatonic Substitutions* in their entirety can be effectively summarized by means of the following relations:

$$VIm7 \leftrightarrow Imaj7 \leftrightarrow IIIm7 \tag{1}$$

$$VII\emptyset \leftrightarrow IIm7 \leftrightarrow IVmaj7 \tag{2}$$

$$Imaj7 \leftrightarrow IIIm7 \leftrightarrow V7 \tag{3}$$

$$IIm7 \leftrightarrow IVmaj7 \leftrightarrow VIm7 \tag{4}$$

$$IIIm7 \leftrightarrow V7 \leftrightarrow VII\emptyset \tag{5}$$

$$IVmaj7 \leftrightarrow VIm7 \leftrightarrow Imaj7 \tag{6}$$

$$V7 \leftrightarrow VII\emptyset \leftrightarrow IIm7 \tag{7}$$

If we set, for example, $I = C$, from the foregoing relations we immediately obtain the following:

$$Am7 \leftrightarrow Cmaj7 \leftrightarrow Em7 \tag{8}$$

$$B\emptyset \leftrightarrow Dm7 \leftrightarrow Fmaj7 \tag{9}$$

$$Cmaj7 \leftrightarrow Em7 \leftrightarrow G7 \tag{10}$$

$$Dm7 \leftrightarrow Fmaj7 \leftrightarrow Am7 \tag{11}$$

$$Em7 \leftrightarrow G7 \leftrightarrow B\emptyset \tag{12}$$

$$Fmaj7 \leftrightarrow Am7 \leftrightarrow Cmaj7 \tag{13}$$

$$G7 \leftrightarrow B\emptyset \leftrightarrow Dm7 \tag{14}$$

The *Diatonic Substitutions* can be described, with obvious meaning of the notation, by resorting to a matrix of chords (that represents the *Diatonic Tensor*):

$$Dia^{Ion}(I) = \begin{bmatrix} I^{\Delta} & 0 & IIIm7 & 0 & 0 & VIm7 & 0 \\ 0 & IIm7 & 0 & IV^{\Delta} & 0 & 0 & VII\emptyset \\ I^{\Delta} & 0 & IIIm7 & 0 & V7 & 0 & 0 \\ 0 & IIm7 & 0 & IV^{\Delta} & 0 & VIm7 & 0 \\ 0 & 0 & IIIm7 & 0 & V7 & 0 & VII\emptyset \\ I^{\Delta} & 0 & 0 & IV^{\Delta} & 0 & VIm7 & 0 \\ 0 & IIm7 & 0 & 0 & V7 & 0 & VII\emptyset \end{bmatrix} \tag{15}$$

Evidently, the *Diatonic Substitutions* can be summarized exploiting the following simple relation:

$$Dia_{ij}^{Ion} \leftrightarrow Dia_{ji}^{Ion} \tag{16}$$

From *(15)*, by setting, for example, *I* = *C*, we obtain:

$$Dia^{Ion}(C) = \begin{bmatrix} C^{\Delta} & 0 & Em7 & 0 & 0 & Am7 & 0 \\ 0 & Dm7 & 0 & F^{\Delta} & 0 & 0 & B\emptyset \\ C^{\Delta} & 0 & Em7 & 0 & G7 & 0 & 0 \\ 0 & Dm7 & 0 & F^{\Delta} & 0 & Am7 & 0 \\ 0 & 0 & Em7 & 0 & G7 & 0 & B\emptyset \\ C^{\Delta} & 0 & 0 & F^{\Delta} & 0 & Am7 & 0 \\ 0 & Dm7 & 0 & 0 & G7 & 0 & B\emptyset \end{bmatrix} \tag{17}$$

In *Figure 1* a useful graphic representation of the *Diatonic Substitutions* is provided.

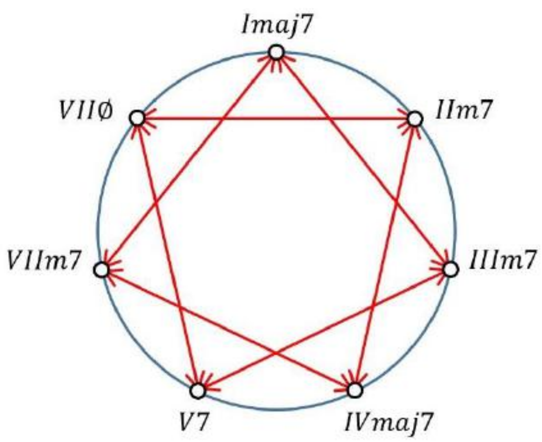

Figure 1. Diatonic Substitutions

III. SECONDARY DOMINANTS

Any chord, even if it arises from a previous harmonic substitution, can be converted into a *Dominant Seventh Chord*. [1] [2] [3] [4] Referring, once again, to the chords obtained from the *Ionian Harmonization*, we have:

$$Imaj7 \rightarrow I7 = V(IV) \tag{18}$$

$$IIm7 \rightarrow II7 = V(V) \tag{19}$$

$$IIIm7 \rightarrow III7 = V(VI) \tag{20}$$

$$IVmaj7 \rightarrow IV7 = V(bVII) \tag{21}$$

$$V7 = V(I) = Primary\ Dominant(I) \tag{22}$$

$$VIm7 \rightarrow VI7 = V(II) \tag{23}$$

$$VII\emptyset \rightarrow VII7 = V(III) \tag{24}$$

If we set, for example, *I* = *C*, from the foregoing relations we immediately obtain the following:

$$Cmaj7 \rightarrow C7 = V(F) \tag{25}$$

$$Dm7 \rightarrow D7 = V(G) \tag{26}$$

$$Em7 \rightarrow E7 = V(A) \tag{27}$$

$$Fmaj7 \rightarrow F7 = V(B^b) \tag{28}$$

$$G7 = V(C) = Primary\ Dominant(I) \tag{29}$$

$$Am7 \rightarrow A7 = V(D) \tag{30}$$

$$B\emptyset \rightarrow B7 = V(E) \tag{31}$$

From *(28)* we immediately deduce that the dominant seventh chord obtained from the fourth degree (*F7*, in the specific case) leads towards a note, to extremely simplify, that does not belong to the *C Ionian Scale* (*Bb*, in the specific case). Consequently, the *Secondary Dominant* that arises from the forth degree is to be regarded as *non-functioning* (or *non-functional*).

IV. TRITONE SUBSTITUTION

Any *Dominant Seventh Chord*, especially in the case it is altered and even if it arises from a previous harmonic substitution, can be replaced by a chord of the same kind (a dominant seventh chord) distant three whole tones from the initial chord. [1] [2] [3] [4] [5] Generally, if we denote with *X* a generic note belonging to the *Chromatic Scale*, and with *t* a *Whole Tone Interval*, we can write:

$$X7 \rightarrow Y7 \tag{32}$$

$$Y = X + 3t \tag{33}$$

If we set, for example, *X* = *C*, from the foregoing relations we immediately obtain the following:

$$C7 \rightarrow G^b7 \equiv F^{\#}7 \tag{34}$$

V. DIMINISHED SUBSTITUTION

Any *Dominant Seventh Chord*, especially if it is provided with the flat ninth and even if it arises from a previous harmonic substitution, can be replaced by a *Diminished Chord* distant a major third, a perfect fifth, a minor seventh or a flat ninth from the initial chord. [2] [3] [4] [5] [6] [7] All the above-mentioned intervals are implicitly regarded as ascending. In other terms, we can concisely write, with obvious meaning of the notation, as follows:

$$X7^{(b9)} \rightarrow Ydim7 \tag{35}$$

$$Y = X + 2t + \frac{3n}{2}t \qquad n = 0,1,2,3 \tag{36}$$

More explicitly, we have:

$$X7^{(b9)} \rightarrow \begin{cases} \langle X+2t\rangle dim7 & n=0, major\ third \\ \langle X+\frac{7}{2}t\rangle\, dim7 & n=1, perfect\ fifth \\ \langle X+5t\rangle dim7 & n=2, minor\ seventh \\ \langle X+\frac{13}{2}t\rangle\, dim7 & n=3, flat\ ninth \end{cases} \quad (37)$$

By setting, for example, $X = C$, from *(37)* we obtain:

$$C7^{(b9)} \rightarrow \begin{cases} Edim7 & n=0, major\ third \\ Gdim7 & n=1, perfect\ fifth \\ B^b dim7 & n=2, minor\ seventh \\ D^b dim7 & n=3, flat\ ninth \end{cases} \quad (38)$$

VI. EXPANSION

The *Dominant 9sus4* and *b9sus4 Chords* can be expressed by resorting to the so-called *Slash Chords*. In very formal terms, with obvious meaning of the notation, we can write:

$$V7^{9sus4}(X) = \frac{II-7(X)}{\langle X+\frac{7}{2}t\rangle} \quad (39)$$

$$V7^{b9sus4}(X) = \frac{II-7b5(X)}{\langle X+\frac{7}{2}t\rangle} \quad (40)$$

By virtue of *(39)* and *(40)*, we can state that, in a certain measure, any *Dominant Seventh Chord* can be imagined as being preceded by a *Minor Seventh Chord* or a *Half-Diminished Chord* distant a descending perfect fourth. [3] Consequently, employing a vertical line to separate two consecutive beats or bars, we have:

$$V7 \rightarrow \begin{cases} II-7 \quad | V7 \\ II-7b5\, | V7 \end{cases} \quad (41)$$

If we set, for example, $X = C$, from *(39)* and *(40)* we obtain:

$$G7^{9sus4} = \frac{Dm7}{G} \quad (42)$$

$$G7^{b9sus4} = \frac{D\emptyset}{G} \quad (43)$$

Coherently with the setting ($X = C$), from *(41)* we have:

$$G7 \rightarrow \begin{cases} Dm7\, |\, G7 \\ D\emptyset\, |\, G7 \end{cases} \quad (44)$$

VII. SOME NOTEWORTHY CASES

Case 1: Rhythm Changes (Bridge)

Let's consider the following simple harmonic progression:

Imaj7	*Imaj7*	*VIm7*	*Vm7*
IIm7	*IIm7*	*V7*	*V7*

Harmonic Progression 1.1

If we set $I = B^b$, we immediately obtain:

B^bmaj7	B^bmaj7	Gm7	Gm7
Cm7	Cm7	F7	F7

Harmonic Progression 1.2

Taking into account the previous progression, let's carry out the following substitutions:

$$bars\ 1{,}2{:}\ B^b maj7 \xrightarrow{diat.} Dm7 \xrightarrow{sec.\ dom.} D7 \quad (45)$$

$$bars\ 3{,}4{:}\ Gm7 \xrightarrow{sec.\ dom.} G7 \quad (46)$$

$$bars\ 5{,}6{:}\ Cm7 \xrightarrow{sec.\ dom.} C7 \quad (47)$$

From the *Harmonic Progression 1.2*, by virtue of *(45)*, *(46)* and *(47)*, we finally obtain:

D7	D7	G7	G7
C7	C7	F7	F7

Harmonic Progression 1.3

Case 2: I'm Getting Sentimental Over You (first 8 bars)

Let's now consider the following harmonic progression:

Imaj7	*V7*	*Imaj7*	*IVm7*
IIm7	*V7*	*Imaj7 VIm7*	*IIm7 V7*

Harmonic Progression 2.1

By setting $I = F$, we immediately obtain:

Fmaj7	C7	Fmaj7	Dm7
Gm7	C7	Fmaj7 Dm7	Gm7 C7

Harmonic Progression 2.2

Taking into account the foregoing progression, let's carry out the following substitutions:

$$bar\ 2{:}\ C7 \xrightarrow{diat.} E\emptyset \xrightarrow{sec.\ dom.} E7 \xrightarrow{exp.} Bm7\, |\, E7 \quad (48)$$

$$bar\ 3{:}\ Fmaj7 \xrightarrow{sec.\ dom.} F7 = V(B^b) \xrightarrow{diat.} A\emptyset \quad (49)$$

$$bars\ 4{,}6{:}\ Dm7 \xrightarrow{sec.\ dom.} D7 \quad (50)$$

$$bars\ 5{,}8{:}\ Gm7 \xrightarrow{sec.\ dom.} G7 \quad (51)$$

From the *Harmonic Progression 2.2*, by virtue of *(48)*, *(49)*, *(50)* and *(51)*, we immediately obtain:

Fmaj7	Bm7 E7	Am7^{b5}	D7
G7	C7	Fmaj7 D7	Gm7 C7

Harmonic Progression 2.3

It is worth underlining that, in some cases, a desired chord (or a harmonic progression in its entirety) can be obtained by following various lines of reasoning. Just to provide an example, it is easy to verify how the third bar of the *Harmonic Progression 2.3* could have been alternatively deduced by resorting to the following simple substitutions:

$$bar\ 3\text{: } Fmaj7 \xrightarrow{diat.} Dm7 \xrightarrow{sec.\ dom.} D7 \tag{52}$$

$$bars\ 4\text{: } Dm7 \xrightarrow{sec.\ dom.} D7 \tag{53}$$

$$bars\ 3{,}4\text{: } D7 \xrightarrow{exp.} A\emptyset \mid D7 \tag{54}$$

Case 3: Stella by Starlight (first 8 bars)

Let's consider the following harmonic progression:

IIm7	*IIm7*	*V7*	*V7*
Imaj7	*Imaj7*	*IIm7*	*V7*

Harmonic Progression 3.1

If we set *I* = *B^b*, we immediately obtain:

Cm7	Cm7	F7	F7
B^{b}maj7	B^{b}maj7	Cm7	F7

Harmonic Progression 3.2

Taking into account the progression we have just obtained, let's carry out the following substitutions:

$$bars\ 1{,}2\text{: } Cm7 \xrightarrow{diat.} A\emptyset \xrightarrow{sec.\ dom.} A7 \xrightarrow{exp.} E\emptyset \mid A7 \tag{55}$$

$$bars\ 3{,}4\text{: } F7 \xrightarrow{exp.} Cm7 \mid F7 \tag{56}$$

$$bars\ 5{,}6\text{: } B^bmaj7 \xrightarrow{sec.\ dom.} B^b7 \xrightarrow{exp.} Fm7 \mid B^b7 \tag{57}$$

$$bar\ 7\text{: } Cm7 \xrightarrow{diat.} E^bmaj7 \tag{58}$$

$$bar\ 8\text{: } F7 \xrightarrow{diat.} Dm7 \xrightarrow{sec.\ dom.} D7 \xrightarrow{trit.} A^b7 \tag{59}$$

From the *Harmonic Progression 3.2*, exploiting *(55)*, *(56)*, *(57)*, *(58)* and *(59)*, we finally obtain:

Em7^{b5}	A7	Cm7	F7
Fm7	B^{b}7	E^{b}maj7	A^{b}7

Harmonic Progression 3.3

The chord obtained in the eight bar (*A^{b}7*) leads towards *B^{b}maj7* (ninth bar, not displayed in the *Harmonic Progression 3.3*). The progression *^{b}VII7 | Imaj7* is commonly named *"Back-Door Solution"*. [5]

Case 4: Easy Living (first 8 bars)

Let's consider the following banal harmonic progression:

Imaj7 VIm7	*IIm7 V7*	*Imaj7 VIm7*	*IIm7 V7*
Imaj7 VIm7	*IIm7 V7*	*Imaj7 VIm7*	*IIm7 V7*

Harmonic Progression 4.1

If we set *I* = *A^b*, from the foregoing progression we obtain:

A^{b}maj7 Fm7	B^{b}m7 E^{b}7	A^{b}maj7 Fm7	B^{b}m7 E^{b}7
A^{b}maj7 Fm7	B^{b}m7 E^{b}7	A^{b}maj7 Fm7	B^{b}m7 E^{b}7

Harmonic Progression 4.2

Taking into account the previous harmonic progression, let's carry out the following substitutions:

$$bar\ 1\text{: } Fm7 \xrightarrow{sec.\ dom.} F7 \xrightarrow{dim.} Adim7 \tag{60}$$

$$bar\ 2\text{: } E^b7 \xrightarrow{diat.} G\emptyset \xrightarrow{sec.\ dom.} G7 \xrightarrow{dim.} Bdim7 \tag{61}$$

$$bars\ 3{,}6\text{: } A^bmaj7 \xrightarrow{diat.} Cm7 \tag{62}$$

$$bar\ 3\text{: } Fm7 \xrightarrow{diat.} A^bmaj7 \xrightarrow{sec.\ dom.} A^b7 \tag{63}$$

$$bar\ 4\text{: } B^bm7 \xrightarrow{diat.} D^bmaj7 \tag{64}$$

$$bar\ 4\text{: } E^b7 \xrightarrow{diat.} Cm7 \xrightarrow{sec.\ dom.} C7 \xrightarrow{trit.} G^b7 \tag{65}$$

$$bar\ 7\text{: } Fm7 \xrightarrow{sec.\ dom.} F7 \tag{66}$$

From the *Harmonic Progression 4.2*, taking into account *(60)*, *(61)*, *(62)*, *(63)*, *(64)*, *(65)* and *(66)*, we finally obtain:

A^{b}maj7 Ao7	B^{b}m7 Bo7	Cm7 A^{b}7	D^{b}maj7 G^{b}7
A^{b}maj7 Fm7	B^{b}m7 E^{b}7	Cm7 F7	B^{b}m7 E^{b}7

Harmonic Progression 4.3

The second chord in the fourth bar (G^b7) leads towards $A^b maj7$: once again, a typical *"Back-Door Solution"*.

Case 5: Giant Steps

Let's now consider the following harmonic progression:

Imaj7 VIm7	*IIm7 V7*	*Imaj7*	*VIm7*
IIm7 V7	*Imaj7 V7*	*Imaj7*	*V7*
Imaj7	*VIm7*	*IIm7*	*V7*
Imaj7	*V7*	*Imaj7*	*V7*

Harmonic Progression 5.1

By setting *I = B*, we immediately obtain:

Bmaj7 $G^\#m7$	$C^\#m7$ $F^\#7$	Bmaj7	$G^\#m7$
$C^\#m7$ $F^\#7$	Bmaj7 $F^\#7$	Bmaj7	$F^\#7$
Bmaj7	$G^\#m7$	$C^\#m7$	$F^\#7$
Bmaj7	$F^\#7$	Bmaj7	$F^\#7$

Harmonic Progression 5.2

Taking into account the previous progression, let's carry out the following substitutions:

$$bar\ 1: G^\#m7 \xrightarrow{sec.\ dom.} G^\#7 \equiv A^b7 \xrightarrow{trit.} D7 \tag{67}$$

$$bars\ 2{,}5{,}11: C^\#m7 \xrightarrow{sec.\ dom.} C^\#7 \equiv D^b7 \xrightarrow{trit.} G7 \tag{68}$$

$$bars\ 2{,}5: F^\#7 \xrightarrow{diat.} A^\#\emptyset \xrightarrow{sec.\ dom.} A^\#7 \equiv B^b7 \tag{69}$$

$$bars\ 3{,}6{,}9{,}15: Bmaj7 \xrightarrow{diat.} D^\#m7 \xrightarrow{sec.\ dom.} D^\#7 \equiv E^b7 \tag{70}$$

$$bars\ 4{,}10: G^\#m7 \xrightarrow{sec.\ dom.} A^b7 \xrightarrow{trit.} D7 \xrightarrow{exp.} Am7 \mid D7 \tag{71}$$

$$bars\ 8{,}14: F^\#7 \xrightarrow{diat.} A^\#\emptyset \xrightarrow{sec.\ dom.} B^b7 \xrightarrow{exp.} Fm7 \mid B^b7 \tag{72}$$

$$bars\ 12{,}16: F^\#7 \xrightarrow{exp.} C^\#m7 \mid F^\#7 \tag{73}$$

From the foregoing harmonic progression, by virtue of *(67)*, *(68)*, *(69)*, *(70)*, *(71)*, *(72)* and *(73)*, we obtain:

Bmaj7 D7	G7 B^b7	E^b7	Am7 D7
G7 B^b7	E^b7 $F^\#7$	Bmaj7	Fm7 B^b7
E^b7	Am7 D7	G7	$C^\#m7$ $F^\#7$
Bmaj7	Fm7 B^b7	E^b7	$C^\#m7$ $F^\#7$

Harmonic Progression 5.3

The *Harmonic Progression* 5.3, clearly, does not represent the desired result yet. In order to obtain the original Coltrane progression, we have to further modify all the chords written in red. On this purpose, we introduce a banal harmonic substitution (often simply named *"Quality Substitution"*) that allows to instantly transform a *Dominant Seventh Chord* into a *Major Seventh*, without resorting to the so-called *"Modal Interchange"*. In this regard, suffice it to bear in mind that the two above-mentioned chords are characterized by the same triad (the same *chordal notes*): in other terms, they only differ in the seventh (that can be considered, at least in a certain sense, as being a *fundamental tension*). It is worth highlighting how, by exploiting this very method, the well-known *"Tadd Dameron* (or *Coltrane*) *Turnaround"* [4] [8] can be obtained. Ultimately, we can write:

$$bars\ 2{,}5{,}11: G7 \xrightarrow{dom.\ to\ maj.} Gmaj7 \tag{74}$$

$$bar\ 3{,}6{,}9{,}15: E^b7 \xrightarrow{dom.\ to\ maj.} E^bmaj7 \tag{75}$$

Taking into account *(74)* and *(75)* we finally obtain:

Bmaj7 D7	Gmaj7 B^b7	E^bmaj7	Am7 D7
Gmaj7 B^b7	E^bmaj7 $F^\#7$	Bmaj7	Fm7 B^b7
E^bmaj7	Am7 D7	Gmaj7	$C^\#m7$ $F^\#7$
Bmaj7	Fm7 B^b7	E^bmaj7	$C^\#m7$ $F^\#7$

Harmonic Progression 5.4

VIII. FINAL REMARKS

Let's start from a banal sequence of four bars (or beats, or sets composed of an equal number of beats) characterized by the same chord (in our case, a *Major Seventh Chord*):

Imaj7	*Imaj7*	*Imaj7*	*Imaj7*

Harmonic Progression 6.1

Although it cannot be formally regarded as a harmonic substitution, we are clearly allowed to resort to the so-called *"Tonicization"* [9] [10] [11] [12] [13] in order to create a first, simple harmonic motion (nothing but an *Authentic Cadence*):

Imaj7	*Imaj7*	*V7*	*V7*

Harmonic Progression 6.2

Let's now carry out the following substitutions:

$$bars\ (beats)\ 3{,}4{:}\ V7 \xrightarrow{exp.} IIm7 \mid V7 \tag{76}$$

$$bar\ (beat)\ 2{:}\ Imaj7 \xrightarrow{diat.} VIm7 \tag{77}$$

From the *Harmonic Progression 6.2*, by virtue of *(76)* and *(77)*, we immediately obtain:

Imaj7	*VIm7*	*IIm7*	*V7*

Harmonic Progression 6.3

It is evident that, starting from a single *Major Seventh Chord*, we can easily obtain a *Major Turnaround*. In other terms, we are implicitly stating that all the harmonic progressions we have obtained in the previous section may be imagined as arising from a single chord. In a certain sense, we could even state that, net of some progressions explicitly based upon the so-called *"Pure Plagal Cadence"* (*IVmaj7 | Imaj7*), all the *Popular Jazz Songs*, as far as the harmonic structure is concerned, may be considered as originating from a single chord (a *Major Seventh Chord* or, exploiting the harmonization of whatever minor scale, a *Minor Seventh*). It is worth underlining that we have explicitly referred to the *"Pure Plagal Cadence"*, since the "*Authentic"* or *"Extended"* one (*IVmaj7 | V7 | Imaj7*) can be immediately deduced, by simply resorting to a banal *Diatonic Substitution*, from a *IIm7 | V7 | Imaj7* harmonic progression which, in turn, can be obtained from a single *Major Seventh Chord* by exploiting a *"Tonicization"* and, taking into account *(76)*, an *Expansion*.

ACKNOWLEDGEMENTS

This paper is dedicated to my father, Antonio Cataldo, who unexpectedly passed away on the 11th of June 2016.
I would like to thank my friends Francesco D'Errico, Giulio Martino, and Sandro Deidda, excellent Italian jazz musicians and esteemed teachers at the Conservatory of Salerno, for their precious suggestions.

REFERENCES

[1] D'Errico, F. (2017). Armonia Funzionale e Modalità – Rudimenti per l'Improvvisazione a Indirizzo Jazzistico. Naples, Italy: Editoriale Scientifica.

[2] Cho, G. J. (1992). Theories and Practice of Harmonic Analysis. Lewiston, NY: Edwin Mellen Press.

[3] Levine, M. (2009). The Jazz Theory Book (Italian Edition by F. Jegher). Milan, IT: Curci Jazz.

[4] Lawn, R., Hellmer, J. L. (1996). Jazz: Theory and Practice. Los Angeles, CA: Alfred Publishing Co. Inc.

[5] Coker, J. (1997). Elements of the Jazz Language for the Developing Improvisor. Los Angeles, CA: Alfred Publishing Co. Inc.

[6] Cataldo, C. (2017). The Art of Improvising: the Be-Bop Language and the Dominant Seventh Chords. Art and Design Review, 5, 181-188. http://doi.org/10.4236/adr.2017.53014

[7] Cataldo, C. (2017). Il Linguaggio Be-Bop e gli Accordi di Settima di Prima Specie [The Be-Bop Language and The Dominant Seventh Chords]. Journal of Science, Humanities and Arts (JOSHA) 4(4). https://dx.doi.org/10.17160/josha.4.4.340

[8] Coker, J., Casale, J., & Campbell, G. (1982). Patterns for Jazz – A Theory Text for Jazz Composition and Improvisation: Treble Clef Instruments. Los Angeles, CA: Alfred Publishing Co. Inc.

[9] Dobbins, B. (2010). Jazz Arranging and Composing – L'Approccio Lineare (Italian Ed. by Roberto Spadoni). Italy: Volontè & Co.

[10] Cataldo, C. (2017). The Art of Improvising: the Be-Bop Language and the Minor Seventh Chords. Art and Design Review, 5, 213-221. https://doi.org/10.4236/adr.2017.54017

[11] Cataldo, C. (2017). Il Linguaggio Be-Bop e gli Accordi di Settima di Seconda Specie [The Be-Bop Language and The Minor Seventh Chords]. Journal of Science, Humanities and Arts (JOSHA), 4(4). https://dx.doi.org/10.17160/josha.4.4.339

[12] Cataldo, C. (2017). The Art of Improvising: the Be-Bop Language and the Major Seventh Chords. Art and Design Review, 5, 222-229. https://doi.org/10.4236/adr.2017.54018

[13] Cataldo, C. (2017). Il Linguaggio Be-Bop e gli Accordi di Settima di Quarta Specie [The Be-Bop Language and The Major Seventh Chords]. Journal of Science, Humanities and Arts (JOSHA), 4(4). https://dx.doi.org/10.17160/josha.4.4.341