Estimation and Analysis of Channel Distortion and Carrier Frequency Offset (CFO) in MIMO-OFDM Systems

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Abstract—Wireless communication is a key area of research with increasing demand of high data rate applications at a low cost. Multiple input multiple output - orthogonal frequency division multiplexing (MIMO-OFDM) is the most promising technology for high spectral efficiency and for very high data throughput. Despite of being an emerging technology, it has to face a major challenge of carrier frequency offset (CFO) error, which causes inter carrier interference that ultimately leads to loss of orthogonality. This paper provides an intuitive knowledge of the concept used in the estimation of Channel distortion and CFO.

Keywords—CFO, FFT, IFFT, MIMO, MSE, OFDM, SNR.

I. INTRODUCTION

With the growing mobile radio communication systems, Orthogonal Frequency Division Multiplexing (OFDM) has been considered as the essence of most 4G communication systems, fixed Wi-Fi system, mobile Wi-Fi system fixed WiMAX system mobile WiMAX system and Long Term Evolution (LTE system)[1]. OFDM is a Multi-carrier Modulation (MCM) scheme in which a large number of closely spaced orthogonal sub-carrier signals are used to carry data on several parallel data streams or channels. Unlike Frequency Division Multiple Access (FDMA), OFDM uses the spectrum much more efficiently by spacing the channels closer together. OFDM is a promising technology of the future communication system.

In recent years, direct conversion receiver has drawn a lot of attention due to its low power consumption and low implementation cost. However, some mismatches in direct conversion receiver can seriously degrade the system performance, such as in-phase and quadrature-phase (I/Q) imbalance and carrier frequency offset (CFO). The I/Q imbalance is due to the amplitude and phase mismatches between the I and Q-branch of the local oscillator whereas the CFO is due to the mismatch of carrier frequency at the transmitter and receiver. It is known [5] that CFO can cause a serious inter-carrier interference (ICI) in OFDM systems. Channel distortion is an important factor which affects MIMO-OFDM communication system and can be estimated using training sequences in the channel. There have been many reports in the literature on the estimation of channel distortion and CFO [1]-[6]. In [3] assuming that the channel frequency response is smooth, a frequency-domain estimation method has been proposed to estimate the channel distortion and CFO. Recently, exploiting the fact that the size of the DFT matrix is usually larger than the channel length in OFDM systems, a time-domain method was proposed in [4] for the estimation of channel distortion and CFO. Both the frequency-domain and time-domain methods need only one OFDM block for training sequence and can achieve a good performance. For CFO estimation, a low complexity maximum likelihood (ML) technique was proposed in [6]. Recently, there have been a lot of interests in combining the OFDM systems with the multiple-input multiple-output (MIMO) technique. These systems are known as MIMO OFDM systems. Many methods have been proposed to deal with the channel estimation in MIMO OFDM systems. One of these methods is to send the training sequences (known to the receiver) from the transmitter. The design of training sequences for MIMO OFDM systems was investigated in [9]-[12]. In [9], the author derived a special class of the optimal training sequences in one OFDM block. The general solution for the optimal sequences and the conditions for their existence were derived in [10] and [11] respectively. However these methods assume that there is no mismatch of the local oscillators. Several methods have been developed for the estimation of the channel distortion and CFO mismatches [13]-[21]. In [16], the authors derived the Cramer-Rao bound (CRB) for the joint estimation of CFO and channel response for MIMO systems. The design of training sequence for MIMO OFDM systems in the presence of CFO was investigated in [17][18]. In [19], the authors used orthogonal training sequences for CFO estimation. In [20], the authors
studied the blind carrier estimation method in MIMO OFDM systems. Both the data-aided and the non-data-aided methods were investigated in [23]. Most of these methods need more than one training block to achieve a good performance. In this paper, we extend the method in [1] for the estimation of CFO and channel response in MIMO-OFDM systems. By introducing a concept called channel residual energy (CRE), we propose a new estimation method that needs only one OFDM block for training sequences.

II. MIMO-OFDM SYSTEM ARCHITECTURE

Multiple input multiple output (MIMO) is the use of multiple antennas at both the transmitter and receiver to improve performance of the communication system. MIMO technology has gained lots of attention in wireless communication systems because it offers significant increases in data throughput and link range without additional bandwidth. The combination of MIMO with OFDM is best technology for fourth generation (4G) wireless communications. However, the high data rate performance of the MIMO OFDM system is very sensitive to CFO which introduces ICI.

MIMO-OFDM system is a combination of MIMO and OFDM techniques to achieve the high spectral efficiency and increased data rate. MIMO-OFDM system transmits independent modulated signals from multiple antennas simultaneously [4]. At the receiver, input data is received, MIMO decoding on each of subchannels extracts the data from all the transmitted antenna data on all the subchannels. From the figure 1 $N_t$ is the number of transmitter antennas and $N_r$ is the number of receiver antennas. The MIMO OFDM system transmitter has $N_t$ parallel transmission paths which are very similar to the single antenna OFDM system. Each and every branch channel perform serial to parallel conversion, N-point IFFT (Inverse Fast Fourier Transform), pilot insertion and cyclic prefix (CP) or cyclic extension before the final transmission. In MIMO OFDM system $N_t$ different signals are transmitted simultaneously over the channel paths ($N_t \times N_r$) and each received signal is a combination of transmitted signal and distortion noise [5]. The receiver must estimate and correct the possible symbol errors and frequency offsets. At the receiver CP is removed and N-point FFT is performed per receiver branch. In this paper, the estimation of channel distortion and CFO is based on the single carrier processing that implies MIMO detection has to be done per OFDM subcarrier. The received signals of subcarrier $k$ are routed to the $k^{th}$ MIMO decoder to recover all the $N_t$ transmitted data signals on the subcarrier. After the transmitted symbol the transmission antenna is combined and then proceeded for the subsequent operations like demodulation and decoding.

III. CFO: AN OVERVIEW

The OFDM systems are very sensitive to CFO and timing, therefore, before demodulating the OFDM signals at the receiver side, the receiver must be synchronized to the time frame and carrier frequency which has been transmitted. The degradation in the performance of MIMO-OFDM is due to the existence of CFO and channel distortion. CFO occurs when the local oscillator signal for down conversion in the receiver does not synchronize with carrier signal contained in the received signal. CFO always exists because transmitter and receiver never oscillate at identical frequencies. Therefore there is always a difference between the carrier frequencies that is generated in the receiver with the one that is generated in transmitter; this difference is called frequency offset i.e. $f_{offset} = f_c - f_c'$

Where, $f_c$ is the carrier frequency in the transmitter and $f_c'$ is the carrier frequency in receiver.

In order to help the synchronization, the signals that are transmitted, have the references parameters that are used in
receiver for synchronization. However, in order to synchronize with the transmitter, it needs to know two important factors:

(i) Prior to the FFT process, where it should start sampling the incoming OFDM symbol from.
(ii) How to estimate and correct any CFO.

After estimating the symbol boundaries in the receiver and when the presence of the symbol is detected, the next step is to estimate the frequency offset.

IV. CHANNEL DISTORTION AND CFO ESTIMATION USING PROPOSED METHOD

4.1 ESTIMATION USING CHANNEL RESIDUAL ENERGY CONCEPT

This paper introduces a method to estimate optimum values of Channel Distortion and CFO by minimizing the channel residual energy. The Channel Residual Energy (CRE) is the total amount of energy present in a channel at a particular time. This energy is reduced when the channel is engaged in a transmission or reception.

The figure 1 [17] shows the mathematical signal model of MIMO OFDM system. The input signal $S_j$ is input of the $M$-point IFFT, after performing the IFFT operation obtain the $M \times 1$ vector $X_j$. The insertion of Cyclic Prefix of length $L-1$ is done after IFFT operation, then the signal is transmitted from $i$th transmit antenna, at the receiver the transmitted signal recovered.

1. Let the channel impulse response from $i$th transmit antenna to the $j$th receive antenna be $h_{ji}(n)$. Let assume the length of all channels are $\leq L$ and the length of the Cyclic Prefix (CP) is $L-1$. Therefore, there is no interference between the adjacent OFDM channels after CP is removed. The received vector at the $j$th receive antenna is written as:

$$r_j = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} + n_j$$

Here, $H_{ji}$ is an $M \times M$ matrix with the first column $h_{ji}(0) \ldots h_{ji}(L-1)$ and $n_j$ is $M \times 1$ matrix that is blocked version of channel noise. On employing FEQ, transmitted signal can be recovered.

2. OFDM system always suffers from CFO because it is unavoidable in OFDM system. This can only be minimized. Normalized CFO is defined as

$$0_j = \frac{\Delta f}{f_s} = \frac{\Delta f_j}{MT}$$

Where, M is the size of the FFT matrix and T is the sample spacing between carrier signals.

3. The vector signal affected due to CFO is

$$y_j = E_j r_j$$

Where, $r_j$ is the desired baseband vector and $E_j$ is the $M \times M$ diagonal matrix is defined by

$$E_j = \text{diag} \left[ 1, e^{j \frac{2 \pi}{M} \theta_1}, \ldots, e^{j \frac{2 \pi}{M}(M-1) \theta_j} \right]$$

4. Suppose, there is also Channel Distortion at the receiver along with CFO then, the received signal due to channel distortion becomes;

$$W_j = p_j y_j + q_j y_j^*$$

Where, $p_j$ and $q_j$ are the channel distortion parameters at the receiver.

5. Channel distortion is related to amplitude $\epsilon_j$ and phase mismatch $\phi_j$ as

$$p_j = \frac{1 + \beta_j e^{-j \phi_j}}{2} \quad \text{and} \quad q_j = \frac{1 - \beta_j e^{j \phi_j}}{2}$$

6. The received signal then becomes

$$W_j = p_j E_j r_j + q_j E_j^* r_j^*$$

This received signal is distorted due to presence of $E_j$ which destroys the orthogonality of subcarrier.

7. To recover this distorted signal, channel distortion parameter $\lambda_j$ is:

$$\lambda_j = \frac{q_j}{p_j}$$

**CASE 1:** If channel distortion parameter $\lambda_j$ is known at the receiver side then scaled baseband signal is shown as:

$$p_j y_j = \frac{W_j - \lambda_j W_j^*}{1 - |\lambda_j|^2}$$

**CASE 2:** If carrier frequency offset parameter $\theta_j$ is also known at the receiver, the desired baseband vector is:
\[ p_j y_j = \mathbf{E}_j^* \frac{w_j - \hat{\lambda}_j w_j}{1 - |\hat{\lambda}_j|^2} \]

But as there is always channel distortion and carrier frequency offset present in the received signal which is unknown so to recover the baseband signal, following procedure can be done.

1. Estimate the MIMO OFDM channel
2. Estimate the optimum values of channel response and CFO

4.1.1 To estimate MIMO channel when there is no channel distortion and CFO

Rewriting the received vector:
\[ \mathbf{r}_i = \begin{bmatrix} \mathbf{H}_{i,0} \mathbf{H}_{i,1} \ldots \ldots \mathbf{H}_{i,N_t-1} \end{bmatrix} \times \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N_t-1} \end{bmatrix} + \mathbf{n}_j \]

Where, \( \mathbf{X} \) is \( N \times 1 \) matrix. This can be written in a simpler form as:
\[ \mathbf{r} = \mathbf{A} \mathbf{c} + \mathbf{n} \]

Where, channel gain is \( \mathbf{c} = [c_0^T, c_1^T \ldots \ldots \ldots c_{N_t-1}^T]^T \)

Received vector \( \mathbf{A} = [A_0, A_1 \ldots \ldots \ldots A_{N_t-1}] \)

\( \mathbf{n} \) = Gaussian channel noise

Here, \( \mathbf{c} \) is identifiable if \( \mathbf{A} \) has full column rank. To fulfill this condition so that channel identifiability is guaranteed, append zeros to the matrix which results:
\[ \mathbf{d}_k = \begin{bmatrix} c_k^T \\ 0 \end{bmatrix} \text{ k= 1, 2, 3,.............N_t-1} \]

On collecting all \( \mathbf{d} \) vectors, it becomes
\[ \mathbf{d} = \begin{bmatrix} d_0^T \\ d_1^T \ldots \ldots \ldots d_{N_t-1}^T \end{bmatrix}^T \]

Suppose \( \mathbf{A} \) is a matrix such that the matrix \( \mathbf{B} \) is invertible
\[ \mathbf{B} = [A_0A_0^T, A_1A_1^T \ldots \ldots \ldots A_{N_t-1}A_{N_t-1}^T] \] and therefore
\[ \mathbf{d} = \mathbf{B}^{-1} \mathbf{r} \]

Hence, the received signal is: \( \mathbf{r} = \mathbf{B} \mathbf{d} \)

4.1.2 Estimation of channel response and CFO

When CFO is not present that is \( \mathbf{E} = \mathbf{I} \), where \( \mathbf{I} \) is identity matrix. Therefore scaled received signal is:
\[ p_j y_j = \frac{w_j - \hat{\lambda}_j w_j}{1 - |\hat{\lambda}_j|^2} \]

Here, MIMO channel response is obtained as:
\[ p_j \hat{\mathbf{d}} = \mathbf{B}^{-1} \mathbf{r}_j = \mathbf{B}^{-1} \mathbf{E}_j^* \frac{w - \hat{\lambda}_j \mathbf{w}^*}{1 - |\hat{\lambda}_j|^2} \]

When channel distortion is estimated, the first entries of \( \hat{\mathbf{d}} \) are because of channel and last entries are present because of noise.

For high SNR as always required these entries should be small because these entries represent the energy present in the channel. To explain this, a concept called channel residual energy is defined as:
\[ \text{CRE} = \sum_{l=0}^{N_t-1} \sum_{l'=0}^{L-1} |[p \hat{\mathbf{d}}]_l|^2 \]

Where, \( \rho = \frac{M}{N_t} \)

Errors in the entries of \( \hat{\mathbf{d}} \) results high CRE. On minimizing the error in these entries results more accurate estimation of CFO without knowing the channel response. For the estimation of CRE, a matrix is defined as:
\[ \mathbf{U} = \begin{bmatrix} 0 & l_{p-1} & 0 & 0 & \ldots & 0 \\ 0 & 0 & l_{p-1} & 0 & \ldots & 0 \\ 0 & 0 & 0 & l_{p-1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & l_{p-1} \end{bmatrix} \]

Suppose here that \( \rho > l \) so that matrix \( \mathbf{U} \) is a non zero matrix which is multiplied with \( p \hat{\mathbf{d}} \) to obtain CRE as:
\[ \text{CRE} = \mathbf{U} \mathbf{B}^{-1} \mathbf{w} - \hat{\lambda} \mathbf{w} \]

The above equation is written on the assumption that the channel distortion is very less than unity.

It is known that optimal value of channel distortion parameter can be obtained on minimizing CRE. Hence, on differentiating the above equation to obtain optimum value of \( \lambda \), we get:
\[ \hat{\lambda}_{opt} = \frac{ \left( UB^{-1} w \right) \left( UB^{-1} w^* \right) }{ \left[ UB^{-1} w^* \right] } \]

Using above two equations, minimum CRE also gives the optimum value of CFO estimation parameter which is shown as:
\[ \epsilon_{opt} = \min \left( \frac{ \left[ UB^{-1} w \right] - \left( UB^{-1} w \right) }{ \left[ UB^{-1} w^* \right] } \right) \]
### 4.2 Estimation Using Two Repeated Training Sequences

When there are two consecutive repeated training sequences are present in an OFDM block, their individual optimum values are calculated first and then their average is taken out which is considered as optimum value of channel distortion or CFO as a whole. This method of estimation of channel distortion and CFO is called two step estimation method because in the first step initial values are calculated and in the next step these values are tracked to achieve more accurate results. The two received vectors of form $\frac{M}{2} \times 1$ are written as:

$$w_a = p y + q y^* + n_a$$
$$w_b = p e^{j \pi \lambda} y + q (e^{j \pi \lambda} y)^* + n_b$$

Where, $a$ and $b$ are the two block indices.

**Step 1:** To obtain the initial values of channel distortion and CFO estimation parameters, consider $\lambda = 0$. For practical systems, it is very close to zero. So $\epsilon = \frac{\pi}{\lambda} \angle \{w_a, w_b\}$ obtained using optimum CFO equation as described earlier. Now, the initial value of $\lambda$ is obtained on substituting this value in the above given equation of optimum value of channel distortion. At last their average is taken as:

$$\lambda = \frac{1}{2} (\lambda_a + \lambda_b)$$

**Step 2:** If the initial estimate of $\epsilon$ is out of the range of $[-1, 1]$ that means the frequency offset is high. Therefore, after subtracting its integer part from the previously estimated value, its value is again estimated using the same procedure as described in step 1. The channel distortion is calculated then by:

$$\lambda_a = \frac{\| UB^{-1} w \| ( UB^{-1} w^* )^* }{ \| UB^{-1} w^* \|^2 }$$
$$\lambda_b = \frac{\| UB^{-1} w \| ( UB^{-1} w^* )^* }{ \| UB^{-1} w^* \|^2 }$$

On taking the average of these two, the resultant channel distortion parameter is obtained.

### V. SIMULATION RESULTS AND DISCUSSION

The performance of the proposed method of estimation of CFO and Channel Distortion of MIMO-OFDM System is evaluated through simulations. For Estimation of CFO and Channel Distortion for MIMO-OFDM system, simulation parameters are taken as: the channel length $L = 64$, number of channels $M = 1024$, and length of the Cyclic Prefix is $L-1 = 63$. The modulation technique used is Phase Shift Keying (PSK), so the training data are PSK symbols and the noise is AWGN. The length of the training sequence is 64 units. The simulations results are shown in two cases as:

**Case 1:** $N_f = 3, \ N_r = 3, \ \epsilon = u[-1, 1]$  
**Case 2:** $N_f = 4, \ N_r = 1, \ \epsilon = u[-3, 3]$  

Where, $N_i$ is the number of transmitting antennas and $N_r$ is the number of receiving antennas. For simulation, different number of transmitting and receiving antennas are used, but remaining all parameters are same for both the cases.

### 5.1 CHANNEL DISTORTION FOR MIMO-OFDM SYSTEM

![Fig. 4: Mean Square Error of channel distortion in the case 1](image)

![Fig. 5: Mean Square Error of channel distortion in the case 2](image)
Case 1 and Case 2. In Case 1; 3 transmit and 3 receiver antennas are used and in Case 2; 4 transmit and 1 receiver antennas are used. On the x-axis is Mean Squared Error (MSE) and on y-axis Signal to Noise Ratio (SNR). From these figures, it is observed that proposed estimation method provide a good performance and is robust for different types of mismatch parameters. It is also shown that the proposed method outperform the Channel Distortion- CFO method in both the given cases.

5.2 CFO ESTIMATION FOR MIMO-OFDM SYSTEM

The above figures 6 and 7 show the MSE of the estimation of CFO. It is noted that MSE of Case 2 is higher than MSE of Case 1. This is so because, there are more transmit antennas and more channels in the estimation. The MSE is defined as the average error within an OFDM block. In Case 1; the channel distortion- CFO estimation method performs well with smaller value of SNR. In Case 2; the method which was proposed shows better results.

VI. CONCLUSION

In this paper, CFO acquisition and tracking algorithms are proposed for MIMO-OFDM systems to improve the estimation accuracy. When only one OFDM block is available for training, the first method is able to give an accurate estimation of CFO and Channel Distortion. When two repeated training sequences are available in OFDM block, low complexity two step approach is used to solve the estimation problem. In future, comparison and analysis of different values of CRE can be done for various channels, to choose the best channel for transmission.

REFERENCES


