Design of two-channel QMFB using Marquardt optimization method with cosine modulation

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Abstract**—***Marquardt optimization method is used with the cosine modulation to design the two channel Quadrature mirror filter bank (QMFB). A low pass FIR filter is designed using the above method and then the other filters for the QMF is designed using this low pass prototype filter. Objective function that has to be minimized in this paper is an equation formed by summing the pass-band error, stop-band residual energy, square error of the filter bank at Π/2 and reconstruction ripple. As compared to the existing method the proposed algorithm gives better performance in terms of number of iteration required and the total computation time used by CPU.*

Keywords**— *CMFB, cosine modulation, NOI, NPR, Unconstrained optimization method.***

I. INTRODUCTION

QMF banks has a very wide range of applications such as in sub-band coding of speech and image signals [1–6], speech and image compression [7,8], trans-multiplexers [9–11], equalization of wireless communication channels [12], source coding for audio and video signals [13], design of wavelet bases [14], sub-band acoustic echo cancellation [15], and discrete multi-tone modulation systems [16].

The most frequently used filter bank among all the M-band filter banks is cosine-modulated filter bank (CMFB) because the designing is easy and more realizable than that of any other filter banks [3, 6]. In CMFB all the filters are cosine modulated version of a low-pass prototype filter. So the design of whole filter bank reduces to the design of a single prototype filter. Implementation of CMFB consists of one prototype and a discrete cosine transform (DCT). Near perfect reconstruction (NPR) finite impulse response (FIR) CMFB avoids computation of large matrix sets. There are two types of CMFB one is with perfect reconstruction [7] and the other is pseudo-QMF [8]. Unlike PR filter banks, in pseudo-QMF aliasing is canceled approximately and the distortion is approximately a delay [3, 6] and the approximation improves with the filter order. It could be also called a special case of perfect reconstruction QMF bank.

It is found in many signals that their energy is dominantly concentrated in a particular region of frequency. To save the bandwidth in such signals, signal can be compressed and decimated. But simple way of compression affects the quality of the signal. So Quadrature Mirror Filter (QMF) comes as a solution to this problem as it saves the bandwidth and increases efficiency without compromising the quality of the signal.

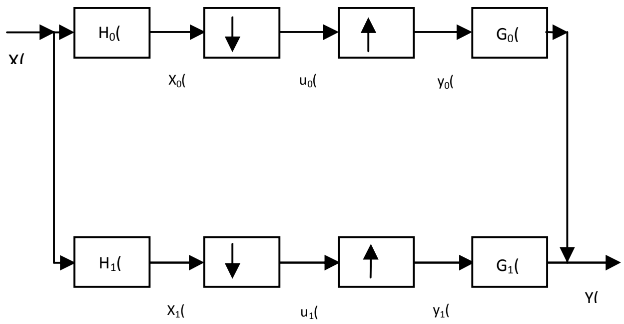

Fig.1: Basic structure of two channel QMF

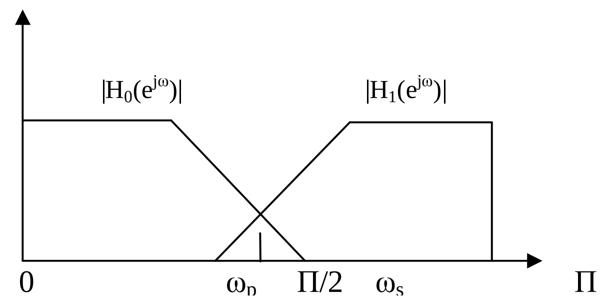

Fig.2: Frequency response of the filter

Figure.1. shows the typical two channel QMF bank and figure.2. shows the frequency response of this filter. This filter splits the signal into two sub-bands using high-pass and low-pass filters $H_1(Z)$ and $H_0(Z)$ respectively. These signals are decimated, coded and transmitted on the transmitter side. These signals are decoded, interpolated and then finally passed though the filters to recombine the signals. The reconstructed signal is not exact replica of the transmitted signal, it suffers from three types of distortions: amplitude distortion (AMD), phase distortion (PHD) and aliasing distortion (ALD).

Most of the researchers stresses on the elimination or minimization of these distortions to obtain the perfect

reconstruction (PR) or near perfect reconstruction (NPR) system [4].
Aliasing can be cancelled completely by choosing the appropriate synthesis filters with respect to the analysis filter phase distortion can be eliminated by using the linear phase FIR filters [5-6]. Amplitude distortion can be reduced using computer aided techniques [7].
Most of the researchers stresses on the elimination or minimization of these distortions to obtain the perfect reconstruction (PR) or near perfect reconstruction (NPR) system [8].

II. ANALYSIS OF QMFB

General equation for the two channels QMF is given as

$$Y(Z)=\frac{1}{2}[H_0(Z)G_0(Z)+H_1(Z)G_1(Z)]X(Z)+\frac{1}{2}[H_0(-Z)G_0(Z)+H_1(-Z)G_1(Z)]X(-Z) \quad (1)$$

The second term in the above equation represents the alias component in the output.
Alias component X(-Z) can be removed by choosing the synthesis filter such that to make the second term zero such as

$$G_0(Z)=H_1(-Z) \text{ and } G_1(Z)=-H_0(-Z) \quad (2)$$

After removing the alias component from the equation (1) overall transfer function is given by

$$T(Z)=\frac{Y(Z)}{X(Z)}=\frac{1}{2}[H_0(Z)G_0(Z)+H_1(Z)G_1(Z)] \quad (3)$$

Y(n) suffers from amplitude distortion (AMD)(if |T(Z)| is not constant for all value of ω,) and it would suffer from phase distortion (PHD)(if T(Z) doesn't has linear phase). So to eliminate AMD |T(Z)| should be all pass and to eliminate PHD T(Z) should be linear phase.
When all the filters involved are linear-phase filters transfer function of the QMF T(Z) would behave like linear phase filter that would remove the phase distortion in the QMF.
The relation of analysis and synthesis to each other is given as [9-10]

$$H_1(Z)=H_0(-Z), G_0(Z)=H_0(Z), G_1(Z)=-H_1(Z)=-H_0(-Z) \quad (4)$$

Using equation (3) and (4), we get

$$T(Z)=\frac{1}{2}\left[H_0^{\,2}(Z)-H_0^{\,2}(-Z)\right] \quad (5)$$

So it's clear from the equation (5) now to get the linear-phase transfer function we need to have linear-phase low-pass linear phase filter.
One of the most frequently used filter bank among all the M-band filter banks is cosine-modulated filter bank (CMFB) because the designing is easier and more realizable than that of other filter banks [11-12].
It is one of the efficient techniques which provide minimum computational cost for the design of filter banks. In CMFB all the filters are cosine modulated version of a low-pass prototype filter. In this technique all the analysis filters and synthesis filters are simultaneously generated by the low-pass prototype filter. So the design of whole filter bank reduces to the design of a single prototype filter [13-18]. Implementation of CMFB consists of one prototype and discrete cosines transform (DCT).
All of these filters can be designed using either the near perfect reconstruction (NPR) or perfect reconstruction (PR) [19]. Many prominent authors has been studied the cosine modulated filter banks with PR and NPR conditions and find that NPR type is more realizable, computationally efficient and more simple in comparison to the PR [14],[20-22].

III. DESIGN USING PROPOSED ALGORITHM

Cosine modulated-QMF bank is designed in two steps, in first step a low pass prototype filter is designed selecting appropriate objective function and applying suitable optimization method. In the second step cosine modulation is applied and all other filters are obtained using the low pass prototype filter. Here objective function used in this paper is an equation that is combination of pass-band error, stop-band residual energy, square error of the filter bank transfer function and reconstruction ripple denoted by E_p, E_s, E_t and rr respectively.
Objective function B_i is given as
B_i=y+t+v+n; (6)
Where, y= L*E_p, t=j*E_s, v=k*E_t and n=g*rr
Where L, j, k and g are constants.
Pass-band error is expressed as

$$E_p = b_i^{\,T} * q_1 * b_i \quad (7)$$

Where

$$q_1 = \frac{1}{\pi}\int_0^{\omega_p}(1-c)(1-c)^T\,d\omega \quad (8)$$

E_s is stop-band residual energy of the prototype filter which is given by

$$E_s = b_i * p_1 * b_i^{\,T} \quad (9)$$

Where

$$p_1 = \frac{1}{\pi}\int_{\omega_s}^{\pi} c * c^T\,d\omega \quad (10)$$

E_t is the square error of the prototype filter given as

$$E_t = \left(\left(b_i^{\,T} * d\right) - H_r 1\right)^2 \quad (11)$$

Where

$$c = [\cos\omega((N-1)/2)\cos\omega((N-1)/2-1)....\cos(\omega/2)]^T \quad (12)$$

d=c evaluated at $\omega = \pi/2$, H_{r1} is the amplitude function and $H_{r1}=(1/2)^{1/2}H_{r0}$.

Where $H_{r\omega}=b_i*c$

rr is the reconstruction ripple of the prototype filter

$$rr=\max_{\omega}\left|10*\log\left|T_0\left(e^{j\omega}\right)\right|\right|-\min_{\omega}\left|10*\log\left|T_0\left(e^{j\omega}\right)\right|\right| \quad (13)$$

Objective function defined by the equation (6) is optimized using Marquardt optimization method.

Cosine modulated filter bank

After designing the low-pass filter all filters of analysis and synthesis sections are obtained by cosine modulation of the prototype filter using the following relations.

$$H(k,n)=2*(real(a(k))*n1+imag(a(k))*n2)*h(n) \quad (14)$$

Where n1 and n2 is given as

$$n1=\cos(pi*(2*k-1)*(2*n-1)/(4*nbands)) \quad (15)$$

$$n2=\sin(pi*(2*k-1)*(2*n-1)/(4*nbands)) \quad (16)$$

Where 'nbands' is equals to the number of sub-bands in the Quadrature mirror filter.

There are two main advantages of cosine modulated filter banks are, first one is the cost of whole analysis filters reduces to the cost of one prototype filter plus the cost of modulating and second one is we need not optimize so many parameters. The number of parameters need to be optimize becomes fewer.

IV. OPTIMIZATION ALGORITHM

Here we are optimizing the objective function using Marquardt optimization method. If b_t is the minimum value at the t^{th} step then b_{t+1} at $t+1^{th}$ step can be calculated by using Marquardt optimization method as follows

$$b_{t+1}=b_t-[J_t]^{-1}\nabla\phi_t \quad (17)$$

where

$$J_t=H_t+a_tI, \quad (18)$$

Where $\nabla\phi_t$ is the gradient of the objective function, H_t is the Hessian matrix, a_t is a t^{th} stage iteration constant and I is N/2 x N/2 identity matrix.

where

$$H_t=2(\phi+\beta D) \quad (19)$$

where β is a constant and $D=d*d^T$

$d=\cos(\omega(((N-1)/2)-N/2))$ at $\omega=\pi/2$)

and $\nabla\phi_t=2(\phi+\beta D)h$ (20)

Steps for optimization

(1) Set initial values of α, e_1, $a>1$, $b<1$, and N.

(2) Set initial design vector $h_0=[h_0(0)h_0(1)h_0(2).....h_0(N/2)-1]T$ such that energy is unit remains within a specified tolerance, i.e.,

$$u=\left|1-2\sum h_0^2(k)\right|<e_1 \quad (21)$$

(3) Set the iteration number t= 0, and $b_t = 2h_t$.

(4) Compute $\nabla\phi_t$ using Eq. (12) at the design vector b_t .

(5) Compute the Hessian matrix H using Eq. (19) and the matrix J_t using Eq. (18).

(6) Compute the new or improved approximation

$$b_{t+1}=b_t-[J_t]^{-1}\nabla\phi_t$$

(7) Compute the value of u, at the point b_{t+1}, and if value is not satisfied then choose the optimum point as b_t , stop the procedure and go to step (10).

(8) If the condition is satisfied at the point b_{t+1}, compute the objective function ϕ_{t+1} at the point b_{t+1}. If $\phi_{t+1}\geq\phi_t$, choose the optimum point as b_t , stop the procedure and go to step (10). If $\phi_{t+1}<\phi$, set $\phi_t=\phi_{t+1}$ and $b_t = b_{t+1}$.

(9) Set the new iteration number as t = t +1, $a = a \times b$ and go to step (3).

(10) Compute $h_0 = (1/2)b_t$ and stop the procedure.

V. RESULT AND DISCUSSION

A new two channel Quadrature mirror filter with a new objective function is designed and compared with the previous one. Here we see that performance of filter with new objective function is even better in terms of CPU time and number of iteration required in the optimization.

Fig. 3: Gain of the analysis filter

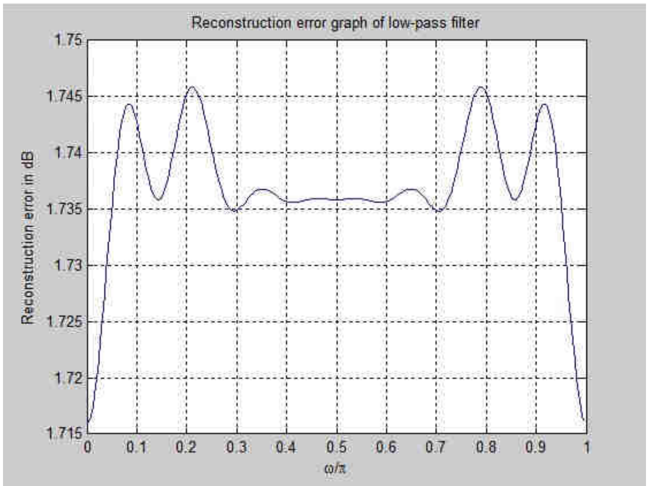

Fig. 4: Reconstruction error graph of the filter

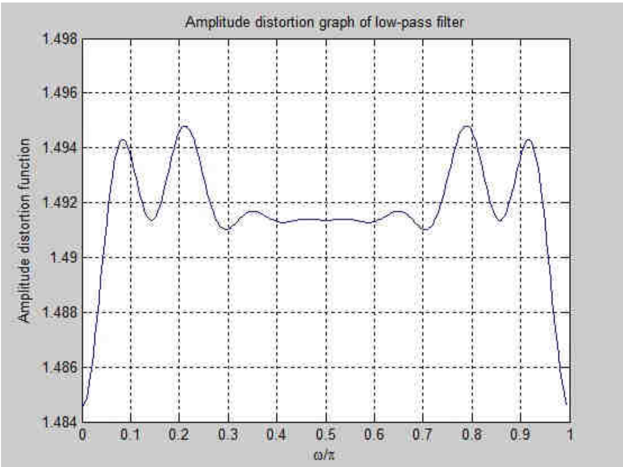

Fig. 5: Amplitude distortion graph of the filter

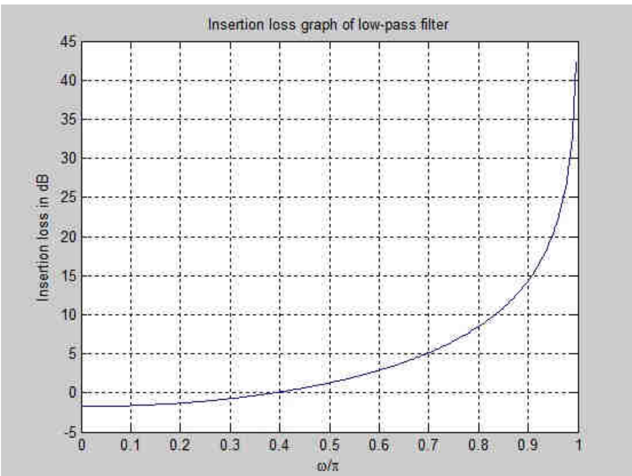

Fig. 6: Insertion loss graph of filter

Table 1Performance parameter of the designed QMFB

Filter order	Bands	CPU time	NOI
20	2	0.109082	34
32	2	0.012186	18
40	2	0.008717	15
60	2	0.007720	10

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