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Formulation of Approximate, Generalized Field Data Based Mathematical Models, and Its Reliability Evaluation, Optimization and Sensitivity Analysis for PVC Manufacturing Process

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Abstract**—***This paper describes an approach for formulation of approximate, generalized field data based mathematical model (FDBM) for the process of PVC pipe manufacturing at some industries. The present work is aimed at establishing mathematical relationship between the responses and inputs at the operation of PVC pipe manufacturing process using single screw extruder. For this purpose various small scale PVC pipes manufacturing industries are visited. The operation of PVC pipe extrusion is studied. The study is focused on an extrusion line starting from its electric motor, extruder hopper, barrel, extruder screw up to the extruder die. First of all the various dependent variables in form of responses and the independent variables in the form of inputs are decided. The categorization of these variables are made in terms of pi terms viz. Π1 Π2 Π3 as independent and ΠD1, ΠD2, ΠD3, ΠD4 as dependent variables. Then the field observations are taken and accordingly data collection process is completed. After this step, an approximate, generalized field data based mathematical models are developed. This work presents an approach to check the reliability of models, which is executed by comparing error frequency graphs of various mathematical models formed. After that the influence of the various independent pi terms in the models are studied by analyzing the indices of the various pi terms. Through the technique of sensitivity analysis, the change in the value of a dependent pi term caused due to an introduced change in the value of independent individual pi term is evaluated. The ultimate objective of this work is not merely developing the mathematical models but to find out the best set of independent variables, which will result in maximization or minimization of the objective functions. This is achieved by applying the technique of optimization. Thus the objects of these models are tested to optimize the inputs required for satisfying the various responses. The comparative analysis is made of the outputs of the network with observed data and the data calculated from the mathematical models. This modeling and simulation approach enables entrepreneur of small scale PVC pipes manufacturing industries to get system wide view obtained by deliberately making local changes in their manufacturing system. They can predict its impact on performance of their machines. With the help of the models, one can find a method to improve the productivity of the industry. The results obtained from experiments are also analyzed by the development of different polynomial mathematical models and its related graphs. Recommendations with respect to improvement in the current operation are suggested and future changes are proposed.*

Keywords—Field databased mathematical modeling, sensitivity analysis, and optimization

I. INTRODUCTION TO PVC PIPE MANUFACTURING PROCESS

The extruder can be considered as one of the core piece of machinery in the polymer processing industry. To extrude means to thrust out a polymer material in any kind of form with a desired cross section through a die. The shape of material will depend on the die opening, and it will change to some extent as it exits from the die. The extruded output is commonly referred to as the extrudate [3]. Pipe and profile production line consists (figure1) of an extruder which is equipped with a die depending on the end product and also a calibration device [8]. After the extruder, the material is run through a cooling pool, nip rolls and a cutting saw. During the manufacturing process of PVC pipes, the molten plastic from extruder is led to a

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circle die towards the calibration device, where the final shape and size are determined [8].

Fig. 1: PVC pipe Manufacturing

Most extruders are equipped with screw as their main mixing component. Screw extruders are classified as single or multiple screw extruders. Single screw extruders are the most common type of extruders used in the polymer industry, because of its straight forward design, relatively low costing, and its reliability, they are most often used (Mad-dock, [4]). The extruder is a very important machine in the plastic industry, as compared to injection molding machines, they are used to produce product of continues profile. Extruder's main components are a hopper at one end from which the material to be extruded is fed, a tubular barrel, usually electrically heated; a revolving screw, ram or plunger within the barrel, and a die at the opposite end for shaping the extruded mass [5]. Extruders may be divided into three general types—single screw, twin or multiple screw, and ram—each type has several variations. The different part that makes up an extruder is reviewed below in figure 2.

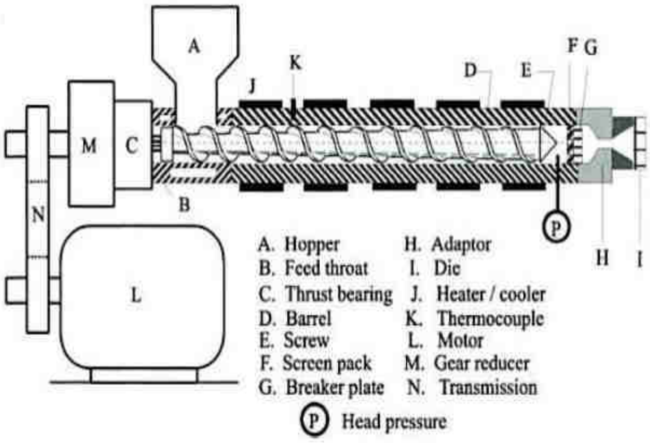

Fig. 2: Component of a typical single screw extruder

A typical extrusion line (figure 3) consists of the material feed hopper, basic extruder (drive, gearbox and screws), the extrusion die, the calibration units, the haul-off, the saw (or other cutting device), and finally the treatment devices for final finishing and handling [1].

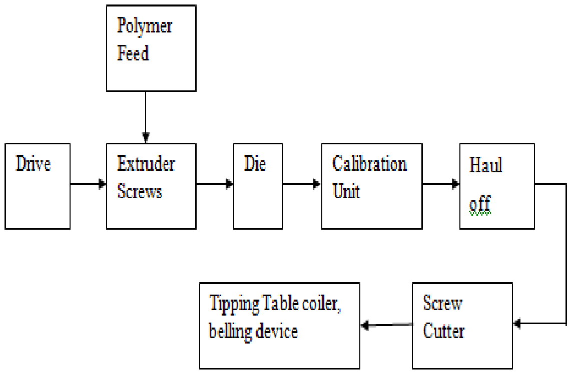

Fig. 3: Process flow of Extrusion Line

The hopper holds the raw plastic material (in either powder or granule form) and continuously feeds this into the extruder, which has a heated barrel containing the rotating screw. This screw transports the polymer to the die head and simultaneously the material is heated, mixed, pressurized and metered. At the die the polymer takes up the approximate shape of the article and is then cooled either by water or air to give the final shape. As the polymer cools it is drawn along by haul-off devices and either coiled (for soft products) or cut to length (for hard products).

II. METHODOLOGY FOR FORMULATION OF APPROXIMATE, GENERALIZED FIELD DATA BASED MATHEMATICAL MODEL

When one is studying any completely physical phenomenon but the phenomenon is very complex to the extent that it is not possible to formulate a logic based model correlating causes and effects of such a phenomenon, then one is required to go in for the field data based models. In view of the dynamic nature of the context under investigation (which reveals complex phenomenon), it is decided that to formulate a field data based models for dependent process parameters ($\Pi D1$), pipe dimensions ($\Pi D2$), pipe weight ($\Pi D3$) and productivity ($\Pi D4$). These models are established adopting methodology of experimentation [5]. It is planned to collect the data by taking extensive observations in the process of PVC pipe extrusion process by actually visiting and PVC pipe manufacturing industries. The planning is carried out by using the classical plan of experimentation [6]. The response data is collected based on the entire generalized models. The approach adopted for formulating approximate, generalized field data based mathematical model suggested by Hilbert (1961) for any complex phenomenon involves following steps

1. Identification of independent and dependent variables or quantities.

2. Reduction of independent variables adopting dimensional analysis.
3. Formulation of the model.
4. Reliability of the model.
5. Model optimization.
6. Sensitivity Analysis of the models.

Identification of variables

The term variables are used in a very general sense to apply any physical quantity that undergoes change. If a physical quantity can be changed independent of the other quantities, then it is an independent variable. If a physical quantity changes in response to the variation of one or more number of independent variables, then it is termed as dependent or response variable. The dependent or the response variables in this case are: Dependent process parameters symbolically represented as ΠD1, PVC pipe dimensions symbolically represented as ΠD2, Weight of the PVC pipes symbolically represented as ΠD3, Productivity symbol symbolically represented as ΠD4. There are many independent variables involved in this system. grouped in as: variables related with electric motor used symbolically represented as Π1, specifications of the extruder machine symbolically represented as Π2, quantity and quality of the raw material related used symbolically represented as Π3.

Table 1: Independent variables related with electric motor

Sr. No	**Code**	**Name of the independent variables**	**MLT indices**	**Type of variable**	**Remark**
1	C1	Motor Power (HP)	$M^1L^2 T^{-3}$	Independent	Electric motor related variables (Π1)
2	C2	weight of the electric motor	$M^1 L^0 T^0$	Independent	
3	C3	Distance of electric motor from the extruder machine	$M^0 L^1 T^0$	Independent	
4	C4	Motor speed (RPM)	$M0L^1 T^{-1}$	Independent	
5	C5	Torque (N-m)	$M^1 L^2 T^{-2}$	Independent	
6	C6	Acceleration due to gravity m/s^2	$M^0 L^1 T^{-2}$	Independent	

Table 2: Independent variables related with specifications of the extruder machines

Sr. No	**Code**	**Name of the independent variables**	**MLT indices**	**Type of variable**	**Remark**
1	A1	Extruder machine length (mm)	$M^0 L^1 T^0$	Independent	Extruder machine specifications (Π2)
2	A2	Extruder machine width (mm)	$M^0 L^1 T^0$	Independent	
3	A3	Extruder machine height (mm)	$M^0 L^1 T^0$	Independent	
4	A4	Barrel centerline fr. floor (mm)	$M^0 L^1 T^0$	Independent	
5	A5	Hopper capacity (kg)	$M^1 L^0 T^0$	Independent	
6	A6	Hopper height (mm)	$M^0 L^1 T^0$	Independent	
7	A7	Screw outside diameter (mm)	$M^0 L^1 T^0$	Independent	
8	A8	Screw inner diameter (mm)	$M^0 L^1 T^0$	Independent	
9	A9	Screw pitch (mm)	$M^0 L^1 T^0$	Independent	
10	A10	Barrel length (mm)	$M^0 L^1 T^0$	Independent	
11	A11	Barrel diameter (mm)	$M^0 L^1 T^0$	Independent	
12	A12	Weight of the extruder m/c (kg)	$M^0 L^1 T^0$	Independent	
13	A13	Die diameter or size (mm)	$M^0 L^1 T^0$	Independent	
14	A14	Die length (mm)	$M^0 L^1 T^0$	Independent	

Table 3: Independent variables related with raw material

Sr.No	**Code**	**Name of the independent variables**	**MLT indices**	**Type of variable**	**Remark**
1	W1	Resin wastage (kg)	$M^1 L^0 T^0$	Independent	
2	W2	Dust (kg)	$M^1 L^0 T^0$	Independent	Raw materia

3	W3	filter (gm)	$M^1 L^0 T^0$	Independent	l related variables or data (Π3)
4	W4	Chemical wax (kg)	$M^1 L^0 T^0$	Independent	
5	W5	TBLS powder (gm)	$M^1 L^0 T^0$	Independent	
6	W6	Steric acid (gm)	$M^1 L^0 T^0$	Independent	
7	W7	Wastage raw mat. size (mm)	$M^0 L^1 T^0$	Independent	
8	W8	Powder size (mm)	$M^0 L^1 T^0$	Independent	
9	W9	Filter material size (mm)	$M^0 L^1 T^0$	Independent	

Table 4: Dependent variables related process parameters

Sr.No	Code	Name of the independent variables	MLT indices	Type of variable	Remark
1	D1	Screw speed (RPM)	$M^0L^1 T^{-1}$	Dependent	Dependent process parameters (ΠD1)
2	D2	Melt viscosity (N-s/m^2)	$M L^{-1} T^{-1}$	Dependent	
3	D3	Melt density (kg/m^3)	$M L^{-3}$	Dependent	
4	D4	Extruder pressure (MPA)	$M L^{-1} T^{-2}$	Dependent	
5	D5	Die pressure (MPA)	$M L^{-1} T^{-2}$	Dependent	
6	D6	Extruder temperature (^{0}C)	$M^0 L^0 T^0$	Dependent	
	D7	Die temperature (^{0}C)	$M^0 L^0 T^0$	Dependent	

Table 5: Dependent variables related with PVC pipe dimensions

Sr.No	Code	Name of the independent variables	MLT indices	Type of variable	Remark
1	P1	Pipe diameter (mm)	$M^0 L^1 T^0$	Dependent	Pipe dimensions. (ΠD2)
2	P2	Pipe wall thickness (mm)	$M^0 L^1 T^0$	Dependent	

Table 6: Dependent variables related with weight of PVC pipe

Sr.No	Code	Name of the independent variables	MLT indices	Type of variable	Remark
1	Y1	Pipe weight (kg)	$M^1 L^0 T^0$	Dependent	Pipe weight (ΠD3)

Table 7: Dependent variables related with productivity

Sr.No	Code	Name of the independent variables	MLT indices	Type of variable	Remark
1	Y1	Processing time (sec)	$M^0 L^0 T^1$	Dependent	productivity (ΠD4)
2	T2	Productivity	$M^0 L^0 T^0$	Dependent	

Reduction of variables using dimensional analysis

Dimensional analysis was carried out to established dimensional equations, exhibiting relationships between dependent Π terms and independent Π terms using Buckingham Π theorem. Dimensional analysis can be used primarily as an experimental tool to combine many experimental variables into one [1]. The various independent and dependent variables of the system with their symbols and dimensional formulae are given in nomenclature. There are several quite simple ways in which a given test can be made compact in operating plan without loss in generality or control. The best known and the most powerful of these is dimensional analysis. In the past dimensional analysis was primarily used as an experimental tool whereby several experimental variables could be combined to form one. The field of fluid mechanics and heat transfer were greatly benefited from the application of this tool [3]. Almost every major experiment in this area was planned with its help. Using this principle modern experiments can substantially improve their working techniques and be made shorter requiring less time without loss of control. Deducing the dimensional equation for a phenomenon reduces the number of independent variables in the experiments [4]. The exact mathematical form of this dimensional equation is the targeted model. This is achieved by applying Buckingham's Π theorem [5] [6]. Following table summarizes reduction of variables into dimensionless pi terms by using technique of dimensional analysis.

Table 8: Reduction of independent and dependent variables into pi (Π) terms

Pi term	Code	Description of Pi terms	Pi term equation
Π1	C6	Acceleration due to gravity mm/s2	$\Pi 1 = \frac{C6 \text{ x } C3 \text{ x } C2}{C5}$
	C3	Distance of electric motor from the extruder machine	
	C2	Mass or weight of the electric motor (kg)	
	C5	Torque on electric motor (N-mm)	
Π2	A12	Weight of extruder machine (kg)	$\Pi 2 = \frac{A12}{A5} \text{ x } \frac{A1}{A2} \times \frac{A3}{A4} \text{ x } \frac{A7}{A8} \times \frac{A6}{A9} \text{ x } \frac{A10}{A11} \times \frac{A14}{A13}$
	A5	Hopper capacity (kg)	
	A1	Extruder machine length (mm)	
	A2	Extruder machine width (mm)	
	A3	Extruder machine height (mm)	
	A4	Barrel centerline from floor (mm)	
	A7	Screw outside diameter (mm)	
	A8	Screw inside diameter (mm)	
	A6	Hopper height (mm)	
	A9	Screw pitch (mm)	
	A10	Barrel length (mm)	
	A11	Barrel diameter (mm)	
	A14	Die length (mm)	
	A13	Die diameter or size (mm)	
Π3	W1	Resin wastage (kg)	$\Pi 3 = \frac{W9}{W7} \text{ x } \frac{W7}{W8} \times \frac{W5}{W6} \text{ x } \frac{W3}{W4} \times \frac{W1}{W2}$
	W2	Dust (kg)	
	W3	Filter (gm))	
	W4	Chemical wax (gm)	
	W5	TBLS powder (gm)	
	W6	Steric acid (gm)	
	W7	Wastage raw material size (mm)	
	W8	Powder size (mm)	
	W9	Filter material size (mm)	
ΠD1	D1	Screw speed (m/s)	$\Pi D1 = \frac{D4}{D1} \text{ x } \frac{D1}{D5} \times \frac{D2}{D3} \text{ x } \frac{D3}{D2} \times \frac{D7}{D6}$
	D2	Melt viscosity (N-s/m2) or (kg/m-s)	
	D3	Melt density (kg/m3)	
	D4	Extruder pressure (kg/ms2)	
	D5	Die pressure (kg/ms2)	
	D6	Extruder temperature (0C)	
	D7	Die temperature (0C)	
	D8	Pipe diameter or size (mm)	

	D9	Pipe wall thickness (mm)	
	D10	Pipe weight (kg)	
	D11	Processing time (sec)	
	D12	Productivity	
ΠD2	P1	Pipe diameter (mm)	ΠD2 $=\frac{P1}{P2}$
	P2	Pipe wall thickness (mm)	
ΠD3	Y1	Pipe weight (kg)	ΠD3 = Y1
	Y2	Processing time (sec)	
ΠD4	T2	Productivity	ΠD4 = $\frac{Y1}{Y2}$x Y2/Y1

The main purpose of this technique is making experimentation shorter without loss of control. As per dimensional analysis [1], response variables dependent process parameters (ΠD1), pipe dimensions (ΠD2), pipe weight (ΠD3) and productivity (ΠD4) are written in the function form as:

ΠD1 = f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ------------ (1)

ΠD2 = f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ------------ (2)

ΠD3 = f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ----------- (3)

ΠD4 = f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ---------- (4)

Three independent pi terms (Π1, Π2, Π3,) and four dependent pi terms (ΠD1, ΠD2, ΠD3, ΠD4,) have been in the design of experimentation and are available for the model formulation. Independent Π terms = (Π1, Π2, Π3), Dependent Π terms = (ΠD1, ΠD2, ΠD3, ΠD4,). Each dependent Π is assumed to be function of the available independent Π terms, For Process Parameters (ΠD1), ΠD1 = k1 x $(\Pi1)^{a1}$ x $(\Pi2)^{b1}$ x $(\Pi3)^{c1}$ ------------ (5)

For Pipe Dimensions (ΠD2), ΠD2 = K2 x $(\Pi2)^{a2}$ x $(\Pi2)^{b2}$ x $(\Pi3)^{c2}$ ------------ (6)

For Pipe Weight (ΠD3), ΠD3 = k3 x $(\Pi3)^{a3}$ x $(\Pi3)^{b3}$ x $(\Pi3)^{c3}$ ------------ (7)

For Productivity (ΠD4), ΠD4 = k4 x $(\Pi4)^{a4}$ x $(\Pi4)^{b4}$ x $(\Pi4)^{c4}$ ------------ (8)

Procedure for developments of the model for dependent pi term ΠD1 in general form

ΠD1 = k1 x $(\Pi1)^{a1}$ x $(\Pi2)^{b1}$ x $(\Pi3)^{c1}$

Taking log on the both sides of equation for ΠD1, getting eight unknown terms in the equations,

Log ΠD1 = log k1+ a1log Π1+ b1log Π2+ c1log Π3 ------- (9)

Let, Z1= log ΠZ1, K1 = log k1, A = log Π1, B = log Π2 C = log Π3

Putting the values in equations 9, the same can be written as

Z1 = K1+ a1 A + b1 B + c1 C ----- (10)

Equation 9 is a regression equation of Z on A, B, and C in an n dimensional co-ordinate system.

This represents a regression hyper plane [7]. To determine the regression hyper plane, determines a1, b1, c1, d1, e1 and f1 in equation 9 so that

∑Z1 = nK1 + a1*∑A + b1*∑B + c1*∑C

∑Z1*A = K1*∑A +a1*∑A*A + b1*∑B*A + c1*∑C*A

∑Z1*B = K1*∑B +a1*∑A*B + b1*∑B*B + c1*∑C*B

∑Z1*C = K1*∑C +a1*∑A*C+ b1*∑B*C + c1*∑C*C

In the above set of equations the values of the multipliers K1, a1, b1, c1 are substituted to compute the values of the unknowns (viz. K1, a1, b1, and c1). The values of the terms on L H S and the multipliers of K1, a1, b1, and c1 in the set of equations are calculated. After substituting these values in the above equations one will get a set of 4 equations, which are to be solved simultaneously to get the values of K1, a1, b1, c1,. The above equations can be verified in the matrix form and further values of K1, a1, b1, c1, can be obtained by using matrix analysis.

X1 = inv (W) x P1 ---------- (11)

The matrix method of solving these equations using 'MATLAB' is given below. W = 4 x4 matrix of the multipliers of K1, a1, b1, c1, d1, e1, f1 and g1 P1 = 4 x 1 matrix of the terms on L H S and X1 =4 x 1 matrix of solutions of values of K1, a1, b1, and c1 Then, the matrix obtained is given by, matrix

$$Z_1 \quad x \begin{bmatrix} 1 \\ A \\ B \\ C \end{bmatrix} = \begin{bmatrix} n & A & B & C \\ A & A^2 & BA & CA \\ B & AB & B^2 & CB \\ C & AC & BC & C^2 \end{bmatrix} x \begin{bmatrix} K_1 \\ a_1 \\ b_1 \\ c_1 \end{bmatrix}$$

Development of model for dependent pi term ΠD1 in clubbed Form

In this type of model all the Pi terms i.e.π_1,π_2,π_3 are multiplied (clubbed) together and then using regression analysis mathematical model is formed.

The mathematical clubbed model is shown below. For the dependent pi term ΠD1, we have,

ΠD1 = f (Π1*Π2*Π3) ^{a1}, Where 'f' stands for "function of" and a probable exact mathematical form for this phenomenon could be

ΠD1 = K1 * (Π1*Π2*Π3) ^{a1} ------------ (1)

The same procedure as mentioned above is adopted for response variables and after solving this matrix in MATLAB we can get the mathematical model

Development of models for dependent pi term in various combination forms as

In this type of model all the Pi terms i.e. π_1, π_2, π_3 are taken together in various combinations and multiplied (clubbed) together and then using regression analysis mathematical model is formed.

Model – 1, ΠD1 = f (Π1 * Π2 / Π3) ^{a}

Model – 2, ΠD1 = f (Π2 * Π3 / Π1) ^{a}

Model – 3, ΠD1 = f (Π1 * Π3 / Π2) ^{a}

There are two unknown terms in the equation (1), viz. constant of proportionality K1 and indices a1. It is decided to solve this problem by curve fitting technique. The same procedure as mentioned above is adopted for response variables and after solving this matrix in MATLAB we can get the mathematical model. The various models developed for all the response variables are shown in following table.

Table 9: Models developed for dependent pi term ΠD1

Sr No	Forms of models	Mathematical Equation of the model
1	General Form	$(\pi_{D1}) = 23.77935114.\{(\pi_1)^{0.4389}.(\pi_2)^{-0.7474}.(\pi_3)^{0.4155}\}$
2	Clubbed Model	$(\pi_{D1}) = 1.41.\{(\pi_1.\pi_2.\pi_3)^{0.0494}\}$
3	Combination Model 1	$(\pi_{D1}) = 1559.19.\{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{-0.4237}$
4	Combination Model 2	$(\pi_{D1}) = 3.44\ \{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{0.0507}$
5	Combination Model 3	$(\pi_{D1}) = 1.79\{\left(\frac{\pi_1.\pi_3}{\pi_2}\right)^{0.2231}$

Table 10: Models developed for dependent pi term ΠD2

Sr No	Forms of models	Mathematical Equation of the model
1	General Form	$(\pi_{D2}) = 4.87E+01.\{(\pi_1)^{0.4416}.(\pi_2)^{-0.4445}.(\pi_3)^{0.1504}\}$
2	Clubbed Model	$(\pi_{D2}) = 5.59E+01.\{(\pi_1.\pi_2.\pi_3)^{-0.0017}\}$
3	Combination Model 1	$(\pi_{D2}) = 2.45E+02.\{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{-0.1147}$
4	Combination Model 2	$(\pi_{D2}) = 6.08E+01\{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{-0.0136}$
5	Combination Model 3	$(\pi_{D2}) = 3.83E+01\{\left(\frac{\pi_1.\pi_3}{\pi_2}\right)^{0.0654}$

Table 11: Models developed for dependent pi term ΠD3

Sr No	Forms of models	Mathematical Equation of the model
1	General Form	$(\pi_{D3}) = 3.15E+00.\{(\pi_1)^{0.6958}.(\pi_2)^{-0.8655}.(\pi_3)^{0.5223}\}$

2	Clubbed Model	$(\pi_{D3}) = 2.42E-01.\{(\pi_1.\pi_2.\pi_3)^{0.1037}\}$
3	Combination Model 1	$(\pi_{D3}) = 1.64E+04.\{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{-0.6176}$
4	Combination Model 2	$(\pi_{D3}) = 1.48E+00.\{\left(\frac{\pi_2.\pi_3}{\pi_1}\right)^{0.1159}$
5	Combination Model 3	$(\pi_{D3}) = 6.72E-01.\{\left(\frac{\pi_1.\pi_3}{\pi_2}\right)^{0.3706}$

Table 12: Models developed for dependent pi term ΠD4

Sr No	**Forms of models**	**Mathematical Equation of the model**
1	General Form	$(\pi_{D4}) = 4.33E-01.\{(\pi_1)^{-0.0572}.(\pi_2)^{0.0948}.(\pi_3)^{-0.3847}\}$
2	Clubbed Model	$(\pi_{D4}) = 9.40E+00.\{(\pi_1.\pi_2.\pi_3)^{-0.1911}\}$
3	Combination Model 1	$(\pi_{D4}) = 4.96E-06.\{\left(\frac{\pi_1.\pi_2}{\pi_3}\right)^{-0.6176}$
4	Combination Model 2	$(\pi_{D4}) = 4.77E-01.\{\left(\frac{\pi_2.\pi_3}{\pi_1}\right)^{-0.2474}$
5	Combination Model 3	$(\pi_{D4}) = 4.63E-01.\{\left(\frac{\pi_1.\pi_3}{\pi_2}\right)^{-0.4556}$

Co-efficient of determination for all response variables

This is a statistical method that explains how much of the variability of a factor can be caused or explained by its relationship to another factor. Coefficient of determination is used in trend analysis. It is computed as a value between 0 (0 percent) and 1 (100 percent). Higher the value; better is the fit. Coefficient of determination is symbolized by r^2 because it is square of the coefficient of correlation symbolized by r. The coefficient of determination is an important tool in determining the degree of linear-correlation of variables ('goodness of fit') in regression analysis and also called r-square. It is calculated using relation shown below:

$R^2 = 1- \sum Yi\text{-}fi)^2 / \sum (Yi\text{-}Y)^2$...... (1)

Where, Yi= Observed value of dependent variable for i^{th} Experimental sets (Experimental data), fi = Observed value of dependent variable for i^{th} predicted value sets (Model data), Y= Mean of Yi and R^2 = Co-efficient of Determination. Following tables shows these calculations. Following table gives the values of coefficient of determination of all the models.

Table 13: Co-efficient of determination for all models

Response Variable	**Value of R^2 for General Model**	**Value of R^2 for Clubbed Model**	**Value of R^2 for various forms of Model**		
	(π_D) =kx $(\pi_1)^a$x $(\pi_2)^b$x $(\pi_3)^c$	**(π_D) =kx $(\pi_{1x}\pi_{2x}\pi_3)$**	**(π_D) =kx $(\pi_{1x}\pi_2/\pi_3)$**	**(π_D) =kx $(\pi_{2x}\pi_3/\pi_1)$**	**(π_D) =kx $(\pi_{1x}\pi_3/\pi_2)$**
ΠD1	**0.997372**	0.97811	0.983866	0.971888	0.988601
ΠD2	**0.9891**	0.982517	0.983513	0.980564	0.986903
ΠD3	**0.9917**	0.942953	0.949759	0.931199	0.979912
ΠD4	**0.9861**	0.982915	0.94785	0.979408	0.977572

From the calculations of all the value of R^2 for all the models above, it is clear that for value of R^2 of general models are nearer to 1 than clubbed; combined forms of models. Also from the table above it is found that values of general models indicates a nearly perfect fit, and therefore, these models are supposed to be a reliable model for future forecasts. Hence the reliable models from the calculations of co-efficient of determinations are

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(1) $(\pi_{D1}) = 23.77935114.\{(\pi_1)^{0.4389}.(\pi_2)^{-0.7}$

(2) $(\pi_{D2}) = 4.87E+01.\{(\pi_1)^{0.4416}.(\pi_2)^{-0.444}$

(3) $(\pi_{D3}) = 3.15E+00.\{(\pi_1)^{0.6958}.(\pi_2)^{-0.865}$

(4) $(\pi_{D4}) = 4.33E-01.\{(\pi_1)^{-0.0572}.(\pi_2)^{0.0948}.(\pi_3)^{0.0...}\}$

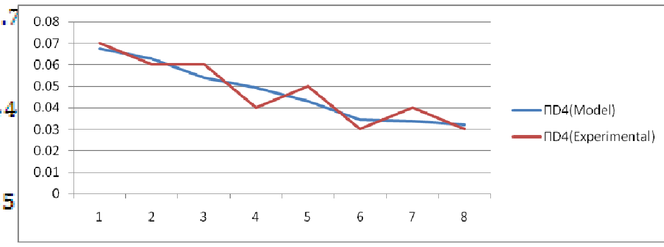

Fig. 7: Dependent variable ΠD4 experimental vs. ΠD4 model having R2 0.9861

Comparison between actual values and values obtained by models

Computed values based on above sited mathematical model could be readily possible by just putting the values of corresponding Pi terms. The graphical representation between the actual values of dependent terms and values obtained by model with coefficient of determination are shown in comparative form as below.

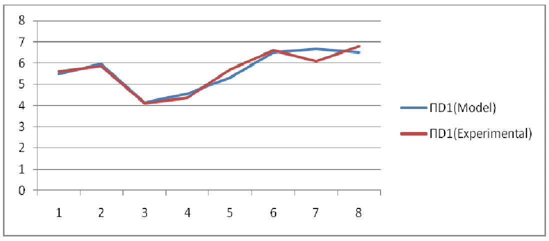

Figure 4: dependent variable ΠD1 experimental vs. ΠD1 model having R^2 0.997372

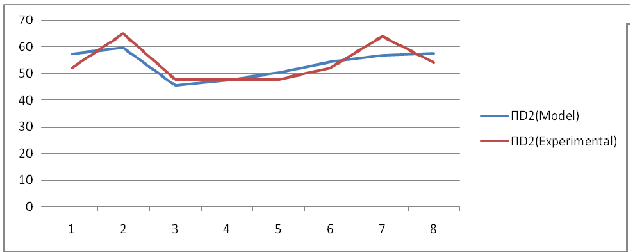

Fig. 5: Dependent variable ΠD2 experimental vs. ΠD2 model having R^2 0.9891

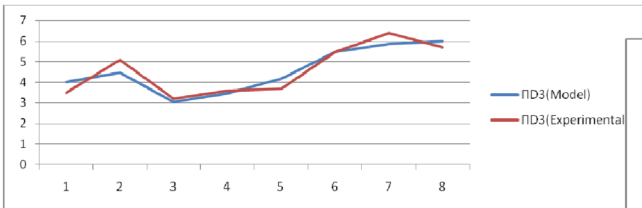

Fig. 6: Dependent variable ΠD3 experimental vs. ΠD3 model having R^2 0.9917

Determination of Reliability of Models

Reliability of model is established using relation Reliability =100-% mean error and Mean error = [(∑xi*fi)/∑ xi] where, xi is % error and fi is frequency of occurrence. Therefore the reliability of General model and Clubbed Model are equal to %and %respectively. Following table shows the reliability evaluated for all models mentioned above.

Table 14: Reliability of the models

Response Variable	Reliability for General Model
	$(\pi_D) = k \times (\pi_1)^a \times (\pi_2)^b \times (\pi_3)^c$
ΠD1	96.5
ΠD2	92.5
ΠD3	93.375
ΠD4	89

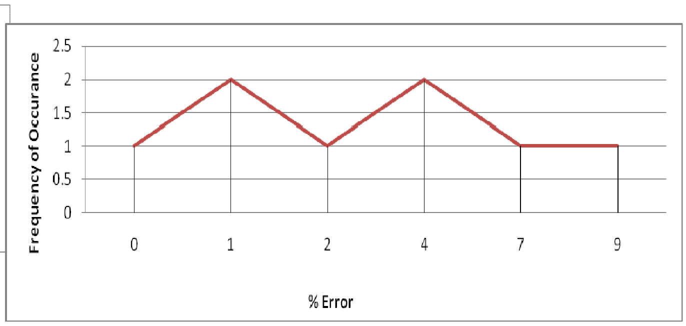

Fig. 8: Graph between % of Error and frequency occurrence of error for model ΠD1

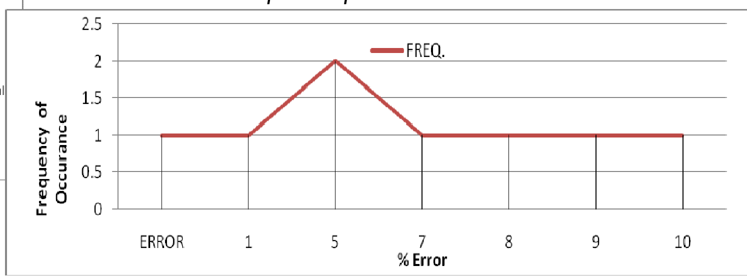

Fig. 9: Graph between % of Error and frequency occurrence of error for model ΠD2

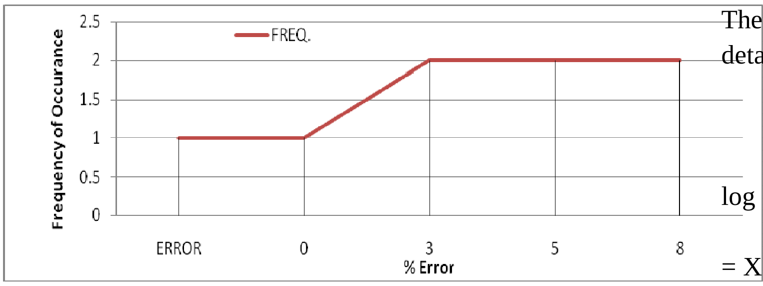

Fig. 10: Graph between % of Error and frequency occurrence of error for general ΠD3

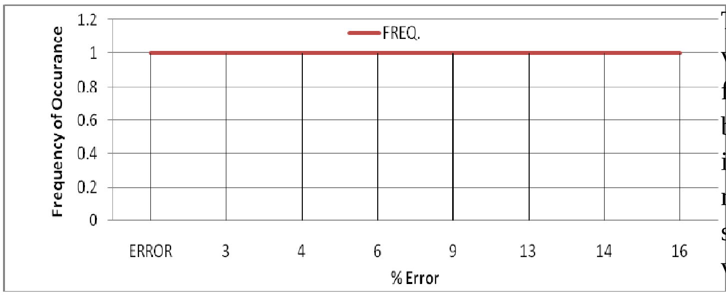

Fig 11: Graph between % of Error and frequency occurrence of error for general ΠD4

Estimation of limiting values of response variables

The ultimate objective of this work is to find out best set of variables, which will result in maximization or minimization of the response variables. In this section attempt is made to find out the limiting values of response variables. To achieve this, limiting values of independent π term viz. $\pi1$, $\pi2$, $\pi3$ are put in the respective models.. The limiting values of these response variables are compute for PVC pipe manufacturing is as given in table.

Table 15: Limiting Values of Response Variables

Max and Min. of Response π terms	PVC Pipe Extrusion Operation			
	For Process Parameters (ΠD1)	For Pipe Dimensions (ΠD2)	For Pipe Weight (ΠD3)	For Productivity (ΠD4)
Maximum	18.04018475	105.6015922	19.59909588	0.076940501
Minimum	1.977707371	31.20033668	1.22794984	0.028114943

Model Optimization:

Four mathematical models have been developed for the phenomenon. The ultimate objective of this work is not merely developing the models but to find out best set of independent variables, which will result in maximization or minimization of the objective functions. This can be achieved by taking the log of both the sides of the model. The linear programming technique is applied which is detailed as below.

$$\Pi D1 = k1 \times (\Pi1)^{a1} \times (\Pi2)^{b2} \times (\Pi3)^{c1}$$

Taking log of both the sides of the Equation, one has

Log ΠD1 = log k1 x a1 log (Π1) + b2 log (Π2) + c1 log (Π3)

Log ΠD1 = Z, log k1 = k1, log (Π1) = X1, log (Π2) = X2, log (Π3) = X3

Then the linear model in the form of first degree polynomial can be written as under

$Z = K + a \times X_1 + b \times X_2 + c \times X_3$

Thus, Equation constitutes for the optimization or to be very specific for maximization for the purpose of formulation of the problem. The constraints can be the boundaries defined for the various independent pi terms involved in the function. During the experimentation the ranges for each independent pi terms have been defined, so that there will be two constraints for each independent variable as under. If one denotes maximum and minimum values of a dependent pi term ΠD1 by $\Pi D1_{max}$ and $\Pi D1_{min}$ respectively then the first two constraints for the problem will be obtained by taking log of these quantities and by substituting the values of multipliers of all other variables except the one under consideration equal to zero. Let the log of the limits be defined, as C_1 and C_2 {i.e. $C_1=\log(\Pi D1_{max})$ and $C_2=\log(\Pi D1_{min})$. Thus, the Equations of the constraints will be as under.

$1 \times X_1 + 0 \times X_2 + 0 \times X_3 \leq C1$

$1 \times X_1 + 0 \times X_2 + 0 \times X_3 \geq C2$

The other constraints can be likewise found as under

$0 \times X_1 + 1 \times X_2 + 0 \times X_3 \leq C3$

$0 \times X_1 + 1 \times X_2 + 0 \times X_3 \geq C4$

$0 \times X_1 + 0 \times X_2 + 1 \times X_3 \leq C5$

$0 \times X_1 + 0 \times X_2 + 1 \times X_3 \geq C6$

After solving this linear programming problem one gets the minimum value of Z. The values of the independent pi terms can then be obtained by finding the antilog of the values of Z, X1, X2, and X3. The actual values of the multipliers and the variables are found. This can be solved as a linear programming problem using the MS Solver available in MS Excel. In this case there is model corresponding to PVC pipes extrusion process. These models have nonlinear form; hence it is to be converted into a linear form for optimization purpose. This can be achieved by taking the log of both the sides of the model and the linear programming technique is applied**.** On solving the above problem by using MS solver we get values of X_1, X2, X3, and Z. Thus Π_{D1min} = Antilog of Z and corresponding to this value of the Π_{D1min} the values of the independent pi terms are obtained by taking the antilog of X_1,X_2,X_3 and Z. Similar procedure is adopted to optimize the models for Π_{D2}, Π_{D3}, Π_{D4}, the optimized values of Π_{D1}, Π_{D2}, Π_{D3}, Π_{D4}, are tabulated in the following table.

Table 16: Optimize values of response variables for PVC pipe extrusion process

	For Process Parameters (ΠD1) min		For Pipe Dimensions (ΠD2)max	
	Log values of π terms	**Antilog of π terms**	**Log values of π terms**	**Antilog of π terms**
Z	0.296162032	1.977707	2.023670466	105.6015921
X_1	3.905218393	8039.30291	4.183706782	15265.35054
X_2	5.213337061	163431.9872	4.619807134	41668.4297
X_3	2.653212514	450	3.602059991	4000
	For Pipe Weight (ΠD3) min		For Productivity (ΠD4) max	
	Log values of π terms	Antilog of π terms	Log values of π terms	Antilog of π terms
Z	0.089180627	1.22794984	-1.113844993	0.0769405
X_1	3.905218393	8039.30291	3.905218393	8039.30291
X_2	5.213337061	163431.9872	5.213337061	163431.9872
X_3	2.653212514	450	2.653212514	450

Sensitivity Analysis

The influence of the various independent π terms has been studied by analyzing the indices of the various π terms in the models. The technique of sensitivity analysis, the change in the value of a dependent π term caused due to an introduced change in the value of individual π term is evaluated. In this case, of change of ± 10 % is introduced in the individual independent π term independently (one at a time).Thus, total range of the introduced change is ± 20 %. The effect of this introduced change on the change in the value of the dependent π term is evaluated .The average values of the change in the dependent π term due to the introduced change of ± 10 % in each independent π term. This defines sensitivity. Nature of variation in response variables due to increase in the values of independent pi terms is given in following figure.

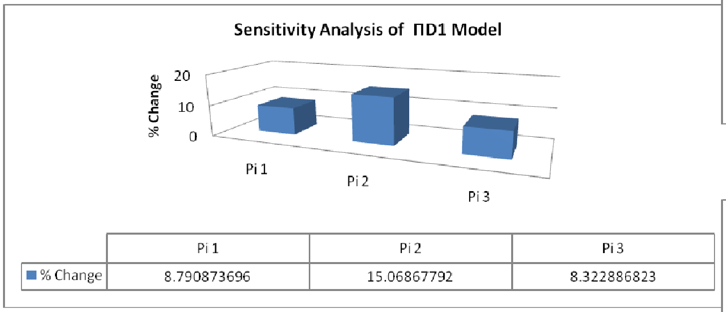

Fig. 12: Sensitivity analysis of ΠD1 model

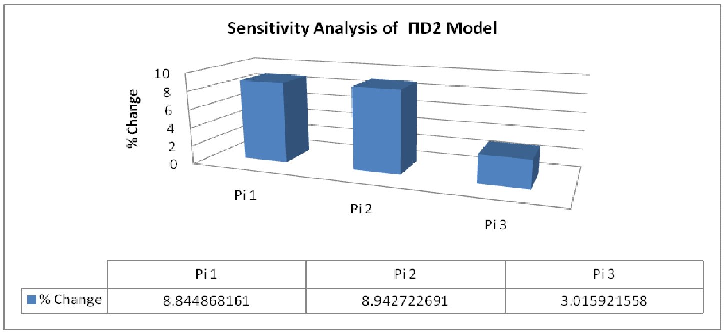

Fig. 13: Sensitivity analysis of ΠD2 model

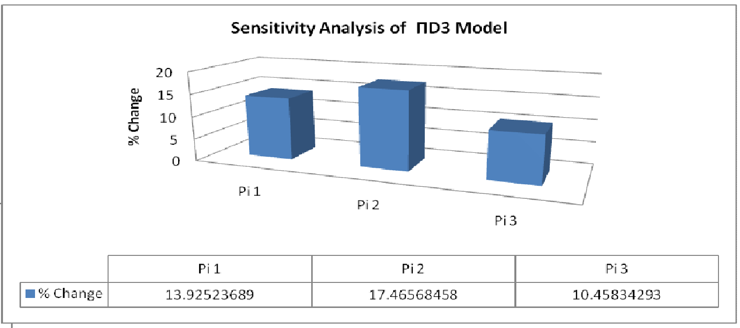

Fig. 14: Sensitivity analysis of ΠD3 model

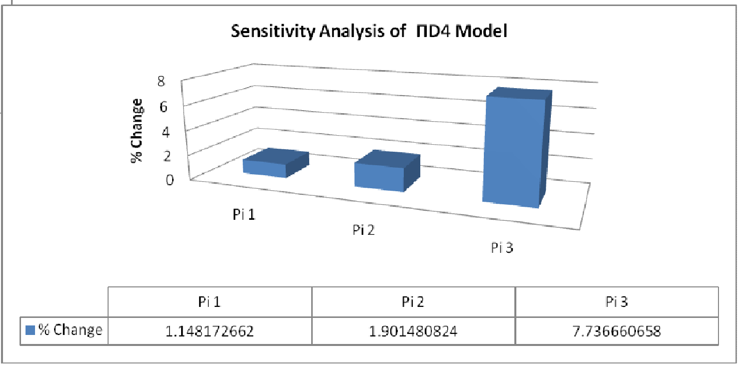

Fig. 15: Sensitivity analysis of ΠD4 model

III. CONCLUSION

The present paper gave an illustration of how dimensional analysis (DA) can be applied to significantly reduce the number of independent variables used to optimize the dependent process parameters of the extruder machines, pipe dimensions, pipe weights and the productivity as response variable using field data based mathematical modeling. Using dimensional analysis 29 around number of independent variables has been reduced to 03 dimensionless pi terms and around 14 number of dependent pi terms into 04 dimensionless pi terms. This can greatly help in formulation of approximate, generalized field data based mathematical models in easier manner. Thus in this way the dimensional equations established in reduced or compact mode made the complete experimentation process less time taking having generation of optimum data. The conclusion in the form of interpretation of model is being reported in terms of several aspects viz. (1) Order of influence of various inputs (causes) on outputs (effects) (2) Relative influence of causes on effect (3) Interpretation of curve fitting constant K. The value of curve fitting constant in the model for (ΠD1) is 23.77. This collectively represents the combined effect of all variables as variables related with electric motors, specifications of the extruder machine and quantity of the raw materials used. Further, as it is positive, this indicates that these causes have strong influence on the dependent process parameters of the extruders (ΠD1). The absolute index of π1 is the highest Viz. 0.4389. Thus, this is term related to specifications related with electric motor which is the most influencing π term in this model. The absolute index of π3 is 0.4155, which is related with quantity of raw material used. This is very close to absolute index of π1 0.4389. This indicates that pi term π3 is also very much influencing term in this model. The absolute index of π2 is negative viz. – 0.7474. This pi term is related with specifications of the extruder machines. This negative index indicating that dependent process parameters are inversely proportional to specifications of the extruder machines. The value of curve fitting constant in the model for (ΠD2) is 4.87E + 01. This collectively represents the combined effect of all variables such variables related with electric motors, specifications of the extruder machine and quantity of the raw materials used. Further, as it is positive, this indicates that these causes have strong influence on the dependent process parameters of the extruders (ΠD2). The absolute index of π1 is the highest Viz. 0.44416. Thus, this is term related to specifications related with electric motor which is the most influencing π term in this model. The absolute index of π3 is 0.15404, which is related with quantity of raw material used. This indicates that pi term π3 is less influencing term than π1 in this model. The absolute index of π2 is negative viz. – 0.4445. This pi term is related with specifications of the extruder machines. This negative index indicating that pipe dimensions are inversely proportional to specifications of the extruder machines. The value of curve fitting constant in the model for (ΠD3) is 3.15E + 00. This collectively represents the combined effect of all variables such variables related with electric motors, specifications of the extruder machine and quantity of the raw materials used. Further, as it is positive, this indicates that these causes have strong influence on the dependent process parameters of the extruders (ΠD3).The absolute index of π1 is the highest Viz. 0.6958. Thus, this is term related to specifications related with electric motor which is the most influencing π term in this model. The absolute index of π3 is 0.5223, which is related with quantity of raw material used. This indicates that pi term π3 is also very much influencing term after π1 in this model. The absolute index of π2 is negative viz. – 0.8655. This pi term is related with specifications of the extruder machines. This negative index indicating that weight of the PVC pipes are inversely proportional to specifications of the extruder machines. The value of curve fitting constant in the model for (ΠD4) is 4.33E - 01. This collectively represents the combined effect of all variables as variables related with electric motors, specifications of the extruder machine and quantity of the raw materials used. The absolute index of π2 is the highest Viz. 0.0948. Thus, this is term related to specifications related with extruder machines which is the most influencing π term in this model. The absolute index of π1 is - 0.0572 which is related with specifications of the electric motors used. This indicates that pi term π1 is also less influencing term after in this model. The absolute index of π3 is negative viz. – 0.3847. This pi term is related with quantity of the raw material used. This negative index indicating that productivity of the PVC pipe manufacturing process is proportional to quantity of raw material used. Thus from these models “Intensity of interaction of inputs on deciding Response” can be predicted. The optimization methodology adopted is unique and rigorously derives the most optimum solution for field data available for PVC pipe manufacturing process.

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