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Some Properties of Max-Min Composition

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Abstract*—This paper discusses several properties of max-min composition, with particular emphasis on symmetry, max-min transitivity, and inverse composition. Furthermore, the paper presents mathematical proofs of these properties and verifies them through numerical examples using a 3×3 order of matrix.

I. INTRODUCTION

In classical set theory, relations define the connections between elements of two or more sets, typically as a subset of the Cartesian product of the sets. However, in fuzzy set theory, fuzzy relations generalize this concept by introducing degrees of membership. This allows partial or uncertain relationships between elements, making fuzzy relations more flexible for modeling real-world scenarios involving uncertainty and vagueness. A fuzzy relation is a mapping that assigns each pair of elements from two sets a value between 0 and 1, representing the strength or degree of relation between those elements.

Whereas, the composition of fuzzy relations is a method to combine two or more fuzzy relations to derive a new relationship. This concept extends the idea of relational composition from classical set theory to fuzzy set theory, allowing for modeling of complex, uncertain and imprecise systems. In real-world scenarios, relationships between elements are often indirect. For example, if there is a fuzzy relation between sets P and Q and another between Q and R, then the composition helps to establish a fuzzy relation between P and R. Fuzzy relation composition plays a crucial role in systems where indirect, uncertain, or gradual relationships need to be effective.

The concept of fuzzy relations is an extension of the fuzzy set theory introduced by Zadeh [1]. In 2012, Dorugade Namdev et al. [2] presented a simple approach to predict life of a component indirectly from imprecise information by exploring the compositional rule of inference of fuzzy relations. They also developed a MATLAB program to compute the degree of success of the outcome.

Gowrishankar et al. [3] presented a systematic exposition of fundamental operations on fuzzy relations, with particular emphasis on max-min and max-product compositions supported by illustrative examples. The authors further analyze key algebraic properties—including zero, identity, equality, inequality, subset, associativity, union, intersection, and distributivity and substantiate these properties through numerical verification using 2×2 matrix representations, supplemented by exercise problems with detailed solutions. Shakhatreh and Qawasmeh [4] presented some concepts and definitions related to the max-min composition of fuzzy relations. They also proved max-min composition of three fuzzy relations using associativity. Mangijaobi [5] introduced associative properties in max-product composition of three binary fuzzy relations and also presented the reversal law of two binary fuzzy relations holds under max-product composition. Further, an example is also provided by using a 3×3 order of matrix. Bezdek and Harris [6] explored some connections between fuzzy partitions and similarity relations. Also finally they showed that every fuzzy c-partition can induce pseudo-metrics on finite data sets, providing a theoretical basis for fuzzy clustering. Related work can be found in the references [7,8,9,10,11,12,13].

This paper discusses several properties of max-min composition. It presents formal mathematical proofs of these properties and verifies them through numerical examples using a 3×3 order fuzzy relation matrix.

II. PRELIMINARIES

First, we will discuss some definitions and preliminaries concepts related to fuzzy sets, fuzzy relations and max-min composition.

Definition 2.1 [10]. If U is a universe of discourse and u be any particular element of U, then a fuzzy set $\tilde{P}$ defined on U may be written as a collection of ordered pairs $\tilde{P}=\{(u,\mu_{\tilde{P}}(u)):u\in U\}$ where, each pair $(u,\mu_{\tilde{P}}(u))$ is called a singleton.

Definition 2.2 [4]. Let U and V be nonempty sets. Then $\tilde{P}=\{((u,v),\mu_{\tilde{P}}(u,v)):(u,v)\in U\times V\}$ is called a fuzzy relations in $U\times V$.

Definition 2.3 [4]. Let U, V and W be nonempty sets, $\tilde{P}_1$ be a fuzzy relation in $U\times V$, and $\tilde{P}_2$ be a fuzzy relation in $V\times W$. Then $\tilde{P}_1\circ\tilde{P}_2=\{(u,\text{w}),\ \max_{v\in V}\{\min\{\mu_{\tilde{P}_1}(u,v),\mu_{\tilde{P}_2}(v,w)\}:u\in U,v\in V,w\in W\}$ is called max-min composition of $\tilde{P}_1$ and $\tilde{P}_2$.

III. RESULTS

Now we prove the following theorems

Theorem 3.1. If $\tilde{P}_1$ and $\tilde{P}_2$ are symmetric, then $\tilde{P}_1\circ\tilde{P}_2$ is symmetric if $\tilde{P}_1\circ\tilde{P}_2=\tilde{P}_2\circ\tilde{P}_1$.

Proof: Here we need to prove that

$\mu_{\tilde{P}_1\circ\tilde{P}_2}(u,w)=\mu_{\tilde{P}_2\circ\tilde{P}_1}(w,u)$ for all $u,w\in U$.

We have,

$\tilde{P}_1(u,v)=\tilde{P}_1(v,u)[\because \tilde{P}_1$ is symmetric]

Similarly, $\tilde{P}_2(v,w)=\tilde{P}_2(w,v)[\because \tilde{P}_2$ is symmetric]

We have

$$\mu_{\tilde{P}_1\circ\tilde{P}_2}(u,w)=\max_{v\in U}\{\min\{\mu_{\tilde{P}_1}(u,v),\mu_{\tilde{P}_2}(v,w)\}\}\text{ for all }u,w\in U. \quad (1)$$

$$\mu_{\tilde{P}_2\circ\tilde{P}_1}(w,u)=\max_{v\in U}\{\min\{\mu_{\tilde{P}_2}(w,v),\mu_{\tilde{P}_1}(v,u)\}\}$$
$$=\max_{v\in U}\{\min\{\mu_{\tilde{P}_2}(v,w),\mu_{\tilde{P}_1}(u,v)\}\} \quad (2)$$

$[\because \tilde{P}_2$ and $\tilde{P}_1$ are both symmetric]

From (1) and (2)

$\mu_{\tilde{P}_1\circ\tilde{P}_2}(u,w)=\mu_{\tilde{P}_2\circ\tilde{P}_1}(w,u)$

$\therefore \tilde{P}_1\circ\tilde{P}_2=\tilde{P}_2\circ\tilde{P}_1$

$\therefore$ It is obviously $\tilde{P}_1\circ\tilde{P}_2$ is symmetric.

Example3.1.1.

If $\tilde{P}_1=\begin{bmatrix}.8&.5&.3\\.5&.7&.6\\.3&.6&.9\end{bmatrix}$, and $\tilde{P}_2=\begin{bmatrix}.6&.4&.5\\.4&.9&.7\\.5&.7&.8\end{bmatrix}$ are symmetric,

then $\tilde{P}_1\circ\tilde{P}_2$ is symmetric if $\tilde{P}_1\circ\tilde{P}_2=\tilde{P}_2\circ\tilde{P}_1$.

Proof: We have to prove that $\tilde{P}_1\circ\tilde{P}_2=\tilde{P}_2\circ\tilde{P}_1$.

Then by using max-min composition of fuzzy relation, we have,

$$\tilde{P}_1\circ\tilde{P}_2=\begin{bmatrix}.8&.5&.3\\.5&.7&.6\\.3&.6&.9\end{bmatrix}\circ\begin{bmatrix}.6&.4&.5\\.4&.9&.7\\.5&.7&.8\end{bmatrix}$$
$$=\begin{bmatrix}.6&.5&.5\\.5&.7&.7\\.5&.7&.8\end{bmatrix} \quad (3)$$

Here, we see that $\tilde{P}_1\circ\tilde{P}_2$ is symmetric.

Again,

$$\tilde{P}_2\circ\tilde{P}_1=\begin{bmatrix}.6&.4&.5\\.4&.9&.7\\.5&.7&.8\end{bmatrix}\circ\begin{bmatrix}.8&.5&.3\\.5&.7&.6\\.3&.6&.9\end{bmatrix}$$
$$=\begin{bmatrix}.6&.5&.5\\.5&.7&.7\\.5&.7&.8\end{bmatrix} \quad (4)$$

Hence, From equation (3) and (4) we get,

$$\tilde{P}_1 o \tilde{P}_2 = \tilde{P}_2 o \tilde{P}_1.$$

Theorem 3. 2. If $\tilde{P}$ is symmetric, then show that each power of $\tilde{P}$ is symmetric.

Proof: We have

$\mu_{\tilde{P}(u,v)} = \mu_{\tilde{P}(v,u)}[\because \tilde{P}(u,v)$ is symmetric for all $u, v \in U$.

$$\tilde{P}^2 = \tilde{P} \circ \tilde{P}$$
$$= \tilde{P}(u,v) \circ \tilde{P}(u,v)$$
$$= \tilde{P}(u,v) \circ \tilde{P}(v,u)[\because \tilde{P}(u,v) \text{ is symmetric}]$$
$$\mu_{(\tilde{P}\circ\tilde{P})\circ\tilde{P}}(u,v) = \max_{u\in U}\{\min\{\mu_{\tilde{P}\circ\tilde{P}}(u,u), \mu_p(u,v)\}\}$$
$$= \max_{u\in U}\{\min\{\max_{v\in U}\{\min\{\mu_{\tilde{P}}(u,v), \mu_{\tilde{P}}(v,u)\}\}, \mu_{\tilde{P}}(u,v)\}\}$$

Hence, each power of $\tilde{P}$ is symmetric.

Example 3.2.1. If $\tilde{P}$ is symmetric, then show that each power of $\tilde{P}$ is symmetric.

Proof:

Let $\tilde{P} = \begin{bmatrix} .7 & .5 & .3 \\ .5 & .6 & .4 \\ .3 & .4 & .8 \end{bmatrix}$ be a symmetric.

Now, we have to show that each power of power of $\tilde{P}$ is also symmetric.

$$\tilde{P} \circ \tilde{P} = \begin{bmatrix} .7 & .5 & .3 \\ .5 & .6 & .4 \\ .3 & .4 & .8 \end{bmatrix} \circ \begin{bmatrix} .7 & .5 & .3 \\ .5 & .6 & .4 \\ .3 & .4 & .8 \end{bmatrix}$$
$$= \begin{bmatrix} .7 & .5 & .4 \\ .5 & .6 & .4 \\ .4 & .4 & .8 \end{bmatrix}$$

Here, we see that $\tilde{P} \circ \tilde{P}$ is symmetric.

Now,

$$(\tilde{P} \circ \tilde{P}) \circ \tilde{P} = \begin{bmatrix} .7 & .5 & .4 \\ .5 & .6 & .4 \\ .4 & .4 & .8 \end{bmatrix} \circ \begin{bmatrix} .7 & .5 & .3 \\ .5 & .6 & .4 \\ .3 & .4 & .8 \end{bmatrix}$$
$$= \begin{bmatrix} .7 & .5 & .4 \\ .5 & .6 & .4 \\ .4 & .4 & .8 \end{bmatrix}$$

Thus, we observe that $(\tilde{P} \circ \tilde{P}) \circ \tilde{P}$ is symmetric.

Hence, if $\tilde{P}$ is symmetric, then each power of $\tilde{P}$ is symmetric.

Theorem 3. 3. A fuzzy relation $\tilde{P}(U,U)$ is called max-min transitive if $\tilde{P} \circ \tilde{P} \subseteq \tilde{P}$.

Proof: A fuzzy relation $\tilde{P}(U,U)$ is called max-min transitive if

$\mu_{\tilde{P}}(u,w) \geq \max_{v\in U}\{\min \mu_{\tilde{P}}(u,v), \mu_{\tilde{P}}(v,w)\}$, for each pair $(u,w) \in U \times U$.

By the definition of max-min composition, we have

$$\mu_{\tilde{P}\circ\tilde{P}}(u,w) = \max_{v\in U}\{\min \mu_{\tilde{P}}(u,v), \mu_{\tilde{P}}(v,w)\}$$

$\leq \mu_{\tilde{P}}(u,w)[\because$ by definition of max-min transitive

$\therefore \tilde{P} \circ \tilde{P} \subseteq \tilde{P}$

Example 3.3.1. Show that a fuzzy relation $\tilde{P}$ is called max-min transitive if $\tilde{P} \circ \tilde{P} \subseteq \tilde{P}$.

Proof: Let $\tilde{P} = \begin{bmatrix} 1 & .7 & .7 \\ .7 & 1 & .7 \\ .7 & .7 & 1 \end{bmatrix}$ (5)

Now, we shall prove that $\tilde{P}$ is transitive.

$$\tilde{P} \circ \tilde{P} = \begin{bmatrix} 1 & .7 & .7 \\ .7 & 1 & .7 \\ .7 & .7 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & .7 & .7 \\ .7 & 1 & .7 \\ .7 & .7 & 1 \end{bmatrix}$$

$$\therefore \tilde{P} \circ \tilde{P} = \begin{bmatrix} 1 & .7 & .7 \\ .7 & 1 & .7 \\ .7 & .7 & 1 \end{bmatrix} \quad (6)$$

Comparing equation (5) and (6) we have

$$\tilde{P} \circ \tilde{P} = \tilde{P} = \begin{bmatrix} 1 & .7 & .7 \\ .7 & 1 & .7 \\ .7 & .7 & 1 \end{bmatrix}$$

i.e., $\tilde{P} \circ \tilde{P} \subseteq \tilde{P}$.

Hence, $\tilde{P}$ is a max-min transitive fuzzy relation.

Theorem 3.4. Let $\tilde{P}_1$ and $\tilde{P}_2$ be two fuzzy relation in $U \times V, V \times W$ respectively, then prove that $[\tilde{P}_1(U,V) \circ, \tilde{P}_2(V,W)]^{-1} = \tilde{P}_2^{-1}(W,V) \circ, \tilde{P}_1^{-1}(V,U)$.

Proof: We need to prove that

$\mu_{[\tilde{P}_1\circ\tilde{P}_2]^{-1}}(u,w) = \mu_{[\tilde{P}_2]^{-1}\circ[\tilde{P}_1]^{-1}}(w,u)$ for all $u, \in U, v \in V, w \in W$.

By definition of max-min composition, we obtain

$$\mu_{\tilde{P}_1 \circ \tilde{P}_2}(u,w) = \max_{v\in V} \min\{\mu_{\tilde{P}_1}(u,v), \mu_{\tilde{P}_2}(v,w)\}$$

$$\mu_{[\tilde{P}_1 \circ \tilde{P}_2]^{-1}}(u,w) = \max_{v\in V} \min\{\mu_{\tilde{P}_2^{-1}}(w,v), \mu_{\tilde{P}_1^{-1}}(v,u)\}$$

$$= \max_{v\in V} \min\{\mu_{\tilde{P}_2}(v,w), \mu_{\tilde{P}_1}(u,v)\} \quad (7)$$

$$[\because \mu_{\tilde{P}_1^{-1}}(v,u) = \mu_{\tilde{P}_1}(u,v)]$$

$$\mu_{\tilde{P}_2^{-1}}(w,v) \circ \mu_{\tilde{P}_1^{-1}}(v,u) = \max_{v\in V} \min\{\mu_{\tilde{P}_2^{-1}}(w,v), \mu_{\tilde{P}_1^{-1}}(v,u)\}$$

$$= \max_{v\in V} \min\{\mu_{\tilde{P}_2}(v,w), \mu_{\tilde{P}_1}(u,v)\} \quad (8)$$

$$[\because \mu_{\tilde{P}_1^{-1}}(v,u) = \mu_{\tilde{P}_1}(u,v)]$$

From equation (7) and (8), we get

$$\mu_{[\tilde{P}_1 \circ \tilde{P}_2]^{-1}}(u,w) = \mu_{\tilde{P}_2^{-1}}(w,v) \circ \mu_{\tilde{P}_1^{-1}}(v,u)$$

Hence, $[\tilde{P}_1(U,V) \circ \tilde{P}_2(V,W)]^{-1} = \tilde{P}_2^{-1}(W,V) \circ \tilde{P}_1^{-1}(V,U)$.

Example3.4.1. Let

$$\tilde{P}_1 = \begin{bmatrix} 1 & .4 & .7 \\ .3 & .9 & .5 \\ .6 & .2 & 1 \end{bmatrix}, \text{ and } \tilde{P}_2 = \begin{bmatrix} .8 & .5 & .6 \\ .7 & 1 & .4 \\ .3 & .9 & .8 \end{bmatrix},$$

Then prove that $[\tilde{P}_1 \circ \tilde{P}_2]^{-1} = \tilde{P}_2^{-1} \circ \tilde{P}_1^{-1}$.

Proof: we need to prove that

$$[\tilde{P}_1 \circ \tilde{P}_2]^{-1} = \tilde{P}_2^{-1} \circ \tilde{P}_1^{-1}.$$

Then by using max-min composition of fuzzy relation, we have,

$$\tilde{P}_1 \circ \tilde{P}_2 = \begin{bmatrix} 1 & .4 & .7 \\ .3 & .9 & .5 \\ .6 & .2 & 1 \end{bmatrix} \circ \begin{bmatrix} .8 & .5 & .6 \\ .7 & 1 & .4 \\ .3 & .9 & .8 \end{bmatrix}$$

$$= \begin{bmatrix} .8 & .7 & .7 \\ .7 & .9 & .5 \\ .6 & .9 & .8 \end{bmatrix}$$

$$\therefore\ [\tilde{P}_1 \circ \tilde{P}_2]^{-1} = \begin{bmatrix} .8 & .7 & .6 \\ .7 & .9 & .9 \\ .7 & .5 & .8 \end{bmatrix} \quad (9)$$

Then,

$$[\tilde{P}_2]^{-1} \circ [\tilde{P}_1]^{-1} = \begin{bmatrix} .8 & .7 & .3 \\ .5 & 1 & .9 \\ .6 & .4 & .8 \end{bmatrix} \circ \begin{bmatrix} 1 & .3 & .6 \\ .4 & .9 & .2 \\ .7 & .5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} .8 & .7 & .6 \\ .7 & .9 & .9 \\ .7 & .5 & .8 \end{bmatrix} \quad (10)$$

Hence, from equation (9) and (10) we get,

$$[\tilde{P}_1 \circ \tilde{P}_2]^{-1} = [\tilde{P}_2]^{-1} \circ [\tilde{P}_1]^{-1}$$

IV. CONCLUSION

This paper study several properties of max-min composition, including symmetry, max-min transitivity, and inverse composition. These properties are mathematically proved and systematically verified by using numerical example of a 3×3 order of matrix.

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