International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-8, Issue-6; Jun, 2021
Journal Home Page Available: https://ijaers.com/
Article DOI:https://dx.doi.org/10.22161/ijaers.86.11

Error Analysis in the Resolutions of 1st Year High School Students in the Study of Notable Products and Polynomial Factoring

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Received: 19 Mar 2021;
Received in revised form:
30 Apr 2021;
Accepted: 25 May 2021;
Available online: 13 Jun 2021

***Keywords*— Algebra, Error Analysis, High School, Mathematical Education.**

***Abstract*— *The present research had as objectives: to analyze successes and mistakes committed in the resolutions of 36 students of a class of the 1st year of High School, of a Pernambuco state school in the city of Petrolina; as well as, to present suggestions for investigations/ explorations, so that teachers who teach Mathematics can contemplate Error Analysis both in the methodological alternative of research, as in teaching, in Mathematical Education. For this aim, a student diagnostic questionnaire was proposed, from which 3 open questions were analyzed, the first sought to probe students' affirmations about the contents of polynomials and the last two of which dealt, in particular, with the contents of Notable Products and polynomial Factorization. From this analysis, 6 categories were created. By results, Categories D, C and B presented, respectively, the highest number of appearances, revealing a large number of mistakes in the distinction between polynomials and equations, in various algebraic manipulations and in the use of practical rules for the development of Notable Products.*

I. INTRODUCTION

The present work is an extension of the scientific initiation research of the first author, by the Institutional Program of Scientific Initiation Scholarships (PIBIC) (PIBIC 2018 - 2019), funded by the National Council for Scientific and Technological Development (CNPq), a research that analyzed the contributions of the game Polinoquiz for the review of polynomial contents, in the 1st year of high school.

Regarding the contents of polynomials, the Parameters for Basic Education of the State of Pernambuco (Pernambuco, 2012) provide that these are initially seen, in the 8th year of elementary school, a constituent to the axis of Algebra and Functions. However, studies show (Booth et al., 2014; Cury, 2018; Guillermo, 1992; Nogueira et al., 2018; Ribeiro, 2001) that, often, these contents generate gaps, whether conceptual and/or procedural, in students, at the end of the final cycles of elementary school. Consequently, these gaps reflect on the learning of algebra in the later years of students' schooling, including higher education.

Thus, it is observed the importance that a teacher who teaches Mathematics knows both to identify the difficulties of students in the contents of polynomials, from their initial contact with such contents, and to choose appropriate teaching strategies in order to be met with such misunderstandings.

Thus, this research aimed to analyze correct answers and mistakes made by students of a class of the 1st year of high school, before their resolutions in a questionnaire containing 3 open questions, addressing, specifically, the contents of Notable Products and Polynomial Factoring. For the analysis of the resolutions of the students present in such questionnaires, we chose to use the trend in

Mathematical Education of Error Analysis, according to what the author Cury (2018) recommends. As well as, we sought to present suggestions for investigations/ explorations, so that teachers who teach Mathematics can contemplate Error Analysis in both the methodological alternative of research and teaching (Cury, 2018).

Regarding the structure of this research, initially, there is a brief presentation of the Analysis of Errors, followed by a discussion on this theme focused on the axis of Algebra and Functions, more specifically, with regard to Notable Products and Polynomial Factoring. Furthermore, the methodological paths of the research are explained. Soon after, the results and discussions are presented. Finally, a conclusion was made.

II. ERROR ANALYSIS AS A METHODOLOGY FOR RESEARCH AND TEACHING IN MATHEMATICS EDUCATION

Error Analysis is a research trend in Mathematics Education that has been consolidated in several countries throughout recent decades. In Brazil, there is a strong representation by the author Helena Noronha Cury who, since the second half of the 1980s, researches and guides in this area.

For Cury (2018), the Analysis of Errors, or rather, the Analysis of written productions of students, consists of a methodological alternative of research and teaching, in Mathematics Education. As a research methodology, it allows the teacher to take an in-depth and comprehensive look at the students' responses, both to the correct answers and to the errors, in the various resolution strategies presented by them, which reveal their forms of appropriation of a certain mathematical content (Cury, 2018; Lima & Moreira, 2019).

Concomitantly with the use of Error Analysis as a research methodology, it can unfold in a teaching methodology, since, in view of the errors and misunderstandings of the students' resolutions, it is possible that the teacher proposes investigations/explorations in the classroom, so that it is possible for students to use the error as a springboard for learning (Cury, 2018).

It is also a communication (often the only way) between the teacher and his students, in which the teacher has the opportunity to know the particularities of his students and to practice a relationship of companionship towards them (Lima & Moreira, 2019).

In this respect, the reflections of the author Luckesi (1999), about the role of error in school practice and in the evaluation of learning, add to the relevance that Error Analysis can assume, to break the paradigm that the mistakes made by students must result in punishments applied by teachers; punishments that generate, for example, feelings of tension, fear, anxiety and guilt, which imply traumas and limitations in learning and that mark the students' own lives.

On the other hand, according to Luckesi (1999), when students' mistakes are seen as sources of virtue, of growth, there is a fertile environment for learning, because: "The errors of learning [...] they serve positively as a starting point for advancement, to the extent that they are identified and understood, and their understanding is the fundamental step for their overcoming" (Luckesi, 1999, p. 57).

Thus, the use of Error Analysis in its entirety by teachers who teach Mathematics, that is, initially as a research methodology and, later, as a teaching methodology, can positively resignify behaviors in the face of errors in school practice.

III. A LOOK AT ALGEBRA ERROR ANALYSIS IN NOTABLE PRODUCTS AND POLYNOMIAL FACTORIZATION

The Common National Curriculum Base (BNCC) of Elementary School (Brasil, 2018, p. 268) recommends, for the thematic unit of Algebra, the development of students, of the so-called Algebraic Thought, concomitant with "the development of a language, the establishment of generalizations, the analysis of the interdependence of quantities and the resolution of problems through equations or ineptitudes". For this, for example, it is necessary that students establish "connections between variable and function and between unknown and equation" (Brasil, 2018, p. 269).

Thus, elementary part of the curricular requirements for the development of this Algebraic Thought comes from the study of polynomials (initial idea, sum operations, subtraction, multiplication, division, including Notable Products and Factoring) which, according to the Parameters for Basic Education of the State of Pernambuco (Pernambuco, 2012), are expected to be seen, initially, in the 8th year of Elementary School, being found in the axis of Algebra and Functions.

From the learning of polynomials, a student can procedurally learn other contents, such as equations, inequations and functions, in his other years of schooling; and also, possibly, in higher education, such as the disciplines of Algebra and Differential and Integral Calculus.

However, studies conducted from the perspective of Error Analysis reveal conceptual and procedural misconceptions in the learning of polynomials, either in the Final Years of Elementary School (Booth et al., 2014; Ribeiro, 2001), in High School (Booth et al., 2014) or higher education (Booth et al., 2014; Cury, 2018; Guillermo, 1992; Nogueira et al., 2018).

Like Booth et al. (2014) point out that many higher education students in the United States present unsatisfactory results in the discipline of Algebra. The authors complement that one of the reasons for such a discipline is particularly challenging, is the fact that there is a deepening of the conceptual misconceptions that students have rooted in previous years of schooling. Hence, we notice the importance and relevance of an effective learning of polynomials in school mathematics.

Still, as for the results of Booth et al.'s research. (2014), these suggest that misconceptions related to the most persistent errors observed in students should be the targets of investigations, inside or outside the classroom; this is because, such misconceptions have shown that they are not simply addressed in a usual approach: a more directive approach is needed. Thus, the potentiality is reinforced, both for teaching and for mathematical learning, of the use of Error Analysis as a methodology of research and teaching.

IV. METHODOLOGICAL PATHS

The present research is classified as the data collection procedures used as field, since it required "the realization of an experiment or the collection of empirical information/data or insertion/intervention in the environment to be studied" (Fiorentini & Lorenzato, 2012, p. 61). Also, regarding the nature of the data, the research had a qualitative approach. This choice proved to be adequate, because "understanding, with the interpretation of the phenomenon" (Gonsalves, 2011, p. 70) was sought.

The subjects involved with the research were 36 students from a 1st year high school class, from a school in the state school of the municipality of Petrolina, state of Pernambuco. Regarding the instrument used in data collection, a student diagnostic questionnaire was proposed, containing open questions. There was no interference from the researchers regarding the assistance to students in the resolution of the questions. In this work, we will only deal with three questions (see Appendix A): the first, concerning the students' statements about polynomials; the second, on Notable Products; and the third, regarding the contents of Polynomial Factoring.

For the interpretation of the data, Questions 2 and 3 of the student diagnostic questionnaire were analyzed through the theoretical perspective of Error Analysis, as a research methodology, proposed by Cury (2018). For this, as well as Cury (2018), a methodology of data analysis called Content Analysis was adopted. Regarding this analysis, the three stages proposed by Bardin (1979 as cited in Cury, 2018) were followed, which are: pre-analysis, exploration of the material and treatment of the results. Thus, 6 categories of analysis were created, based on student resolutions, as observed in Table 1.

Table. 1: Description of the categories of analysis

ANALYSIS CATEGORY	DESCRIPTION OF THE CATEGORIES OF ANALYSIS, WHICH CORRESPOND TO STUDENTS WHO:
Category A	They've reached correct resolutions.
Category B	They misused the practical rules to calculate the Notable Products of the Square Sum of Two Terms, the Square Difference of Two-Terms, or the Product of a Sum and Difference of Two Terms. For example, they summed or subtracted the square from the two terms inside the parentheses, in the calculation of the Square Sum of Two Terms and/or in the Square Difference of Two-Terms
Category C	They performed erroneous algebraic manipulations, either in the grouping of monomials, in the use of the distributive property, in the use of the "signal rule" or by Factoring polynomials using the method of factoring numbers into prime factors.
Category D	They found the right result, however, equaled it to zero in the end. Furthermore, in the Polynomial Factoring, in particular, they matched the polynomial given to zero and/or tried to calculate, whether right or wrong, the roots of this supposed equation.

Category E	Incorrect use of Polynomial Factoring by the cases of Common Factors, Perfect Square Trinomial and Factoring of the Trinomial Type: $x^2 + Sx + P$, in which S = sum and P = product of the two numbers chosen.
Category F	They presented erroneous resolutions, whose analysis was inconclusive.

Also, in the interpretation of the data, during the analysis made, some suggestions of teaching strategies were given that, perhaps, will help teachers who teach Mathematics to propose investigations/explorations, in order to reduce the mistakes made by students. Thus, it would be possible to glimpse, in fact, the Analysis of Errors as a methodology of Mathematics teaching, as Cury (2018) recommends.

V. RESULTS AND DISCUSSIONS

In Question 1, in their first part, 29 students stated that they had already studied the contents of polynomials in their Mathematics classes, as opposed to 7 students, who left the question blank. However, 19 of the 29 students who claimed to have studied the contents of polynomials in the second part of the question said they did not remember such contents. Also, in relation to the second command of Question 1, 10 students presented a variety of statements about the contents of polynomials, such as:

"Yes, they are operations that involve letters and numbers, in which notable products are also presented that make it possible to develop the issues." (Student 2).

"These are operations that involve expressions." (Students 4 and 21).

"Polynomials are geometric shapes, by which we can define their area." (Student 22).

"Yes. They are calculations/resolutions, which use numbers and unknowns for their resolution." (Student 24).

"Polynomials are expressions that can have two or one variables. *I don't remember ☹" (Student 27).

"It's a form of calculation using more than one 'letter' (I forgot the name)." (Student 30).

"Yes. They are variable terms that replace values in a mathematical expression." (Student 31).

"Polynomials are operations involving expressions with variables." (Student 32).

"Yes, in eighth grade. It is an equation that is squared and has two parentheses with numbers, variables and signs." (Student 36).

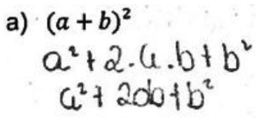
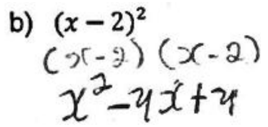
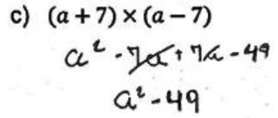
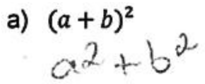
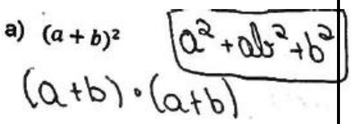
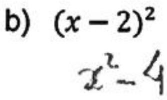
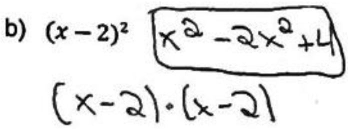
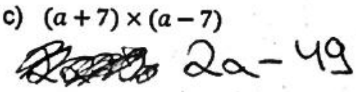
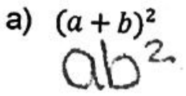
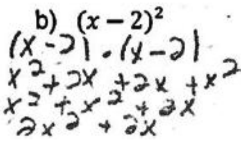
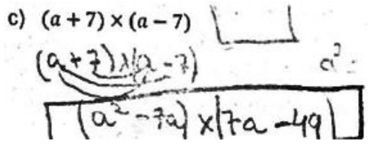
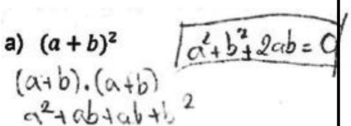
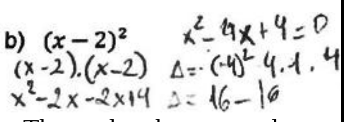
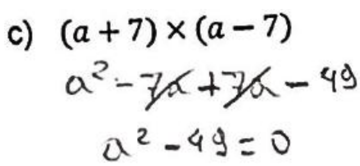
From these statements, it is observed that there are many misconceptions regarding concepts belonging to the contents of polynomials, such as: considering a polynomial as being an equation and, consequently, not distinguishing variable from unknown; this is different from what the BNCC (Brasil, 2018) recommends, because most of these students revealed that they did not establish the connections between variable and function; and between unknown and equation.

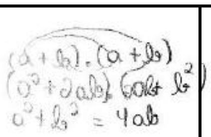
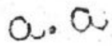
Starting with the analysis of Questions 2 and 3, Table 2 presents a number of blank questions, correct answers and errors by categories; in the latter, specifying the categories found in each item. Note that the number of correct answers per item was low, considering the total of 36 students in the class who answered the questionnaire. Furthermore, for a more detailed analysis of errors of Question 2, Table 3 provides examples for each of the items and categories found in them.

Table. 2: Quantitative of blank questions, correct answers and errors in the student diagnostic questionnaire

QUESTION	BLANK	HITS	ERRORS
2	a) 5 b) 3 c) 2	a) 13 b) 12 c) 13	a) 18 (11 Category B, 4 Category C, 2 Category D and 1 Category F) b) 21 (14 Category B, 1 Category C, 2 Category D and 4 Category F) c) 21 (1 Category B, 15 Category C, 2 Category D and 3 Category F)
3	a) 9 b) 14 c) 10	a) 7 b) 0 c) 2	a) 20 (3 Category C, 9 Category D and 8 Category F) b) 22 (3 Category C, 9 Category D, 7 Category E and 3 Category F) c) 24 (3 Category C, 13 Category D, 6 Category E and 2 Category F)

Table. 3: Examples of student resolutions in Question 02 in each of the categories of analysis

ANALYSIS CATEGORY	**ITEM A**	**ITEM B**	**ITEM C**
Category A	 The student used the Square Sum of Two Terms practical rule.	 The student used the distributive property.	 The student used the distributive property.
Category B	 The student added the square of the two terms inside the parentheses.  The student mistakenly used the practical rule to calculate the Square Sum of Two Terms.	 The student subtracted the square from the two terms inside the parentheses.  The student misused the practical rule to calculate the Square Difference of Two Terms.	 The student misused the practical rule to calculate the Product of a Sum and Difference of Two Terms.
Category C	 The student performed grouping and erroneous potentiation of monomials.	 The student was wrong to use the distributive property and grouping monomials.	 Misuse of distributive property.
Category D	 The student has correctly developed the Square Sum of Two Terms. However, it equaled the result found to zero.	 The student has correctly developed the Square Difference of Two Terms. However, it equaled the result found to zero and began to calculate the roots of the supposed equation.	 The student has correctly developed the Product of a Sum and Difference of Two Terms. However, it equaled the result found to zero.
Category F	It presented erroneous resolution, whose analysis was inconclusive.	It presented erroneous resolution, whose analysis was inconclusive.	It presented erroneous resolution, whose analysis was inconclusive.

	a) $(a+b)^2$ 	b) $(x-2)^2$ 	c) $(a+7)\times(a-7)$ 

In item A, from the analysis of the resolutions of 31 students, five categories were observed:

Category A: Among the 13 students, 8 did algebraic manipulations that showed mastery of distributive property and the notion of grouping similar terms, presenting the resolution:
$(a+b)^2 = (a+b)\times(a+b) = a^2+2ab+b^2$ (whether in this order or not).

Also, it was possible to observe that two students (Table 3) used the practical rule to find the Square Sum of Two Terms. Still, three students presented the resolution $(a+b)^2 = (a+b)\times(a+b)$, which reveal the understanding of the Square Sum of Two Terms as a power; however, without developing it algebraically.

Category B: 11 students who obtained a^2+b^2, $(a.a)+(b.b)$ and a^2+b^2 (Table 3). This type of error was quite common in student resolutions, as well as the studies of Ponte at al. (2009); Ribeiro (2001) and Nogueira et al. (2018), in calculating the Square Sum of Two Terms.

Thus, as a teaching strategy, in order for students to overcome this conceptual misunderstanding, the teacher could propose an arithmetic analysis for this type of operation, before showing it in its algebraic form. For example: $(2+3)^2 = (2+3)\times(2+3) = 5\times5 = 25$, instead of $(2+3)^2 \neq 2^2+3^2 = 4+9 = 13$. In fact, the teacher could also show the practical rule for the resolution of the Square Sum of Two Terms in this arithmetic approach:

$$(2+3)^2 = (2+3)\times(2+3) =$$
$$(2\times2)+(2\times3)+(3\times2)+(3\times3) =$$
$$2^2+2\times(2\times3)+3^3 = 4+12+9 = 25.$$

The teacher should also show the geometric representations of these cases of Notable Products. In addition to relating two mathematical axes (Algebra and Geometry), the study of this representation contributes to a better understanding of equivalent algebraic expressions, as recommended by Ponte et al. (2009), when observing that, for example, $(x+y)^2 \leftrightarrow x^2+2xy+y^2$. This understanding is relevant both for the study of Notable Products and for Polynomial Factoring, especially in cases where these two behave as inverse operations.

There are several methodological alternatives for such geometric representation to be explored, such as the Algeplan didactic material, of accessible preparation, whose main objective is to relate rectangular geometric figures (squares or rectangles) with algebraic expressions (Rosa et al., 2006). To better know about Algeplan, it is recommended to read Rosa et al. (2006).

Still in Category B, 2 students obtained a^2+2a+b^2 and $a^2+ab^2+b^2$ (Table 3). As already said, by teaching strategy, the teacher could approach the origin of this practical rule, showing it arithmetic, algebraic and geometrically.

Through Category B errors, it can be insgiven that, as much as the practical rule can help in the resolution of this type of operation, it is susceptible to forgetfulness or misuse, especially if it has been "learned", initially mechanically, by memorization.

Category C: 4 students presented the result ab^2 (Table 3) revealing an erroneous algebraic manipulation by grouping non-similar terms and trying to square the result found. The error of grouping non-similar terms was also found in the Booth et al. (2014) results, Guillermo (1992) and discussed by Ponte et al. (2009).

Category D: 2 students obtained $a^2+b^2+2ab=0$ (Table 3). This need that many students have to always equal to zero the polynomial found, error, also, observed in the results of Booth et al. (2014) and Ponte et al. (2009), reveals a misunderstanding by not dissociating the concepts of polynomials and equations, whether in the first or second degree; and, consequently, in not distinguishing variable from unknown (Ponte et al., 2009).

Again, there is a distancing from what the BNCC (Brasil, 2018) recommends. In this case, as a teaching strategy, in view of the errors belonging to Category D, in general, the teacher could resume the concepts of polynomials and equations, so that the students perceive the difference between them.

Category F: 1 student started using the distributive property, but presented the following resolution:
$(a+b)\cdot(a+b) \Rightarrow (a^2+2ab)\cdot(2ab+b^2) \Rightarrow a^2+b^2 = 4ab$
(Table 3). It is also noted that this student interpreted the

polynomial obtained as a second-degree equation. In all errors belonging to Category F, because they are very punctual errors, it is recommended that the teacher, if possible, talk in particular with students who present these types of resolution; this so that he understands the problem-solving processes used and so that he can think of teaching strategies to solve the misconceptions in learning.

Then, in item B, of the resolutions of 33 students, 5 categories were also observed:

Category A: 12 students showed mastery of distributive property and grouping (with the exception of one student, who did not group the similar terms): $(x-2)^2 = (x-2) \times (x-2) = x^2 - 4x + 4$ (Table 3). It was observed that two of these students used the practical rule to resolve the Square Difference of Two-Terms.

Still, 3 students presented the resolution $(x-2)^2 = (x-2) \times (x-2)$, which reveals the understanding of the Square Difference of Two-Terms as a power; however, without developing it algebraically.

Category B: 14 students, who presented, among the resolutions, the following: $(x.x) - (2.2)$, $x^2 - 2^2$ (Table 3), $x^2 - 4$ or $x^2 + 4$. Results of this type were also found by Guillermo (1992) and Nogueira et al. (2018). In this case, also, the teaching strategies dictated in Category B of the previous item are used. Also, 2 students obtained: $x^2 - 2x + 4$ and $x^2 - 2x^2 + 4$ (Table 3), which suggest that the students tried to use the practical rule for the resolution of Square Difference of Two-Terms.

Category C: only 1 student, the result of which was $x^2 + 2x + 2x + x^2 \Rightarrow x^2 + x^2 + 2x \Rightarrow 2x^2 + 2x$(Table 3). It is noted that the student presented errors in the use of distributive property, as also observed and discussed in Booth et al. (2014) and in Ponte et al. (2009); in addition to grouping non-similar terms.

Category D: 2 students obtained $x^2 - 4x + 4 = 0$ (Table 3). And in Category F, 4 students presented the following answers: x^2 (Table 3), $-x^2$ and $-2x^2$.

Then, in item C, of the resolutions of 34 students, it was possible to observe 5 categories:

Category A: 13 students who, through algebraic manipulations, showed mastery of distributive property and grouping (with the exception of one student, who did not group) of similar terms, reaching resolutions similar to: $(a+7) \times (a-7) = a^2 - 7a + 7a - 49 = a^2 - 49$ (Table 3). Still, one student hinted that he used the practical rule to resolve the Product of a Sum and Difference of Two Terms.

Category B: one student replied $2a - 49$ (Table 3). He may have tried to use the practical rule for the Product of a Sum and Difference of Two Terms by missing the square of the first term.

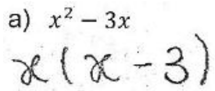
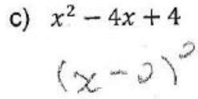
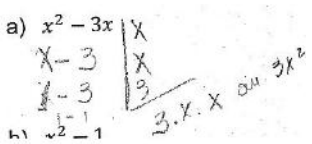
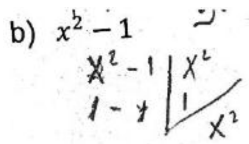
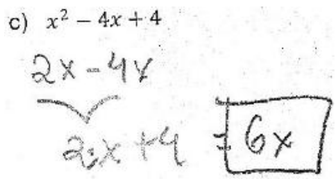
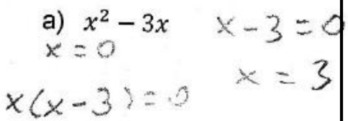
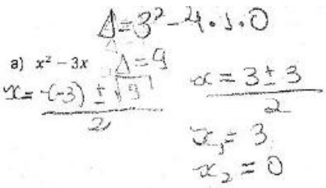
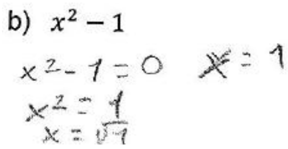
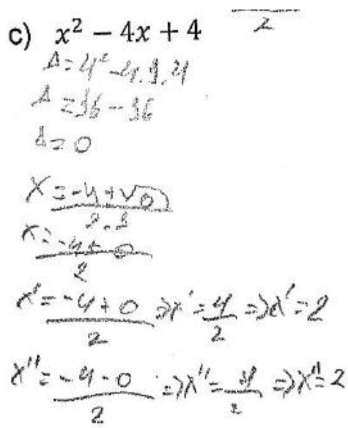
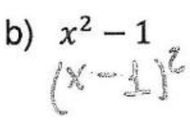
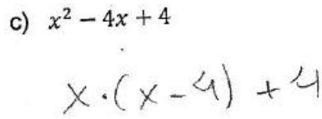
Category C: 15 students presented most of the errors in the use of distributive property, such as the resolution $a^2 - 7a - 7a + a^2 = a^2 + a^2 - 7a - 7a = 2a^2 - 14a$ and the resolution found in Table 3. In addition to these, 2 students mistakenly grouped the monomials of the factors, obtaining $7a \times (-7a)$. Also, three students presented the following resolution: $(a+7) \cdot (a-7) \Rightarrow (a^2 + 7a) \cdot (7a - 49) \Rightarrow a^2 + 14a - 49$

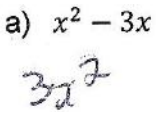
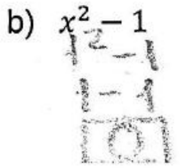
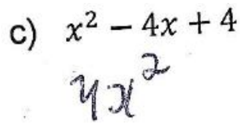
Still, two students who made incorrect use of the power property $a^m \times a^n = a^{m+n}$, presenting the following resolution: $(a+7) \times (a-7) = (a+7)^2$. Finally, a student, who made incorrect use of the Square Difference of Two-Terms and the Square Sum of Two-Terms for the Product of a Sum and Difference of Two Terms, obtaining: $(a+7) \times (a-7) \Rightarrow (a^2 + 2a + 7^2) \times (a^2 - 2a + 7^2)$.

Category D: 2 students, who obtained $a^2 - 49 = 0$ (Table 3). And in Category F, 3 students, who presented the answers: $a\,(7 - 7 + a)$, $a = 49/16a \cdot a$ and $a \cdot a$ (Table 3).

Also, for a more detailed Error Analysis of Question 3, Table 4 provides examples for each of the items and categories found in them.

Table. 4: Examples of student resolutions in the face of Question 3, in each of the categories of analysis

ANALYSIS CATEGORY	**ITEM A**	**ITEM B**	**ITEM C**
Category A	 The Common Factor case was used.	Category not contemplated in this item.	 The case of Perfect Square Trinomials was used.
Category C	 Factored the polynomial using the method of factoring numbers into prime factors.	 Factored the polynomial using the method of factoring numbers into prime factors.	 Performed erroneous grouping of monomials.
Category D	 Factored correctly. However, it equaled the result found to zero and obtained the roots of the supposed equation.  It equaled the polynomial given to zero and calculated the roots of this supposed equation.	 It equated the polynomial given to zero and calculated the roots of this supposed equation.	 It equated the polynomial given to zero and calculated the roots of this supposed equation.
Category E	Category not contemplated in this item.	 Incorrectly used the case of Perfect Square Trinomial Factoring.	 The student misused the case of Common Factor.

Category F	 It presented erroneous resolution, whose analysis was inconclusive.	 It presented erroneous resolution, whose analysis was inconclusive.	 It presented erroneous resolution, whose analysis was inconclusive.

In item A of the resolutions of 27 students, it was possible to observe 4 categories:

Category A: 7 students achieved the expected result for item: $x(x-3)$ (Table 4). It was shown domain of the Common Factor Factoring.

Category C: 3 students, 2 of which understood the act of Polynomial Factoring as a factorization of numbers in prime factors (Table 4) finding varied results, such as $3x^2$. Another student showed knowledge of Common Factor Factoring; however, he performed an erroneous algebraic manipulation, obtaining: $x^2 - 3x = x(x - 3x)$.

Category D: 9 students, especially 2 students who obtained: $x(x-3) = 0$ (Table 4). Of the remaining 7, 3 of these students (Table 4) came to find the two roots of the supposed second-degree equation. Another student obtained $x^2 = \frac{4}{3}$. And in Category F, 8 students presented the following answers: $(x-1)^2$, $(x-3)$, $x = 3$, $3x^2$ (Table 4), $-2x^2$, $2x$ and $-3x = \sqrt{x}$.

In item B of the resolutions of 22 students, it was possible to observe 5 categories:

Category C: 3 students, 2 of which, understood the act of Polynomial Factoring as a factorization in prime factors, finding x^2 (Table 4).

Category D: 9 students and one of them wrote $x^2 - 1 = 0$. Also, 5 students obtained $x = 1$, one of the roots of the supposed second-degree equation found (Table 4). The other students did not find the two roots of the supposed second-degree equation.

Category E: 7 students, 5 of which performed incorrect use of the case of Common Factor Factoring and, the other 2, in the case of Perfect Square Trinomial Factoring, obtaining: $x(x) - 1$; $x(x - 1x)$; $x(x - 0) - 1$ and $(x-1)^2$ (Table 4). In Category F, 3 students presented the following answers: x^2, x and 0 (Table 4).

Finally, in item C of the resolutions of 26 students, it was possible to create 5 categories:

Category A: 2 students achieved the expected result for the item: $(x-2)^2$ (Table 4); without showing, however, how they achieved this result. It is noteworthy that the Polynomial Factoring requested in this item consists of the inverse operation of item B of the previous question, to which these two students developed the Notable Product. Thus, it is possible that the hit of the first is related to the hit of the second.

If this was the case, it is observed that the understanding that $(x-2)^2 \leftrightarrow x^2 - 4x + 4$, in fact, is relevant to the study of Notable Products and Polynomial Factoring, in specific cases in which the latter are reverse operations (Ponte at al., 2009).

Category C: 3 students who understood the act of Polynomial Factoring as a factorization in prime factors, finding $4x^2$; or who performed erroneous groupings between non-similar monomials, obtaining $6x$ (Table 4).

Category D: 13 students, 11 of which obtained at least one of the two roots of the supposed second-degree equation considered (Table 4). Among these, two came to calculate the delta value, but did not continue with the use of quadratic formula.

Category E: corresponds to 6 students, 5 of which made incorrect use of the practical rule of Polynomial Factoring by Common Factor, obtaining: $x(x-4) + 4$ (Table 4); $x(-4x) + 4$ and one of them tried to solve by the case of Factoring of the Trinomial type $x^2 + Sx + P$. Finally, in Category F, 2 students who presented the resolutions $4x^2$ (Table 4); x^2 and x.

In order to demystify the errors observed in this Question 3, by teaching strategies, in addition to those already exposed in the previous question, the teacher could, as suggested in relation to the Notable Products: propose investigations/explorations regarding the arithmetic, algebraic and geometric representations of the cases of Polynomial Factoring. For geometric representation, it is also suggested the use of Algeplan didactic material (Rosa et al., 2006).

After the analysis of Questions 2 and 3, it was observed, from a general perspective, as revealed in Table 5, that the 3 most frequent types of errors were those belonging to Categories D, C and B, respectively.

Table. 5: Quantitative appearances of the categories of analysis.

ANALYSIS CATEGORY	Category A	Category B	Category C	Category D	Category E	Category F
QUANTITATIVE APPEARANCES	47	25	29	34	13	20

The most frequent errors, belonging to Categories D, C and B reinforce the conceptual misconceptions already observed in the student resolutions and discussed: before polynomials, despite correctly developing the Notable Products and correctly factoring the polynomials, the students, in the end, tend to match the result found to zero and to use resolutive processes to find the roots of the supposed equations; also, they presented several inadequate algebraic manipulations, such as the grouping of non-similar monomials and the use of distributive property; as well as, when using erroneously the practical rules for the development of Notable Products.

Finally, regarding the erroneous statements about the content of polynomials, presented in the answers of the 10 students mentioned at the beginning of this topic, in front of Question 1, it was observed in their resolutions that: only 3 of them fully agreed to Question 2 and one another correctly hit only item C of this question; and practically all of them presented erroneous resolutions to Question 3, except for one student who hit item A of the latter. Thus, before this cut out of 10 students, it can be said that conceptual errors were also accompanied by procedural errors; thus, showing a possible relationship between them.

VI. CONCLUSION

After the research, it is understood that it achieved its objectives, since a detailed and categorized analysis of the correct answers and errors present in the student resolutions was made, in view of the questions about Notable Products and Polynomial Factoring, thus exemplifying how Error Analysis works as a research methodology in Mathematics Education. It is notable that, although it is a particular sample, the categories presented are likely to be observed in other contexts and school grades.

Also, some suggestions of investigations/explorations were presented, so that teachers, who teach Mathematics, can contemplate the Analysis of Errors in their bias of teaching methodology. Thus, it is expected that this research can contribute to the daily professional practice of such teachers.

From perspectives of future work, similar Analysis of Errors, in its methodological bias of research and teaching, could be performed by teachers in the year that initially teaches the contents of polynomials, usually in the 8^{th} grade, because the students' mistakes would be decreased more naturally, so that errors, instead of being condemned and punished, would serve as a springboard for learning (Cury, 2018; Luckesi, 1999) and, possibly, students would bring less of these errors to the other years and/or levels of their schooling.

ACKNOWLEDGEMENTS

We thank to CNPq and the Academic Strengthening Program (PFA), for the incentive, including financial, to this research. This support enables the production and sharing of knowledge, from University of Pernambuco (UPE) Campus Petrolina, to society as a whole.

APPENDAGES

Appendix A - Cut-out of the student diagnostic questionnaire proposed to students

Question 01. Have you ever studied polynomials in math classes? Write down everything you know about polynomials.

Question 02. Develop the following Notable Products:

a) $(a+b)^2$; b) $(x-2)^2$; c) $(a+7)\times(a-7)$.

Question 03. Factor the following polynomials:

a) x^2-3x; b) x^2-1; c) x^2-4x+4.

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