Analysis of the Colebrook-White Equation and further approaches to solve Fluid Loss Coefficient Definition Problems

Yan Oliveira Vasconcellos[1], Francisco Jarmeson Silva Bandeira[2], Leonardo Dias Pereira[3], Flavio Maldonado Bentes[4], Marcelo de Jesus Rodrigues da Nóbrega[5], Hildson Rodrigues de Queiroz[6], Fabiano Battemarco da Silva Martins[7], Diego Meireles Lopes[8]

[1]Postgraduate in Supply Engineering (UniBF), Postgraduate in Engineering and Maintenance Management (UniBF). MechanicalEngineer(UNISUAM), Rio de Janeiro, Brazil.
[2]Doctor in Mechanical Engineering (UFRJ), Master in Mechanical Engineering (UFRJ). Professor at the Federal University of Pará (UFPA), Pará, Brazil.
[3]PhD Student in Mechanical Engineering (PUC-Rio), Master in Mechanical Engineering (PUC-Rio). Professor at UNISUAM, Rio de Janeiro, Brazil.
[4]Doctor in Mechanical Engineering (UFRJ), Master in Mechanical Engineering (UnB). Senior Researcher (FUNDACENTRO), Rio de Janeiro, Brazil.
[5]Post-Doctor Senior in Civil Engineering (UERJ), Doctor in Engineering (PUC-Rio), Master in Technology (CEFET/RJ). Professor at UNIGAMA, CEFET-RJ andSanta Úrsula University, Rio de Janeiro, Brazil.
[6]Master in Mechanical Engineering (UFF), Specialist in Petroleum and Natural Gas Engineering (Universidade PETROBRAS), Professor at UNISUAM, Petroleum Engineer (PETROBRAS), Rio de Janeiro, Brazil.
[7]Master in Agricultural and Environmental (UFRRJ), Professor at UNIGAMA and Santa Úrsula University, Rio de Janeiro, Brazil.
[8]PhD Student in Mechanical Engineering and Materials Technology (CEFET-RJ), Master in Mechanical Engineering and Materials Technology (CEFET-RJ) and Professor at Santa Úrsula University (USU), Rio de Janeiro, Brazil.

Abstract**—** *With the advancement of fluid mechanics in engineering, the need to estimate the pressure drop coefficient, becomes necessary for flow loss calculations in order to be able to measure pipe diameter or stipulate flow regimes that are required for a given situation. This coefficient appears in the Darcy-Weisbach formula in equality with the Poiseuille equation and is now measured by the Colebrook-White equation. However, because this equation presents a different characteristic, where the coefficient appears on both sides of the same equation, scholars of the area over time modeled approximations derived from this previous knowledge. In this work we will approach the Colebrook-White principles and their subsequent approaches. The aim of this paper is to analyze the correlations cited, as well as their authors, also analyzing the relative errors between the approximations and the Colebrook-White equation at specific intervals for the relative roughness and the Reynolds number and, from this, to determine which ones. have the lowest relative error.*

Keywords**—fluid mechanics, pressure loss coefficient, Colebrook-White equation and mathematical approximation.***

I. INTRODUCTION

Engineering regarding fluid mechanics has several focuses of study. One of these focuses is on the flow of fluids and their particularities. The focus of this work is on the flow and its concepts, with respect to the flow resistance factor for pressure loss calculations in fluids.

It is discovered through studies that when a fluid gets closer and closer to the "wall" of the pipe, its flow velocity tends to zero, that is, there is a resistance to flow (viscosity of the inner surface of the pipe).

According to Sá Marques and Sousa (1996), the Colebrook-White equation is commonly mentioned in the

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literature on fluid mechanics and has wide applicability by the technicians involved in this specialty, being considered the closest to the physical reality of flows.

To Coban (2012), the Colebrook-White equation is an implicit formula that generates the best result for the pressure loss coefficient in turbulent regime, however, to obtain these results there is a need to perform iterative processes. Such equation has as parameters the relative roughness (ε/D) and the Reynolds number (Re).

Therefore, there are several studies of explicit equations to measure this resistance precisely, replacing the Colebrook-White equation, as closely as possible to the result of the implicit equation.

The calculation of the head loss is the main issue of this work, however there is a problem, since the Colebrook-White equation that is used for this purpose has a characteristic of being implicit, since it presents the coefficient on both sides of the equality.

Due to this characteristic, calculating using this equation becomes complex, as there is a need to perform iterative processes to obtain a result.

With this problem, we seek to gather information, data and authors that otherwise express this modeling, in a less complex way, aiming at an explicit equation and with the results as close as possible to the formula made by Colebrook-White.

For flow in industrial pipelines, knowing how to accurately measure the head loss is essential, as there are several unknowns to be seen, be it the material, the roughness to be worked, the dimensioning, the necessary performance for each case, etc.

According to Resende (2007), the head loss is highlighted, for example, in a hydroelectric power plant, because as the head loss is increased, the generation capacity is decreased.For this, a formula is needed that accurately models this coefficient.

According to Zidan (2015), a suggestion by C.M. White for transition formula which similar to those obtained experimentally for commercial pipes, was simply add together the lower limits of integration y, which satisfy the rough and smooth pipe laws, providing the general formula.

This article has as general objectives to analyze the explicit equations and verify which ones have the lowest and highest average relative error and analyze the relative errors and organize them from the lowest to the highest percentage, since the lowest percentage will have results of coefficient closest to those of Colebrook-White and, by definition, the ideal approximation will be considered. The highest percentage will demonstrate the opposite.

II. THE COLEBROOK-WHITE EQUATION AND ITS APPROXIMATIONS

Below are briefly presented the equations used for study, followed by the calculations to make comparisons.

Colebrook-White equation

To Baqer (2015), the Colebrook equation is an implicit equation that combines experimental results from studies of turbulent flow in rough tubes. The equation is used to iteratively solve the Darcy-Weisbach friction factor “λ”.

According to Soares (2012), we present equation 2.1.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}}\right) \quad (2.1)$$

Moody approach

Pimenta (2017) explains Moody's equation (1947) aspresentedin 2.2.

$$\lambda = 0.0055\left[1 + \left(2 * 10^4\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{\frac{1}{3}}\right] \quad (2.2)$$

Wood approach

Asker, Turgut and Coban (2014), Wood (1966) made correlations validating region extensions for Re > 10^4 and 10^{-5} < (ε/D) <$4x10^{-2}$. Equation 2.3 demonstrates such an approximation.

$$\lambda = a + b * Re^{-c} \quad (2.3)$$

Where:

$$a = 0.53 * (\varepsilon/D) + 0.094 * (\varepsilon/D)^{0.225} \quad (2.4)$$

$$b = 88 * (\varepsilon/D)^{0.44} \quad (2.5)$$

$$c = 1.62 * (\varepsilon/D)^{0.134} \quad (2.6)$$

Churchill approach

According to Brkić (2011), Churchill's approach (1973) is demonstrated in equation 2.7.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.71D} + \left(\frac{7}{Re}\right)^{0.9}\right) \quad (2.7)$$

Eck approach

According to Asker, Turgut and Coban (2014), Eck (1973) performs an approximation expressed in equation 2.8.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.71D} + \frac{15}{Re}\right) \quad (2.8)$$

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Haaland approach

Fox, Mcdonald and Pritchard (2014), Haaland (1984) contributed to the approximation of Colebrook-White's implicit equation to "λ" (Darcy-Weisbach friction factor) and can be expressed in equation 2.9.

$$\frac{1}{\sqrt{\lambda}} = -1.8\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11} + \frac{6.9}{Re}\right] \quad (2.9)$$

Tsal approach

According to Pimenta (2017), Tsal's approximation (1989) is expressed in equation 2.10.

$$A = 0.11\left(\frac{68}{Re} + \frac{\varepsilon}{D}\right)^{0.25} \quad (2.10)$$

Where:

$A \geq 0.018$; $\lambda = A$

$A < 0.018$; $\lambda = 0.0028+0.85A$

Buzzelli approach

According to Asker, Turgut and Coban (2014), Buzzelli (2008) developed the relationship present in equation 2.11.

$$\frac{1}{\sqrt{\lambda}} = A - \left[\frac{A + 2log(B/Re)}{1 + (2.18/B)}\right] \quad (2.11)$$

Where "A" and "B" are expressed in the equations 2.12 and 2.13, respectively below.

$$A = \frac{(0.744ln(Re) - 1.41)}{\left(1 + 1.32\sqrt{\varepsilon/D}\right)} \quad (2.12)$$

$$B = \frac{\varepsilon}{3.7D}Re + 2.51A \quad (2.13)$$

Relative error

To Asker et al (2014), the calculation that will be the basis for the analysis of the approximations in relation to the Colebrook-White equation will be that of the relative error. The relative error can demonstrate how close the result of the coefficient of the explicit equation will be when compared to the coefficient of the equation. Such an equation of relative error can be expressed in equation 2.14.

$$RE = \left(\frac{|\lambda_{CW} - \lambda_{approach}|}{\lambda_{CW}}\right)100 \quad (2.14)$$

Where:

RE = Relative error (%)

λ_{CW} = Friction factor of the Colebrook-White equation (dimensionless)

$\lambda_{approach}$ = Friction factor of the approach (explicit equation) in question (dimensionless)

The following Table 1 illustrates the error percentages and their respective classifications.

Table 1 –Relative error (%) and their classifications

MeanRelativeError (%)	Classification
≤ 0.55	Perfect
0.56 – 1.00	Good
1.10 – 2.00	Regular
2.10 – 3.00	Weak
> 3.00	Terrible

Source: Pimenta (2017)

III. RESULTS AND DISCUSSION

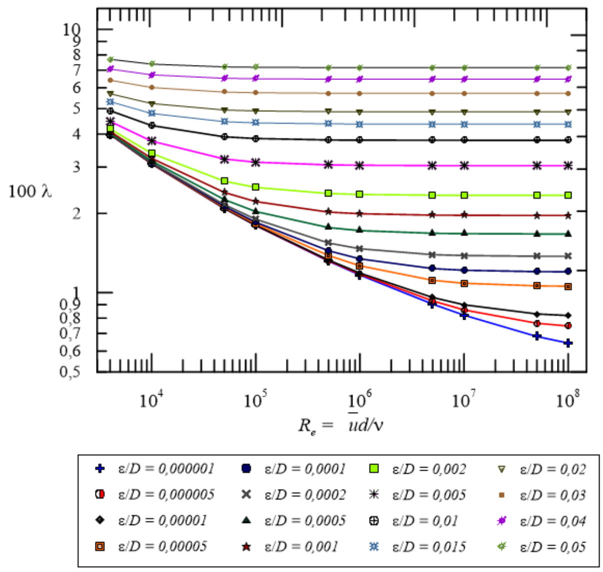
The present work obtained friction factor data considering that the turbulent flow, with Reynolds number greater than four thousand. To obtain the data, sixteen values of relative roughness were used, which correspond from the smooth surface to a rougher surface.

After that, there will be a discussion of relative errors between Colebrook-White and their approximations, to observe the best explicit equations regarding relative errors.

Colebrook-White

The Colebrook-White equation will be used as a reference for comparison with the other explicit equations.For the friction factor calculations, the following parameters were used: $4x10^3 \leq Re \leq 10^8$and $10^{-6} \leq \varepsilon/D \leq 5x10^{-2}$. The graph 1 shows the calculated values of the friction factor for Colebrook - White equation.

Graph 1 –Friction factor with Colebrook-White equation



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Graph 1 shows the values for the friction coefficient in the vertical part (y-axis) and the values of the Reynolds number in the horizontal (x-axis). The respective values of the relative roughness are shown in the lines to which the legend explains their respective colors and values.

It can be seen from Graph 1 that the greater the Reynolds number, the lower the value of the coefficient "λ" for the relative roughness intervals. It is also noticed that there is a tendency for values of "λ" very close to the Reynolds intervals, especially in the periods of $5x10^4 \leq Re \leq 10^8$, where the results approach the equality as the value of the relative roughness grows.

This can be explained by the fact that Equation (2.1), together with the explicit equations, presents a sum of the relative roughness (ε/D) with the Reynoldsnumber (1/Re), since the rest will be just a relation of mathematical operations with constants. This sum, as the relative roughness increases and goes through the Reynolds number intervals, it tends to have a common result. For example, for a relative roughness = $5x10^{-2}$, in the Reynolds number range between $5x10^4 \leq Re \leq 10^8$, the sum will tend to $5x10^{-2}$, as the term "1 / Re" will tend to zero.

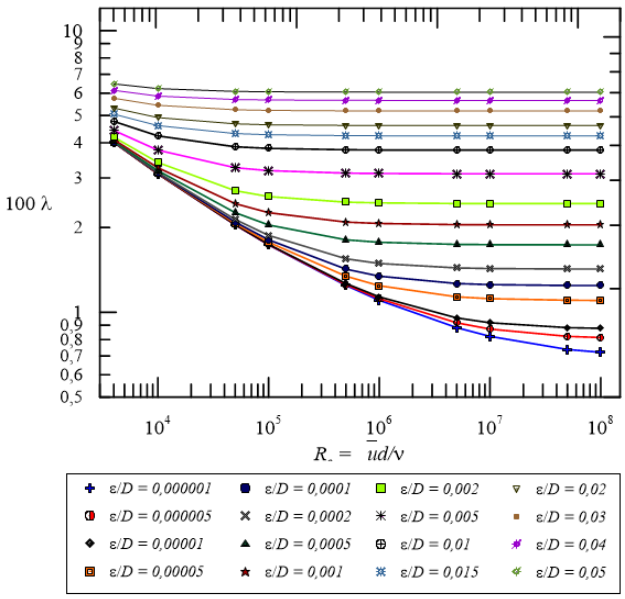
Bandeira (2015) reports that the viscous sublayer presents a thickness which is capable of covering the rough elements, it will not have a significant loss, in this condition it can be said that the flow is in a hydraulically smooth regime. However, the thickness of the viscous sublayer is influenced by the Reynolds number, as the Reynolds number increases, the thickness of the viscous sublayer decreases and for a given high Reynolds number some rough elements emerge significantly, at that moment the friction becomes a function of Reynolds number and roughness as well. For even higher Reynolds values, all the rough elements emerge through the viscous sublayer and the loss of pressure depends on the size of the rough elements, in this condition the flow is in a rough regime.

According to Schlichting (1979), the friction factor varies up to a certain Reynolds number, this is due to the ratio between the protrusions of the surfaces and the height of the boundary layer, however, after a certain point the friction factor stops varying , that is, the friction factor no longer depends on the Reynolds number, this is because the flow has reached a completely rough regime, being possible to visualize in the graph 1 the friction factor remains constantfor each line that represents each relative roughness.

Moody

Calculations will be performed at the intervals above for Equation 2.2. Graph 2 shows the values of λ according to the relative roughness and Reynolds number.

Graph 2 - Friction factor for the Moody equation



It can be seen from Graph 2 that there is a difference with Graph 1 in values for "λ", such discrepancies will be addressed in the error percentage, using Equation 2.14. The behavior and explanation for it are similar to Graph 1, but there are differences in values due to the approximation of the model equations.

Wood

The wood approximation, equation (2.3) was used to obtain data that are shown in Graph 3.

Graph 3 shows the data in which the Wood equation was used, with the behavior of the lines slightly different from the previous graphs, it being possible to observe that for low values of the Reynolds number the results are more different than the Colebrook - white data.

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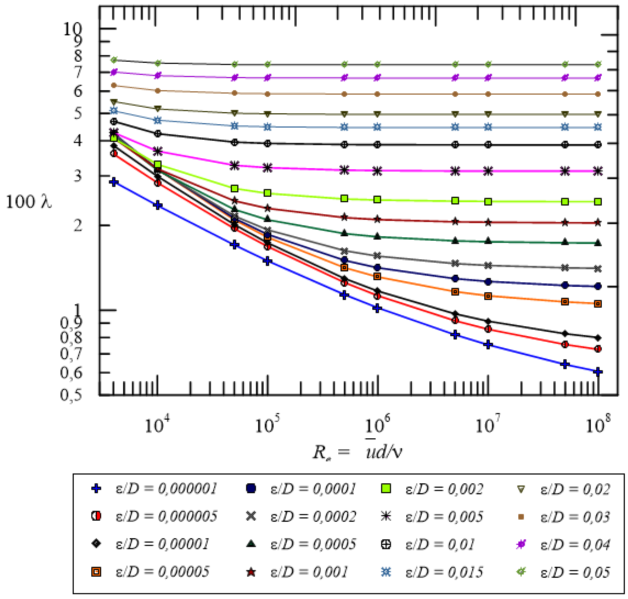
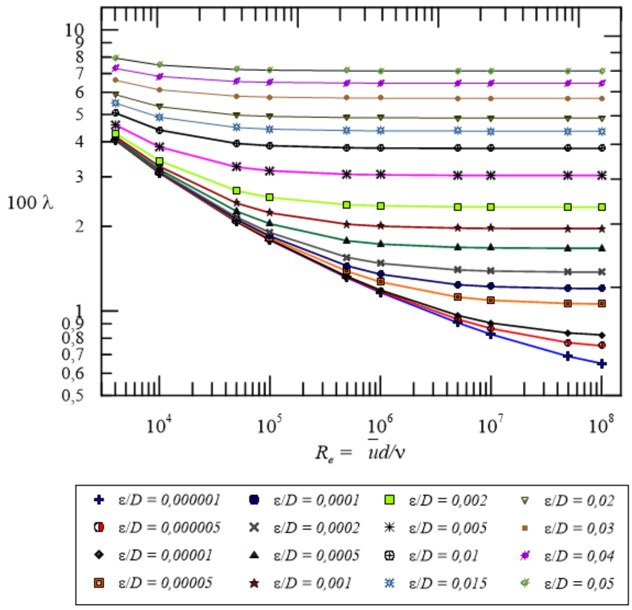
Graph 3 - Friction factor with Wood's equation



Churchill

According to equation (2.7), Graph 4illustrates the friction factor values in the Churchill equation for each interval as shown.

Graph 4 - Friction factor with Churchill's equation



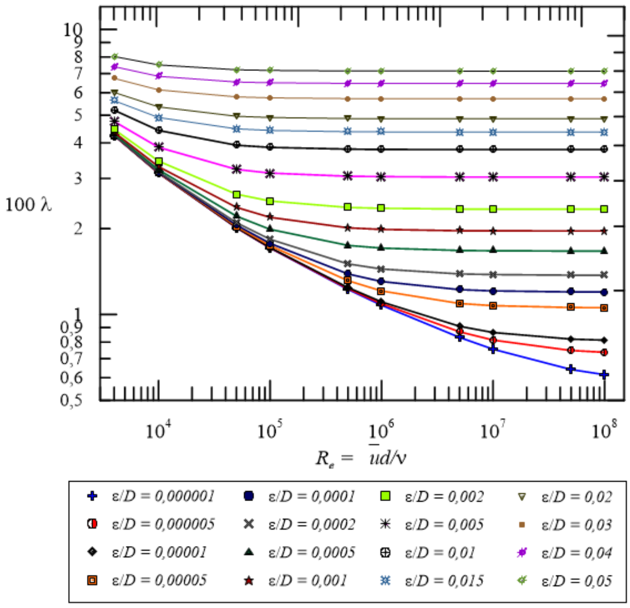
Graph 4 shows the behavior of the lines and margins of values remarkably similar for the coefficient when compared with the Colebrook - White data.

Eck

The Eck model was also simulated with the same conditions as the simulation of the other models.

The Graph 5 contemplates the results of the "λ" coefficient for equation (2.8), for the "Re" intervals and the relative roughness.

Graph 5–Friction factor with Eck equation



It can be seen from Graph 5 that there is the same behavioral similarity of the graphs of coefficient values previously mentioned, and with values of "λ" awfully close to the results of Colebrook-White.

Haaland

For equation 2.9, the following graph 6 is made to demonstrate the values of the friction factor.Graph 6 shows the results of "λ" for the pre-determined "Re" and "ε/D" intervals.

It can be seen from Graph 6 that there is the same behavioral similarity and with coefficient values tending to equality when compared to Graph 1.

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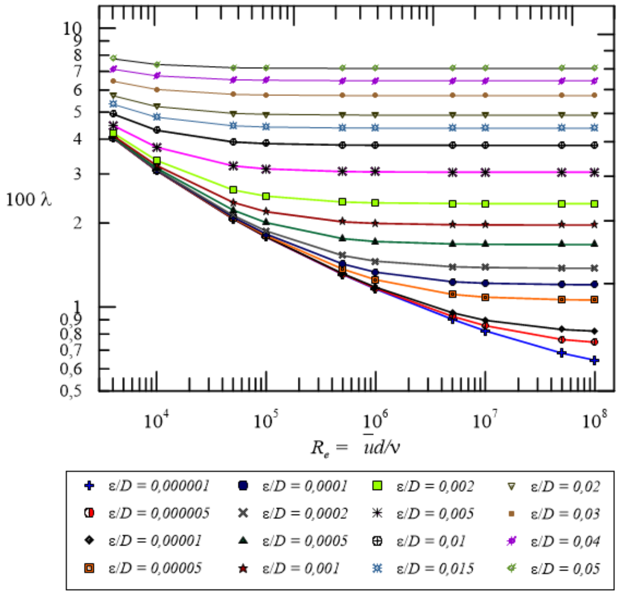
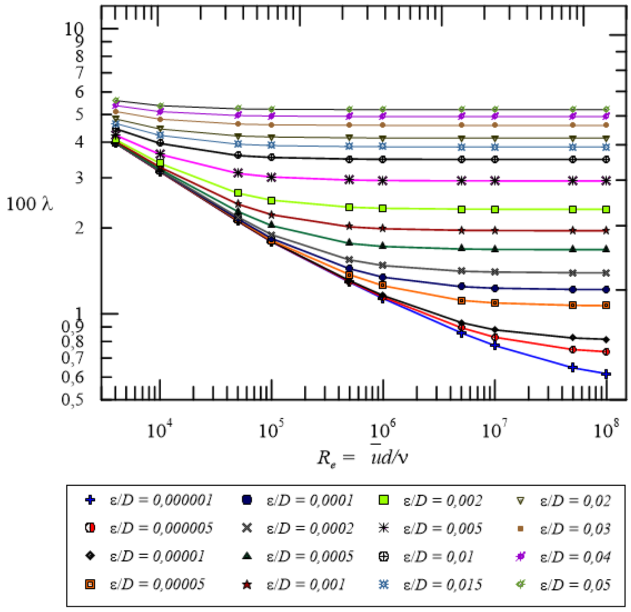
Graph 6–Friction factor with Haaland equation



Tsal

For the equation Tsal(2.10), was used for show the data in the graph 7.

Graph 7–Friction factor with Tsal equation



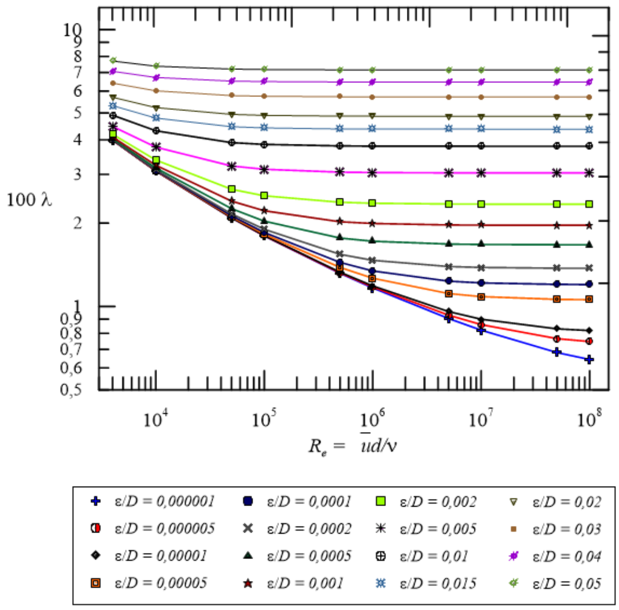
According to the above data, there is a similarity in behavior but there is a considerable discrepancy, it is possible to verify that for low Reynolds numbers the relative roughness present nearest friction factor values than the other methods, it becomes clearer when it is held a comparison with other graphics.

Buzzelli

Buzelli propose the equation (2.11) for determination friction factor. The graph 8 shown the coefficient values for the "Re" and "ε/D".

It can be seen in Graph 8 that it is most similar to Graph 1 in the values of "λ", with low discrepancies will be addressed in the percentage of error, using Equation 2.14. And the low discrepancy makes this approximation method has good results compared to Colebrook - White.

Graph 8–Friction factor withBuzzelli equation



Relative Error

Considering that all the models presented above are an approximation of the Colebrook - White equation, the relative error will occur when comparing the results of each model with Colebrook-White.

The comparison made in the present work lists the results of all models for each relative roughness.

Error for relative roughness of 0.000001

The graph 9 shows the result of all models, including Colebrook - White, for the relative roughness of 0.000001.

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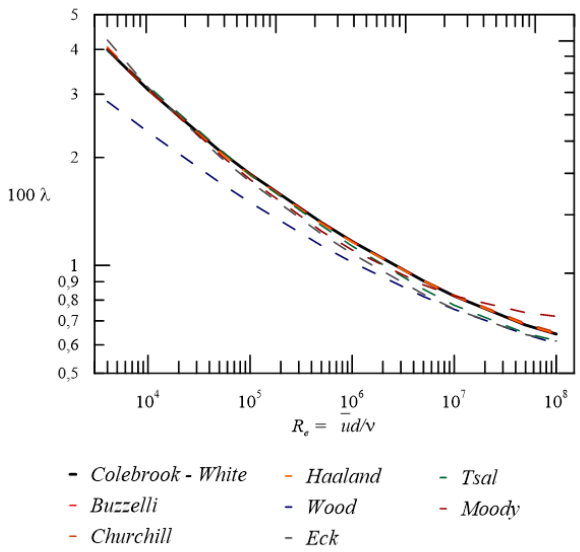
Graph 9–Comparison of the friction factor models for relative roughness of 0.000001



The data shown in graph 9, it is possible to observe that for the relative roughness of 0.000001, Wood's method presented a more discrepant result when compared to the Colebrook-White data.Table 2 shows percentage values of the relative error between all the models.

Table 2 - Values (in %) of the relative errors for the relative roughness de 0,000001

Relative Error (values in %)for ε/D = 0,000001							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	0,6032	28,2335	1,7526	6,4438	1,2910	0,4707	0,0395
1x10⁴	0,4725	23,7204	0,4078	1,5131	0,0076	2,2811	0,0280
5x10⁴	2,2053	18,3812	0,5276	3,5734	0,8608	1,1145	0,0201
1x10⁵	3,5449	16,7771	0,6138	4,9514	0,9308	0,5055	0,0187
5x10⁵	5,4663	13,7958	0,4256	7,0641	0,7398	1,9513	0,0171
1x10⁶	5,4141	12,5954	0,2387	7,6295	0,5739	2,9696	0,0165
5x10⁶	2,5372	9,5795	0,3294	8,1955	0,1499	5,1960	0,0139
1x10⁷	0,0042	8,2094	0,6003	8,0038	0,0032	5,7462	0,0114
5x10⁷	8,2916	5,8529	1,1037	5,9347	0,1845	5,1516	0,0028
1x10⁸	12,0702	5,8164	1,1534	4,4363	0,1956	4,1409	0,0004

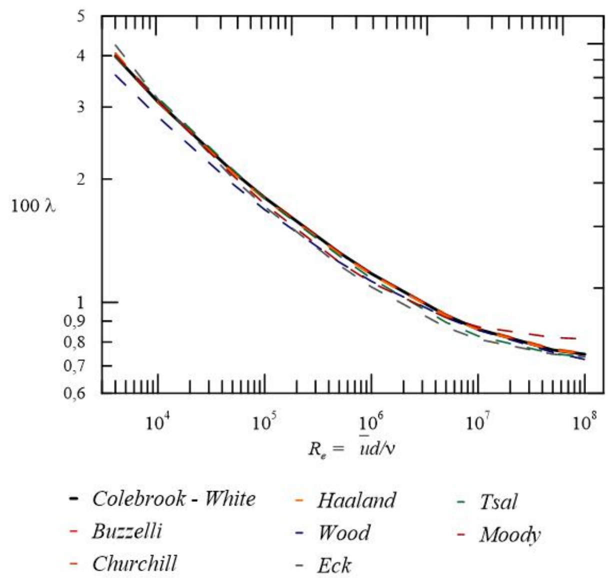
When comparing the results contained in table 1 with table 2, it is possible to conclude, that for the relative roughness condition of 0.000001, the approximation models could be classified as: Moody (0.0042≤RE ≤12.0702) Good for low Reynolds numbers and terrible for high Reynolds numbers; Wood (5.8164 ≤ RE ≤ 28.2335) Terrible; Churchill (0.2387 ≤ RE ≤ 1.7526) between perfect and regular; Eck (1.5131 ≤ RE ≤ 8.1955) between regular and terrible; Halland (0.0032 ≤RE ≤1.2910) between perfect and regular; Tsal (0.4707 ≤ RE ≤ 5.7462) between perfect and terrible; Buzzelli (0.0004 ≤ RE ≤ 0.0395) perfect result.

From the analysis of the graph 9 and the table2, it is possible to verify that the Wood, Moodyand Eck models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.000005

The graph 10 shows the result of all models, including Colebrook - White, for the relative roughness of 0.000005.

Graph 10–Comparison of the friction factor models for relative roughness of 0.000005



The data shown in graph 10, it is possible to observe that for the relative roughness of 0.000005,in general method all have a tendency to next Colebrook-White data.

Table 3 shows percentage values of the relative error between all the models.

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Table 3 - Values (in %) of the relative errors for the relative roughness de 0.000005

Relative Error (values in %)for ε/D = 0.000005							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	0,6022	10,0637	1,7539	6,4438	1,2853	0,475	0,0419
1x10⁴	0,4744	8,0763	0,4105	1,5131	0,0024	2,2756	0,0298
5x10⁴	2,1781	6,7929	0,5177	3,5734	0,8884	1,1186	0,0212
1x10⁵	3,4861	6,503	0,5972	4,9514	0,9738	0,5022	0,0194
5x10⁵	5,1982	5,2293	0,375	7,0641	0,8548	1,8314	0,0159
1x10⁶	4,9456	4,2378	0,1624	7,6295	0,743	2,6919	0,0137
5x10⁶	1,322	1,3542	0,4588	8,1955	0,4815	3,9182	0,0052
1x10⁷	1,3556	0,4532	0,6939	8,0038	0,3712	3,7189	0,0016
5x10⁷	7,3162	1,2884	0,7813	5,9347	0,0763	2,0579	0,0005
1x10⁸	8,8914	2,8841	0,6258	4,4363	0,0313	1,4248	0,0008

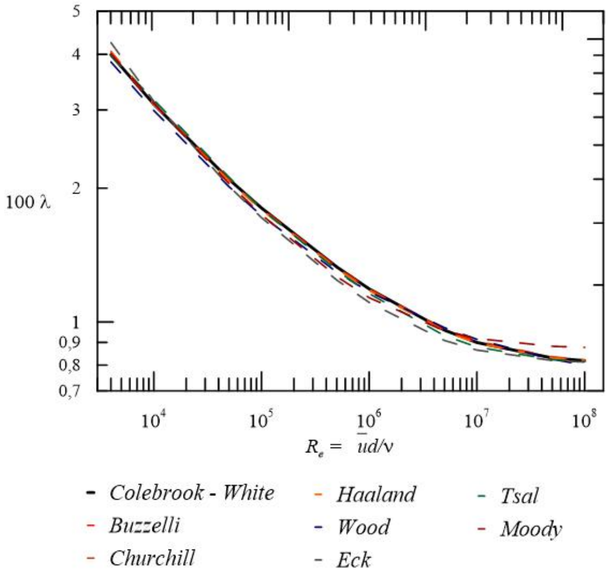
To compare the results contained in table 1 with table 3, it is possible to conclude, that for the relative roughness condition of 0.000005, the approximation models could be classified as: Moody (0.4744 ≤RE ≤ 8.8914) Perfect for low Reynolds numbers and terrible for high Reynolds numbers; Wood (0.4532≤ RE ≤ 10.0637) between perfect and terrible; Churchill (0.1624≤ RE ≤ 1.7539) between perfect and regular; Eck (1.5131 ≤ RE ≤ 8.1955) between regular and terrible; Halland (0.0024≤RE ≤1.2853) between perfect and regular; Tsal (0.4750≤ RE ≤3.9182) between perfect and terrible; Buzzelli (0.0008≤ RE ≤ 0.0419) perfect result.

From the analysis of the graph 10 and the table 3, it is possible to verify that the Wood, Moody and Eck models generate results with greater errors in relation to the Colebrook - White equation, while the Buzzelli models present good approximations.

Error for relative roughness of 0.00001

The graph 11 shows the result of all models, including Colebrook - White, for the relative roughness of 0.00001.

Graph 11–Comparison of the friction factor models for relative roughness of 0.00001



The data shown in graph 11, it is possible to observe that for the relative roughness of 0.00001,in general method all have a tendency to next Colebrook-White data.

Table 4 shows percentage values of the relative error between the models.

Table 4 - Values (in %) of the relative errors for the relative roughness de 0.00001

Relative Error (values in %)for ε/D = 0.00001							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	0,6010	3,6684	1,7555	6,4440	1,2790	0,4803	0,0437
1x10⁴	0,4768	3,1838	0,4140	1,5152	0,0133	2,2688	0,0312
5x10⁴	2,1443	3,7351	0,5055	3,5550	0,9171	1,1236	0,0218
1x10⁵	3,4136	3,8349	0,5768	4,9112	1,0165	0,4984	0,0196
5x10⁵	4,8824	2,6404	0,3158	6,8481	0,9507	1,6910	0,0140
1x10⁶	4,4224	1,4885	0,0782	7,2056	0,8651	2,3813	0,0104
5x10⁶	0,3719	1,2789	0,5476	6,5418	0,5901	2,8287	0,0012
1x10⁷	2,0695	1,6458	0,7100	5,4704	0,4194	2,3309	0,0000
5x1	6,245	0,61	0,5767	2,32	0,0275	0,88	0,001

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0^{7}	4	28		39		61	1
$1x10^{8}$	7,1034	2,3456	0,4120	1,3646	0,0732	0,5188	0,0008

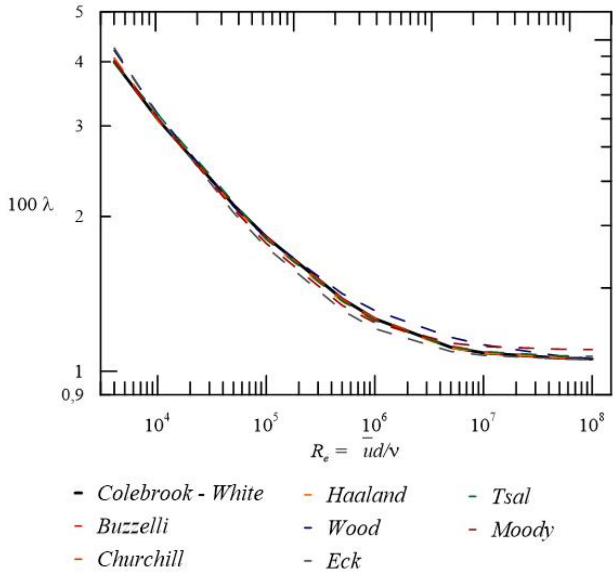
To compare the results contained in table 1 with table 4, it is possible to conclude, that for the relative roughness condition of 0.00001, the approximation models could be classified as: Moody (0.4768 ≤RE ≤ 7.1034) between perfect and terrible; Wood (0.6128≤ RE ≤ 3.8349) between good and terrible; Churchill (0.0782≤ RE ≤ 1.7555) between perfect and regular; Eck (1.3646≤ RE ≤ 7.2056) between regular and terrible; Halland (0.0133≤RE ≤1.2790) between perfect and regular; Tsal (0.4803≤ RE ≤ 2.8287) between perfect and weak; Buzzelli (0.0000≤ RE ≤ 0.0437) perfect result.

From the analysis of the graph 11 and the table 4, it is possible to verify that the Moody and Eck models generate results with greater errors in relation to the Colebrook - White equation, while the Buzzelli models present good approximations.

Error for relative roughness of 0.00005

The graph 12 shows the result of all models, including Colebrook - White, for the relative roughness of 0.00005.

Graph 12–Comparison of the friction factor models for relative roughness of 0.00005



The data shown in graph 12, it is possible to observe that for the relative roughness of 0.00005,in general method all have a tendency to next Colebrook-White data.

Table 5 shows percentage values of the relative error between the models.

Table 5 - Values (in %) of the relative errors for the relative roughness de 0.00005

Relative Error (values in %)for ε/D = 0.00005

Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
$4x10^{3}$	0,5912	5,3121	1,7682	6,4441	1,2365	0,5230	0,0513
$1x10^{4}$	0,4953	2,3521	0,4411	1,5178	0,0829	2,2142	0,0363
$5x10^{4}$	1,8846	0,6382	0,4116	3,5322	1,0725	1,1599	0,0223
$1x10^{5}$	2,8721	0,5349	0,4249	4,8617	1,2216	0,9596	0,0171
$5x10^{5}$	2,9345	2,3326	0,0352	6,5940	1,1960	0,8620	0,0039
$1x10^{6}$	1,7338	3,7910	0,3175	6,7331	1,0199	0,8212	0,0005
$5x10^{6}$	1,8791	4,5521	0,5905	5,2010	0,3647	0,1295	0,0017
$1x10^{7}$	2,9264	3,6698	0,4986	3,8857	0,1349	0,5278	0,0017
$5x10^{7}$	4,0065	0,8903	0,1969	1,2962	0,1217	0,9803	0,0004
$1x10^{8}$	4,1614	0,1426	0,1077	0,7230	0,1601	1,0484	0,0002

To compare the results contained in table 1 with table 5, it is possible to conclude, that for the relative roughness condition of 0.00005, the approximation models could be classified as: Moody (0.4953≤RE ≤4.1614) between perfect and terrible; Wood (0.1426≤ RE ≤ 5.3121) between perfect and terrible; Churchill (0.0352≤ RE ≤ 1.7682) between perfect and regular; Eck (0,7230≤ RE ≤ 6.4441) between good and terrible; Halland (0.0829≤RE ≤1.2365) between perfect and regular; Tsal (0.1295≤ RE ≤ 2.2142) between perfect and weak; Buzzelli (0.0002≤ RE ≤ 0.0513) perfect result.

From the analysis of the graph 12 and the table 5, it is possible to verify that the Moody, Wood and Eck models generate results with greater errors in relation to the Colebrook - White equation, while the Buzzellimodels present good approximations.

Error for relative roughness of 0.0001

The graph 13 shows the result of all models, including Colebrook - White, for the relative roughness of 0.0001.

The data shown in graph 13, it is possible to observe that for the relative roughness of 0.0001,in general method all have a tendency to next Colebrook-White data.

Graph 13–Comparison of the friction factor models for relative roughness of 0.0001.

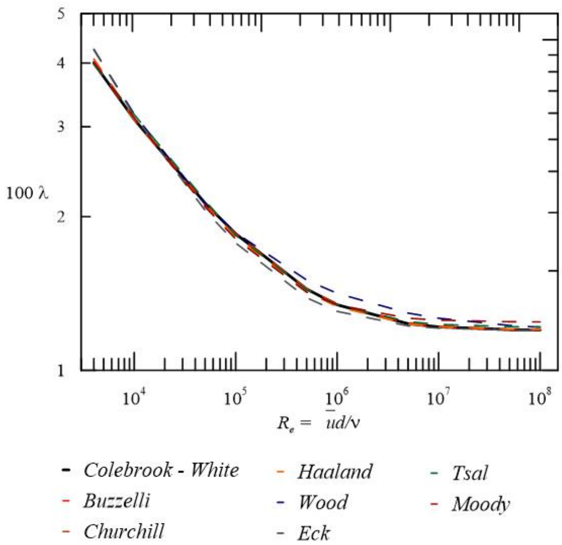


Table 6 shows percentage values of the relative error between the models.

Table 6 - Values (in %) of the relative errors for the relative roughness de 0.0001

Relative Error (values in %)for ε/D = 0.0001							
Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
$4x10^3$	0,5787	6,2406	1,7839	6,4453	1,1921	0,5762	0,0566
$1x10^4$	0,5174	2,3394	0,4744	1,5384	0,1510	2,1462	0,0393
$5x10^4$	1,5845	0,2603	0,3032	3,3563	1,1910	1,1964	0,0205
$1x10^5$	2,2794	0,4551	0,2609	4,4912	1,3439	0,7069	0,0129
$5x10^5$	1,3824	4,1494	0,2851	5,0418	1,1556	0,2866	0,0003
$1x10^6$	0,0674	5,2933	0,4993	4,3414	0,8576	0,0253	0,0004
$5x10^6$	2,5394	4,9264	0,4833	1,8676	0,1668	0,9155	0,0021
$1x10^7$	3,0996	3,9942	0,3505	1,0985	0,0011	1,1423	0,0013
$5x10^7$	3,6148	1,8884	0,1021	0,2877	0,1567	1,3602	0,0002
$1x10^8$	3,6840	1,2183	0,0430	0,1700	0,1782	1,3901	0,0001

To compare the results contained in table 1 with table 6, it is possible to conclude, that for the relative roughness condition of 0.0001, the approximation models could be classified as: Moody (0.0674 ≤RE ≤ 3.6840) between perfect and terrible; Wood (0.2603≤ RE ≤ 6.2406) between perfect and terrible; Churchill (0.0430≤ RE ≤ 1.7839) between perfect and regular; Eck (0,1700≤ RE ≤ 6.4453) between perfect and terrible; Halland (0.0011≤RE ≤1.3439) between perfect and regular; Tsal (0.0253≤ RE ≤ 2.1462) between perfect and weak; Buzzelli (0.0001≤ RE ≤ 0.0566) perfect result.

From the analysis of the graph 13 and the table 6, it is possible to verify that the Woodand Ecktmodels generate results with greater errors in relation to the Colebrook - White equation, while the Churchill and Buzzelli models present good approximations.

Error for relative roughness of 0.0002

The graph 14 shows the result of all models, including Colebrook - White, for the relative roughness of 0.0002.

Graph 14–Comparison of the friction factor models for relative roughness of 0.0002.

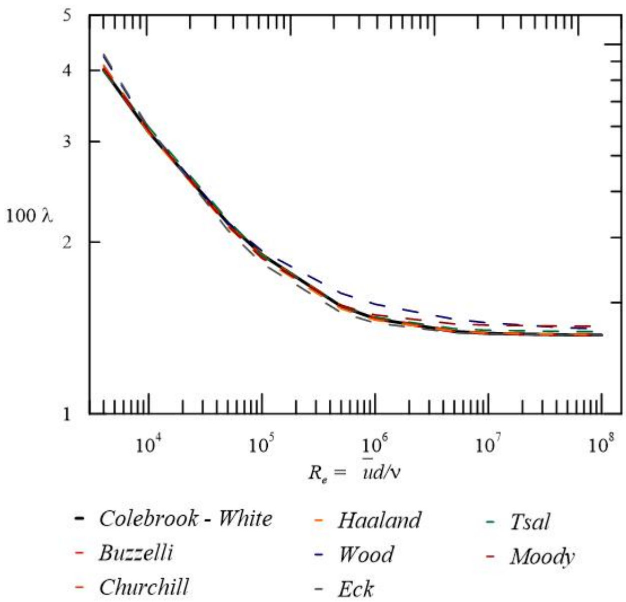


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Table 7 shows percentage values of the relative error between the models.

Table 7 - Values (in %) of the relative errors for the relative roughness de 0.0002

Relative Error (values in %)for ε/D = 0.0002

Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
4x10³	0,5536	5,5207	1,8148	6,4468	1,1169	0,6824	0,0634
1x10⁴	0,5586	1,3325	0,5387	1,5637	0,2578	2,0107	0,0421
5x10⁴	1,0551	0,1287	0,1127	3,1519	1,3197	1,2441	0,0159
1x10⁵	1,3136	1,5749	0,0003	4,0853	1,4205	0,3137	0,0067
5x10⁵	0,3931	5,4281	0,5106	3,8343	0,9325	0,1647	0,0007
1x10⁶	1,5094	6,0242	0,5801	2,9274	0,5699	0,5442	0,0023
5x10⁶	3,0849	5,0383	0,3470	0,9918	0,0122	1,1458	0,0015
1x10⁷	3,3556	4,2927	0,2205	0,5606	0,0889	1,2568	0,0007
5x10⁷	3,5886	2,9150	0,0391	0,1604	0,1771	1,3541	0,0001
1x10⁸	3,6189	2,5250	0,0014	0,1063	0,1887	1,3668	0,0000

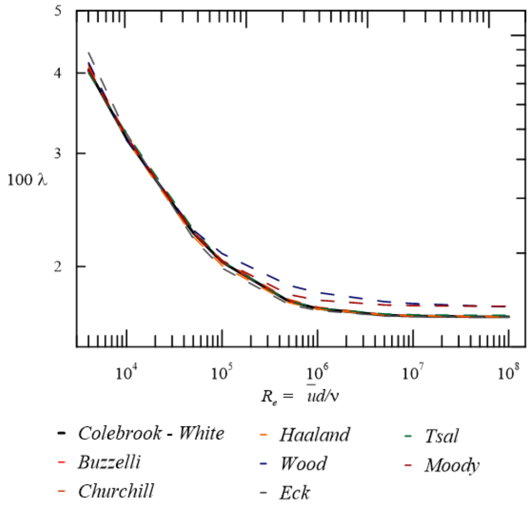
To compare the results contained in table 1 with table 7, it is possible to conclude, that for the relative roughness condition of 0.0002, the approximation models could be classified as: Moody (0.3931 ≤RE ≤ 3.6189) between perfect and terrible; Wood (0.1287≤ RE ≤ 6.0242) between perfect and terrible; Churchill (0.0003≤ RE ≤ 1.8148) between perfect and regular; Eck (0,1063≤ RE ≤ 6.4468) between perfect and terrible; Halland (0.0122≤RE ≤1.4205) between perfect and regular; Tsal (0.1647≤ RE ≤ 2.0107) between perfect and weak; Buzzelli (0.0000≤ RE ≤ 0.0634) perfect result.

From the analysis of the graph 14 and the table 7, it is possible to verify that the Woodand Eck models generate results with greater errors in relation to the Colebrook - White equation, while the Churchill and Buzzelli models present good approximations.

Error for relative roughness of 0.0005

The graph 15 shows the result of all models, including Colebrook - White, for the relative roughness of 0.0005.

Graph 15–Comparison of the friction factor models for relative roughness of 0.0005.



The data shown in graph 15, it is possible to observe that for the relative roughness of 0.0005,in general method all have a tendency to next Colebrook-White data.

Table 8 shows percentage values of the relative error between the models.

Table 8 - Values (in %) of the relative errors for the relative roughness de 0.0005

Relative Error (values in %)for ε/D = 0.0005

Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
4x10³	0,4754	2,7265	1,9038	6,4561	0,9450	0,9977	0,0738
1x10⁴	0,6591	0,6114	0,7158	1,7452	0,4651	1,6082	0,0428
5x10⁴	0,1100	1,0257	0,3036	1,9659	1,3819	1,2363	0,0060
1x10⁵	0,5033	3,0735	0,4591	2,1613	1,2810	0,2975	0,0003
5x10⁵	2,4292	5,8815	0,6182	1,1036	0,4824	0,0822	0,0032
1x10⁶	3,0521	5,9005	0,5045	0,6710	0,1984	0,1608	0,0029
5x10⁶	3,7038	4,9482	0,1862	0,2009	0,1086	0,4178	0,0006
1x10⁷	3,7977	4,5351	0,0957	0,1321	0,1536	0,4552	0,0003

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$5x10^7$	3,8751	3,8925	0,0123	0,0751	0,1908	0,4862	0,0000
$1x10^8$	3,8849	3,7318	0,0323	0,0678	0,1955	0,4901	0,0000

To compare the results contained in table 1 with table 8, it is possible to conclude, that for the relative roughness condition of 0.0005, the approximation models could be classified as: Moody (0.1100 ≤RE ≤ 3.8849) between perfect and terrible; Wood (0.6114≤ RE ≤ 5.9005) between good and terrible; Churchill (0.0123≤ RE ≤ 1.9038) between perfect and regular; Eck (0,0678≤ RE ≤ 6.4561) between perfect and terrible; Halland (0.1086≤RE ≤1.3819) between perfect and regular; Tsal (0.0822≤ RE ≤ 1.6082) between perfect and regular; Buzzelli (0.0000≤ RE ≤ 0.0738) perfect result.

From the analysis of the graph 15 and the table 8, it is possible to verify that the Wood models generate results with greater errors in relation to the Colebrook - White equation, while the Churchill, Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.001

The graph 16 shows the result of all models, including Colebrook - White, for the relative roughness of 0.001.

The data shown in graph 16, it is possible to observe that for the relative roughness of 0.001,in general method all have a tendency to next Colebrook-White data.

Graph 16–Comparison of the friction factor models for relative roughness of 0.001.

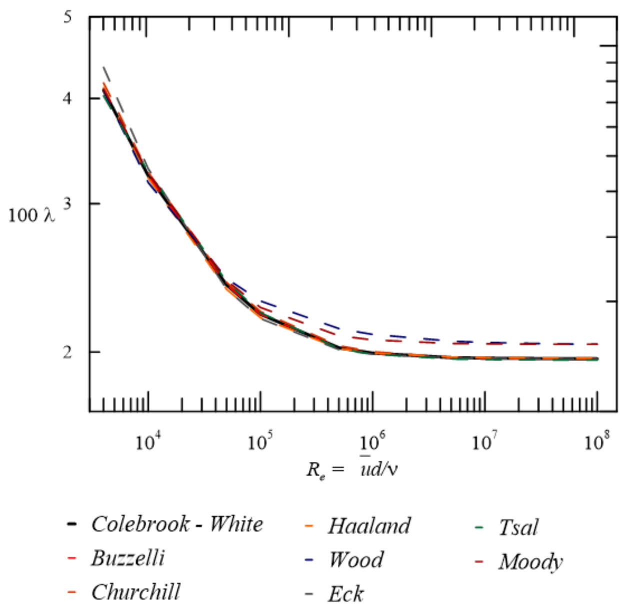

Table 9 shows percentage values of the relative error between the models.

Table 9 - Values (in %) of the relative errors for the relative roughness de 0.001

Relative Error (values in %)for ε/D = 0.001							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
$4x10^3$	0,3368	0,0029	2,0406	6,4625	0,7474	1,5134	0,0797
$1x10^4$	0,7613	1,8312	0,9652	1,9291	0,6390	0,9521	0,0369
$5x10^4$	1,2029	1,7901	0,6925	1,1301	1,2126	0,9331	0,0005
$1x10^5$	1,8726	3,6989	0,7555	1,1525	0,9395	0,4305	0,0010
$5x10^5$	3,3394	5,3661	0,5453	0,4898	0,1889	0,2015	0,0032
$1x10^6$	3,6634	5,2438	0,3761	0,2994	0,0113	0,2909	0,0019
$5x10^6$	3,9619	4,6155	0,0964	0,1164	0,1548	0,3615	0,0003
$1x10^7$	4,0020	4,3975	0,0331	0,0913	0,1774	0,3702	0,0001
$5x10^7$	4,0346	4,0886	0,0367	0,0708	0,1957	0,3771	0,0000
$1x10^8$	4,0387	4,0175	0,0490	0,0683	0,1980	0,3780	0,0000

To compare the results contained in table 1 with table 9, it is possible to conclude, that for the relative roughness condition of 0.001, the approximation models could be classified as: Moody (0.3368≤RE ≤4.0387) between perfect and terrible; Wood (0.0029≤ RE ≤ 5.3661) between perfect and terrible; Churchill (0.0331≤ RE ≤ 2.0406) between perfect and regular; Eck (0,0683≤ RE ≤ 6.4625) between perfect and terrible; Halland (0.0113≤RE ≤1.2126) between perfect and regular; Tsal (0.2015≤ RE ≤ 1.5134) between perfect and regular; Buzzelli (0.0000≤ RE ≤ 0.0797) perfect result.

From the analysis of the graph 16 and the table 9, it is possible to verify that the Wood and Moody models generate results with greater errors in relation to the Colebrook - White equation, while the Buzzelli models present good approximations.

Error for relative roughness of 0.002

The graph 17 shows the result of all models, including Colebrook - White, for the relative roughness of 0.002.

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Graph 17–Comparison of the friction factor models for relative roughness of 0.002.

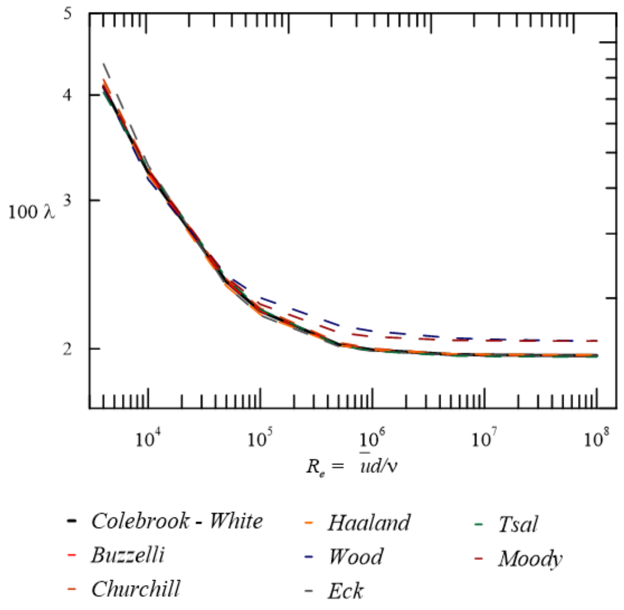


Table 10 shows percentage values of the relative error between the models.

Table 10 - Values (in %) of the relative errors for the relative roughness de 0.002

Relative Error (values in %)for ε/D = 0.002							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	0,0338	2,4737	2,2766	6,4610	0,5105	2,5097	0,0774
1x10⁴	0,7841	2,4322	1,3352	2,1920	0,7339	0,3041	0,0235
5x10⁴	1,9852	2,1561	0,9970	0,3276	0,8390	0,0829	0,0008
1x10⁵	2,6110	3,5440	0,8721	0,3902	0,5182	0,3132	0,0034
5x10⁵	3,4861	4,3863	0,4097	0,1876	0,0037	0,5924	0,0020
1x10⁶	3,6354	4,2863	0,2460	0,1341	0,0976	0,6336	0,0009
5x10⁶	3,7635	3,9678	0,0320	0,0849	0,1790	0,6674	0,0001
1x10⁷	3,7801	3,8715	0,0102	0,0784	0,1896	0,6717	0,0000
5x10⁷	3,7935	3,7445	0,0544	0,0731	0,1982	0,6752	0,0000
1x10⁸	3,7952	3,7174	0,0619	0,0724	0,1992	0,6756	0,0000

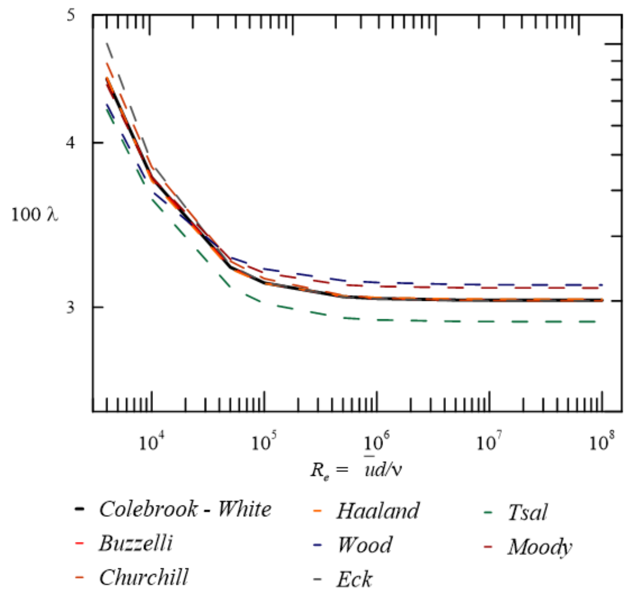
To compare the results contained in table 1 with table 10, it is possible to conclude, that for the relative roughness condition of 0.002, the approximation models could be classified as: Moody (0.0338 ≤RE ≤ 3.7952) between perfect and terrible; Wood (2.1561≤ RE ≤ 4.3863) between weak and terrible; Churchill (0.0102 ≤ RE ≤ 2.2766) between perfect and weak; Eck (0,0724≤ RE ≤ 6.4610) between perfect and terrible; Halland (0.0037≤RE ≤ 0.8390) between perfect and good; Tsal (0.0829≤ RE ≤ 2.5097) between perfect and weak; Buzzelli (0.0000≤ RE ≤ 0.0774) perfect result.

From the analysis of the graph 17 and the table 10 it is possible to verify that the Wood and Moody models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzellimodels present good approximations.

Error for relative roughness of 0.005

The graph 18 shows the result of all models, including Colebrook - White, for the relative roughness of 0.005.

Graph 18–Comparison of the friction factor models for relative roughness of 0.005.



The data shown in graph 18, it is possible to observe that for the relative roughness of 0.005, Moody's, Wood's and Tsal's method presented a more discrepant result when compared to the Colebrook-White data.

Table 11 shows percentage values of the relative error between the models.

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Table 11 - Values (in %) of the relative errors for the relative roughness de 0.005

Relative Error (values in %)for ε/D = 0.005							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
$4x10^3$	1,0089	4,3897	2,7640	6,3793	0,2773	5,2495	0,0517
$1x10^4$	0,0912	2,3136	1,8829	2,5321	0,5484	3,6547	0,0045
$5x10^4$	1,4101	1,7347	1,0699	0,3142	0,2846	3,4520	0,0035
$1x10^5$	1,7454	2,4604	0,7508	0,0925	0,0879	3,5401	0,0034
$5x10^5$	2,0814	2,8519	0,2295	0,0519	0,1335	3,6456	0,0007
$1x10^6$	2,1288	2,8342	0,1091	0,0672	0,1663	3,6614	0,0003
$5x10^6$	2,1677	2,7598	0,0253	0,0788	0,1934	3,6745	0,0000
$1x10^7$	2,1726	2,7378	0,0491	0,0802	0,1968	3,6761	0,0000
$5x10^7$	2,1765	2,7102	0,0729	0,0814	0,1996	3,6775	0,0000
$1x10^8$	2,1770	2,7047	0,0767	0,0815	0,2000	3,6776	0,0000

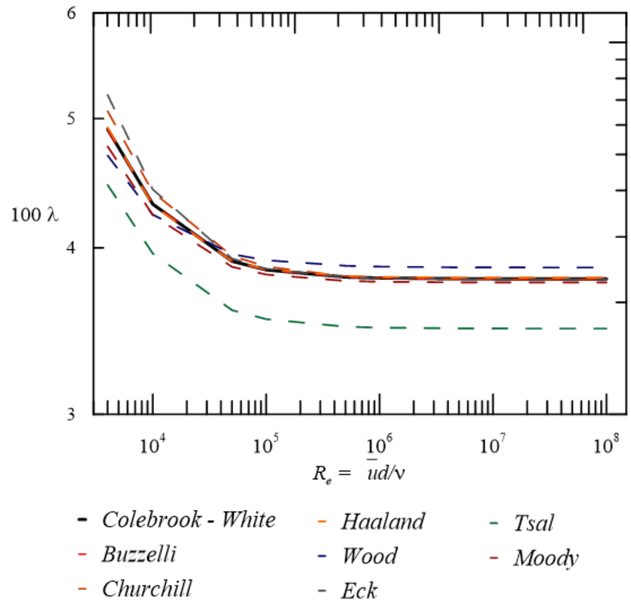
To compare the results contained in table 1 with table 11, it is possible to conclude, that for the relative roughness condition of 0.005, the approximation models could be classified as: Moody (0.0912 ≤RE ≤ 2.1770) between perfect and weak; Wood (1.7347≤ RE ≤ 4.3897) between regular and terrible; Churchill (0.0253≤ RE ≤ 2.7640) between perfect and weak; Eck (0.0519≤ RE ≤ 6.3793) between perfect and terrible; Halland (0.0879≤RE ≤ 0.5484) perfect result; Tsal (3.4520≤ RE ≤ 5.2495) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0517) perfect result.

From the analysis of the graph 18 and the table 11, it is possible to verify that the Tsal and Wood models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzellimodels present good approximations.

Error for relative roughness of 0.01

The graph 19 shows the result of all models, including Colebrook - White, for the relative roughness of 0.01.

Graph 19–Comparison of the friction factor models for relative roughness of 0.01.



The data shown in graph 19, it is possible to observe that for the relative roughness of 0.01, Wood's and Tsal's method presented a more discrepant result when compared to the Colebrook-White data.

Table 12 shows percentage values of the relative error between the models.

Table 12 - Values (in %) of the relative errors for the relative roughness de 0.01

Relative Error (values in %)for ε/D = 0.01							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
$4x10^3$	2,9242	4,4488	3,1583	6,1280	0,3127	9,1533	0,0223
$1x10^4$	1,8730	1,7900	2,1068	2,5781	0,1894	8,1720	0,0000
$5x10^4$	0,9702	1,2223	0,9096	0,4491	0,0047	8,1108	0,0032
$1x10^5$	0,8102	1,6719	0,5639	0,1780	0,0908	8,1594	0,0020
$5x10^5$	0,6692	1,9675	0,1213	0,0376	0,1763	8,2124	0,0003
$1x10^6$	0,6507	1,9857	0,0346	0,0644	0,1882	8,2200	0,0001
$5x10^6$	0,6357	1,9852	0,0560	0,0859	0,1979	8,2263	0,0000
$1x10^7$	0,6338	1,9823	0,0712	0,0885	0,1991	8,2270	0,0000
5x1	0,632	1,977	0,0859	0,090	0,2001	8,227	0,0000

0⁷	3	8		7		7	
1x10⁸	0,6321	1,9769	0,0883	0,0910	0,2002	8,2278	0,0000

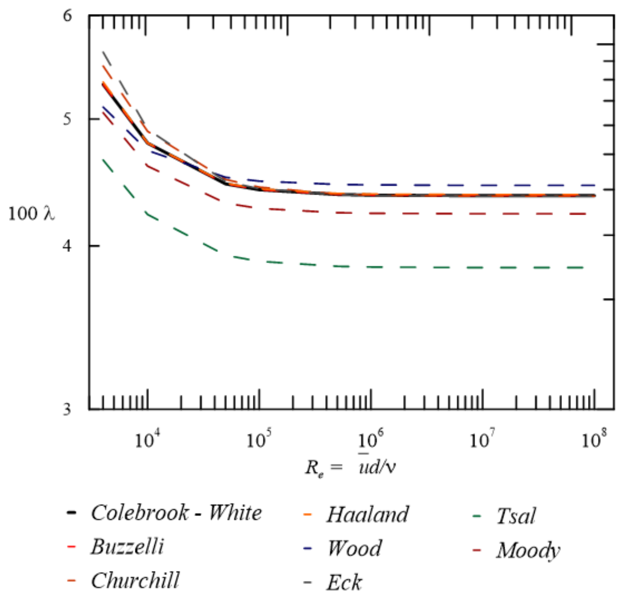
To compare the results contained in table 1 with table 12, it is possible to conclude, that for the relative roughness condition of 0.01, the approximation models could be classified as: Moody (0.6321 ≤RE ≤ 2.9242) between good and weak; Wood (1.2223≤ RE ≤ 4.4488) between regular and terrible; Churchill (0.0346≤ RE ≤ 3.1583) between perfect and terrible; Eck (0.0376≤ RE ≤ 6.1280) between perfect and terrible; Halland (0.0047≤RE ≤ 0.3127) perfect result; Tsal (8.1108≤ RE ≤ 9.1533) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0223) perfect result.

From the analysis of the graph 19 and the table 12, it is possible to verify that the Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.015

The graph 20 shows the result of all models, including Colebrook - White, for the relative roughness of 0.015.

Graph 20–Comparison of the friction factor models for relative roughness of 0.015.



The data shown in graph 20, it is possible to observe that for the relative roughness of 0.015, Moody's and Tsal's method presented a more discrepant result when compared to the Colebrook-White data.

Table 13 shows percentage values of the relative error between the models.

Table 13 - Values (in %) of the relative errors for the relative roughness de 0.015

Relative Error (values in %)for ε/D = 0.015

Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
4x10³	4,8485	3,9048	3,3067	5,8438	0,4302	12,4481	0,0093
1x10⁴	3,9672	1,2816	2,0955	2,4706	0,0311	11,8060	0,0006
5x10⁴	3,3361	1,1658	0,7778	0,4381	0,1103	11,8079	0,0024
1x10⁵	3,2397	1,5211	0,4521	0,1714	0,1504	11,8423	0,0013
5x10⁵	3,1584	1,7846	0,0703	0,0439	0,1895	11,8775	0,0002
1x10⁶	3,1480	1,8111	0,0001	0,0709	0,1948	11,8824	0,0001
5x10⁶	3,1396	1,8272	0,0711	0,0925	0,1992	11,8864	0,0000
1x10⁷	3,1385	1,8283	0,0828	0,0952	0,1997	11,8869	0,0000
5x10⁷	3,1377	1,8286	0,0940	0,0974	0,2002	11,8873	0,0000
1x10⁸	3,1375	1,8285	0,0957	0,0977	0,2002	11,8874	0,0000

To compare the results contained in table 1 with table 13, it is possible to conclude, that for the relative roughness condition of 0.015, the approximation models could be classified as: Moody (3.1375≤RE ≤4.8485) terrible result; Wood (1.1658≤ RE ≤ 3.9048) between regular and terrible; Churchill (0.0001≤ RE ≤ 3.3067) between perfect and terrible; Eck (0.0439≤ RE ≤ 5.8438) between perfect and terrible; Halland (0.0311≤RE ≤ 0.4302) perfect result; Tsal (11.8060≤ RE ≤ 12.4481) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0093) perfect result.

From the analysis of the graph 20 and the table 13 it is possible to verify that the Moody and Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.02

The graph 21 shows the result of all models, including Colebrook - White, for the relative roughness of 0.02.

The data shown in graph 21, it is possible to observe that for the relative roughness of 0.02, Moody's, Wood's and

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Tsal's method presented a more discrepant result when compared to the Colebrook-White data.

Graph 21–Comparison of the friction factor models for relative roughness of 0.02.

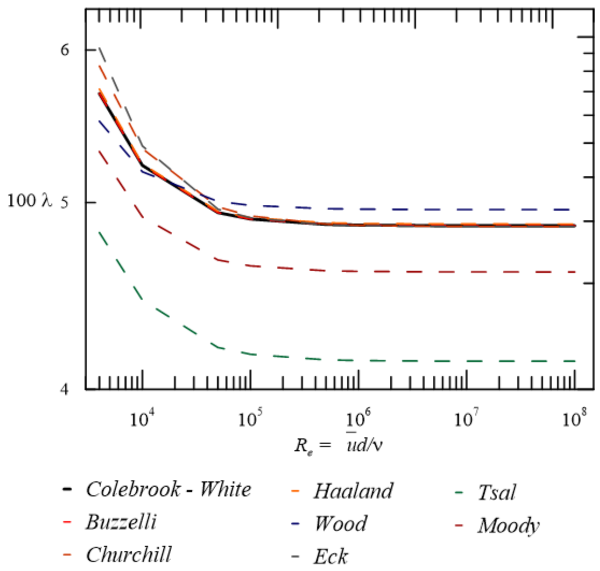

Table 14 shows percentage values of the relative error between the models.

Table 14 - Values (in %) of the relative errors for the relative roughness de 0.02

Relative Error (values in %)for ε/D = 0.02							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	6,6985	3,2524	3,3443	5,5701	0,5380	15,2998	0,0038
1x10⁴	5,9711	0,7635	2,0254	2,3398	0,1671	14,8601	0,0013
5x10⁴	5,5060	1,3407	0,6806	0,4077	0,1621	14,8892	0,0018
1x10⁵	5,4405	1,6463	0,3776	0,1537	0,1787	14,9158	0,0009
5x10⁵	5,3864	1,8877	0,0388	0,0517	0,1955	14,9419	0,0001
1x10⁶	5,3796	1,9159	0,0213	0,0775	0,1979	14,9454	0,0000
5x10⁶	5,3741	1,9368	0,0812	0,0982	0,1998	14,9483	0,0000
1x10⁷	5,3734	1,9391	0,0909	0,1007	0,2001	14,9487	0,0000
5x10⁷	5,3728	1,9408	0,1001	0,1028	0,2003	14,9490	0,0000
1x10⁸	5,3728	1,9410	0,1016	0,1031	0,2003	14,9490	0,0000

To compare the results contained in table 1 with table 14, it is possible to conclude, that for the relative roughness condition of 0.02, the approximation models could be classified as: Moody (5.3728 ≤RE ≤ 6.6985) terrible result; Wood (0.7635≤ RE ≤ 3.2524) between good and terrible; Churchill (0.0213≤ RE ≤ 3.3443) between perfect and terrible; Eck (0.0517≤ RE ≤ 5.5701) between perfect and terrible; Halland (0.1621≤RE ≤ 0.5380) perfect result; Tsal(14.8601 ≤ RE ≤ 15.2998) terrible result; Buzzelli (0.0000 ≤ RE ≤ 0.0038) perfect result.

From the analysis of the graph 21 and the table 14 it is possible to verify that the Moody and Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.03

The graph 22 shows the result of all models, including Colebrook - White, for the relative roughness of 0.03.

Graph 22–Comparison of the friction factor models for relative roughness of 0.03.

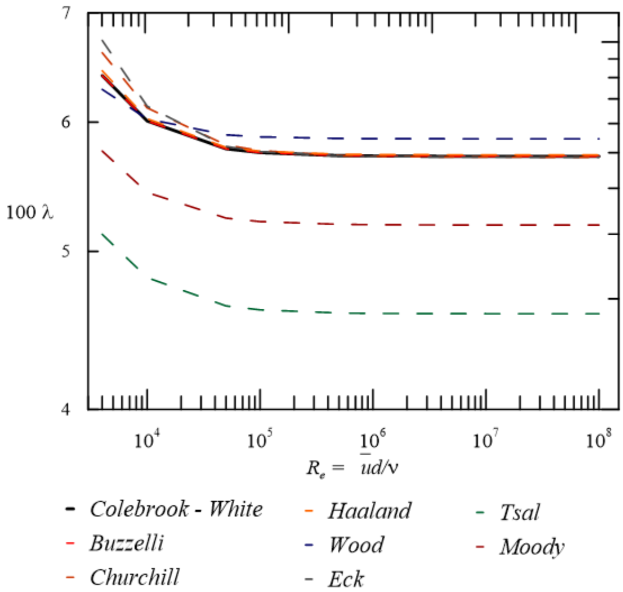

Table 15 shows percentage values of the relative error between the models.

Table 15 - Values (in %) of the relative errors for the relative roughness de 0.03

Relative Error (values in %)for ε/D = 0.03							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
4x10³	10,1094	1,9309	3,2856	5,0913	0,6921	20,0698	0,0004
1x1	9,602	0,28	1,8592	2,10	0,315	19,84	0,001

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0^4	0	88		05	8	40	9
$5x10^4$	9,3179	1,9912	0,5476	0,3475	0,2104	19,8897	0,0012
$1x10^5$	9,2812	2,2406	0,2820	0,1189	0,2044	19,9077	0,0005
$5x10^5$	9,2515	2,4488	0,0002	0,0657	0,2010	19,9246	0,0001
$1x10^6$	9,2478	2,4757	0,0482	0,0888	0,2006	19,9268	0,0000
$5x10^6$	9,2448	2,4975	0,0951	0,1074	0,2004	19,9287	0,0000
$1x10^7$	9,2444	2,5003	0,1026	0,1097	0,2003	19,9289	0,0000
$5x10^7$	9,2441	2,5026	0,1096	0,1116	0,2003	19,9291	0,0000
$1x10^8$	9,2441	2,5029	0,1107	0,1118	0,2003	19,9291	0,0000

To compare the results contained in table 1 with table 15, it is possible to conclude, that for the relative roughness condition of 0.03, the approximation models could be classified as: Moody (9.2441 ≤RE ≤ 10.1094) terrible result; Wood (0.2888≤ RE ≤ 2.5029) between perfect and weak; Churchill (0.0002≤ RE ≤ 3.2856) between perfect and terrible; Eck (0.0657≤ RE ≤ 5.0913) between perfect and terrible; Halland (0.2003≤RE ≤ 0.6921) between perfect and good; Tsal (19.8440≤ RE ≤ 20.0698) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0019) perfect result.

From the analysis of the graph 22 and the table 15, it is possible to verify that the Moody and Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.04

The graph 23 shows the result of all models, including Colebrook - White, for the relative roughness of 0.04.

Graph 23–Comparison of the friction factor models for relative roughness of 0.04.

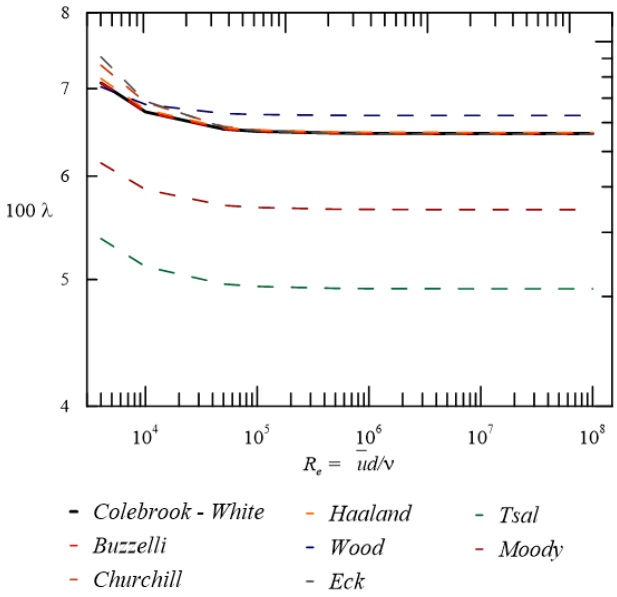

Table 16 shows percentage values of the relative error between the models.

Table 16 - Values (in %) of the relative errors for the relative roughness de 0.04

Relative Error (values in %)for ε/D = 0.04							
Re	**Moody**	**Wood**	**Churchill**	**Eck**	**Haaland**	**Tsal**	**Buzzelli**
$4x10^3$	13,1565	0,6899	3,1681	4,7019	0,7851	23,9818	0,0000
$1x10^4$	12,7861	1,3168	1,7097	1,9083	0,3894	23,8574	0,0020
$5x10^4$	12,5936	2,7824	0,4597	0,2988	0,2315	23,9039	0,0008
$1x10^5$	12,5698	2,9978	0,2213	0,0907	0,2155	23,9174	0,0004
$5x10^5$	12,5508	3,1813	0,0248	0,0770	0,2033	23,9296	0,0000
$1x10^6$	12,5484	3,2056	0,0656	0,0981	0,2018	23,9312	0,0000
$5x10^6$	12,5465	3,2259	0,1051	0,1149	0,2006	23,9325	0,0000
$1x10^7$	12,5462	3,2285	0,1113	0,1170	0,2004	23,9327	0,0000
$5x10^7$	12,5461	3,2307	0,1172	0,1187	0,2003	23,9328	0,0000
$1x10^8$	12,5460	3,2310	0,1180	0,1189	0,2003	23,9328	0,0000

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To compare the results contained in table 1 with table 16, it is possible to conclude, that for the relative roughness condition of 0.04, the approximation models could be classified as: Moody (12.5460 ≤RE ≤ 13.1564) terrible result; Wood (0.6899≤ RE ≤ 3.2310) between good and terrible; Churchill (0.0248≤ RE ≤ 3.1681) between perfect and terrible; Eck (0.0770≤ RE ≤ 4.7019) between perfect and terrible; Halland(0.2003≤RE ≤ 0.7851) between perfect and good; Tsal (23.8574≤ RE ≤ 23.9818) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0020) perfect result.

From the analysis of the graph 22 and the table 16 it is possible to verify that the Moody and Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

Error for relative roughness of 0.05

The graph 24 shows the result of all models, including Colebrook - White, for the relative roughness of 0.05.

The data shown in graph 24, it is possible to observe that for the relative roughness of 0.05, Moody's, Wood's and Tsal's method presented a more discrepant result when compared to the Colebrook-White data.

Graph 24–Comparison of the friction factor models for relative roughness of 0.05.

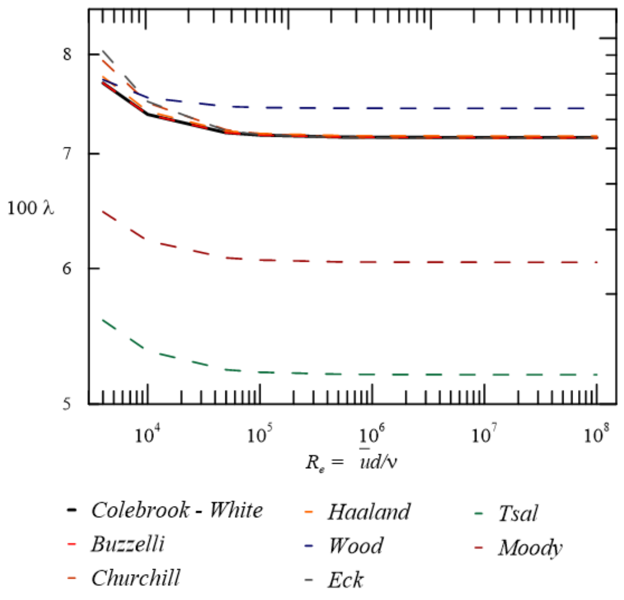

Table 17 shows percentage values of the relative error between the models.

Table 17 - Values (in %) of the relative errors for the relative roughness de 0.05

Relative Error (values in %)for ε/D = 0.05

Re	Moody	Wood	Churchill	Eck	Haaland	Tsal	Buzzelli
$4x10^3$	15,8987	0,4474	3,0408	4,3832	0,8418	27,3066	0,0002
$1x10^4$	15,6174	2,2875	1,5843	1,7546	0,4299	27,2361	0,0019
$5x10^4$	15,4780	3,5909	0,3960	0,2601	0,2425	27,2796	0,0006
$1x10^5$	15,4612	3,7822	0,1781	0,0680	0,2211	27,2902	0,0003
$5x10^5$	15,4479	3,9463	0,0425	0,0865	0,2044	27,2997	0,0000
$1x10^6$	15,4462	3,9683	0,0786	0,1059	0,2024	27,3009	0,0000
$5x10^6$	15,4449	3,9867	0,1132	0,1214	0,2007	27,3019	0,0000
$1x10^7$	15,4447	3,9891	0,1186	0,1234	0,2005	27,3020	0,0000
$5x10^7$	15,4446	3,9911	0,1236	0,1249	0,2003	27,3021	0,0000
$1x10^8$	15,4446	3,9913	0,1244	0,1251	0,2003	27,3022	0,0000

To compare the results contained in table 1 with table 17, it is possible to conclude, that for the relative roughness condition of 0.05, the approximation models could be classified as: Moody (15.4446 ≤RE ≤ 15.8987) terrible result; Wood (0.4474≤ RE ≤ 3.9913) between perfect and terrible; Churchill (0.0425≤ RE ≤ 3.0408) between perfect and terrible; Eck (0.0680≤ RE ≤ 4.3832) between perfect and terrible; Halland (0.2003≤RE ≤ 0.8418) between perfect and good; Tsal (27.2361≤ RE ≤ 27.3066) terrible result; Buzzelli (0.0000≤ RE ≤ 0.0019) perfect result.

From the analysis of the graph and the table it is possible to verify that the Moody and Tsal models generate results with greater errors in relation to the Colebrook - White equation, while the Haaland and Buzzelli models present good approximations.

IV. CONCLUSIONS

It can be seen how the Moody and Tsal method performs high errors as the relative roughness is increased. The Wood method oscillates between good and bad percentages of error, with greater emphasis on the bad results. Churchill and Haaland methods show excellent results for all relative roughness intervals. The Buzzelli method is the model that performed best.

There are several other correlations, statistics, and values for relative roughness (absolute roughness and pipe diameter) and Reynolds number (turbulent fluid) that can be determined.

As future work it is possible to estimate such approximations, statistical calculations and Reynolds values, absolute roughness, and diameters, for a more statistically concrete analysis and / or a more specific analysis depending on the values adopted for relative roughness and Reynolds.

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