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Analysis of stability for uniform rotations of a dumbbell system in an elliptic orbit

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***Abstract*—The evolution of space missions and related systems has been promoting innovation and creation of ideas for new technologies for decades. The search for innovative solutions that combine the optimization of resources and materials guide the recent research in the space area. The objective of this study is to analyze the behavior of two bodies connected by tethers and can be a solution for reducing costs in space missions, one of the concepts that have the potential to fulfill the objective of efficient transport space. In this paper it is discussed the motion of two massive bodies connected by tethers in keplerian motion in a central force field, their viability, the rotational dynamics and the system behavior in the space environment. Models will be created that simulate and explain the dynamics of the object and that analyzes the main parameters for determining the stability and the uniform rotations conditions.*

I. INTRODUCTION

Tethered systems have many areas of application and has been studied in several published articles ([1]–[9]). Beletsky and Levin[1], begins by setting the scene for tethers in space summarizing possible applications and also discussing fact and fiction, analyzing clearly the main parameters and applications for Tethers Systems, as the density of the material the effective forces, orbital dynamics, mechanics models, attitude and possible disturbances for a flexible tethers with end masses, massless and massive variations.

The motion of tethers considering dumbbell oscillations, bodies in the central field of vibrating forces (Burov et al. ([10]–[14]), showing stability solutions for angles on the chaotic dynamics in elliptical orbit, analyzing the problem in another aspect of Moon-tethered pendulum, to considering the uniform rotations of a two-body tethered system in planar motion and the control the length of the tether [15]. In studies ([15]-[21]) were suggested methods of controlling the geometric configurationand Moon-tethered system with variable tether length in restricted three-body problem ([16] and [22]) were studied and demonstrated important results for tethers applications in systems.

II. DYNAMICS OF THE PROBLEM

Consider planar motion of a dumbbell body in the central Newtonian gravitational field (Fig.1). The tether length (l) is small compared to the orbit the system center of mass ($cm_{\otimes}$), moves along an elliptic keplerian orbit.

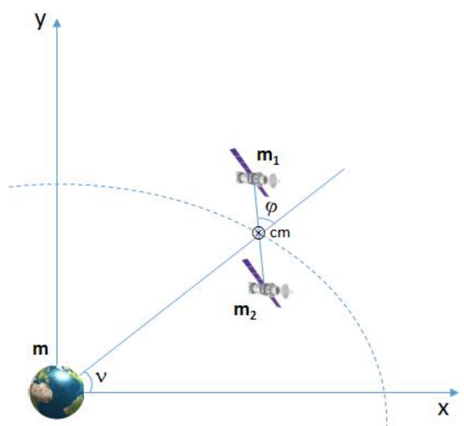

Fig.1: Geometry of the tether system in reference frame.

The body consists of two-point masses (m_1 and m_2) connected by a light tether

$$\rho = \frac{p}{1 + e\cos(\nu)} \tag{1}$$

where p is the focal parameter, e the eccentricity and ν the true anomaly of the orbit. The system coordinates are

$$\begin{cases} x_0 = \rho\cos(\nu) \\ y_0 = \rho\sin(\nu) \\ x_1 = x_0 + l_1\cos(\nu+\varphi) \\ y_1 = y_0 + l_1\sin(\nu+\varphi) \\ x_2 = x_0 - l_2\cos(\nu+\varphi) \\ y_2 = y_0 - l_2\sin(\nu+\varphi) \end{cases} \tag{2}$$

1.1. Potential and Kinetic Energy

The potential energy of the system can be obtained by the following expression,

$$V = -\frac{\mu_0 m_1}{|\vec{r_1}|} - \frac{\mu_0 m_2}{|\vec{r_2}|} \tag{3}$$

where m_i represents the mass of the point; $\vec{r_i}$the position of the point mass with respect to center of the Earth; $\mu_0 = GM$; G is the universal gravitational constant and M the mass of the Earth. Introduced the parameters, μ and m, given by

$$\mu = \frac{m_1}{m} \tag{4}$$

$$m = m_1 + m_2 \tag{5}$$

Substitute on Eq. (3)leads to

$$V = -\mu_0\mu\left(\frac{m}{\sqrt{x_1^2 + y_1^2}} + \frac{1-m}{\sqrt{x_2^2 + y_2^2}}\right) \tag{6}$$

The Eq. (7)is cumbersome and can be simplified, introducing a new parameter $\lambda = \frac{l}{p}$, and assuming $\lambda \ll 1$ the tether length is much smaller than the focal parameter p. The Taylor Series expansion up to the 2^{nd} order of λ for the potential energy is

$$V = -\frac{m\mu_0(1+e\cos(\nu))}{p} + \frac{m(\mu-1)\mu\mu_0(1+e\cos(\nu))^3(1+3\cos(2\varphi))\lambda^2}{4p} \tag{7}$$

The kinetic energy of the system can be written as

$$T = \frac{m_1|\vec{v_1}|^2}{2} + \frac{m_2|\vec{v_2}|^2}{2} \tag{8}$$

$$T = \frac{1}{4}m\left(\frac{2p^2(1+e^2+2e\cos(\nu))\dot{\nu}^2}{(1+e\cos(\nu))^4} - 2(\mu-1)\mu\left(l^2(\dot{\nu}+\dot{\varphi})^2 + \dot{l}^2\right)\right) \tag{9}$$

1.2. Lagrange Equations of Motion

For the subsequent analysis, the generalized coordinates φ and l are used and the system is assumed to be subject to the gravity-gradient forces.

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{dL}{d\varphi} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{l}}\right) - \frac{dL}{dl} = 0 \end{cases} \tag{10}$$

$$l\left(3\mu_0 sin(2\varphi)\left(1+ecos(\nu)\right)^3 + 2p^3(\ddot{\nu}+\ddot{\varphi})\right) + 4p^3\dot{l}(\dot{\nu}+\dot{\varphi}) = 0 \tag{11}$$

The equation is rewritten as a function of the true anomaly ν (Eq. (11)- (12))

$$(\ )' = \frac{d}{d\nu} \tag{12}$$

$$\frac{d}{dt} = \dot{\nu}\frac{d}{d\nu} = \omega_0(1+e\cos(\nu))^2\frac{d}{d\nu} \tag{13}$$

with

$$\omega_0 = \frac{\mu_0}{p^3} \tag{14}$$

The equations of motion of the spacecraft can be written as

$$(1+e\cos(\nu))\varphi'' + 2\left(\frac{l'}{l}(1+e\cos(\nu)) - e\sin(\nu)\right)(\varphi'+1) + 3\cos(\varphi)\sin(\varphi) = 0 \tag{15}$$

considering a movement with uniform rotations,

$$\varphi = \omega\nu + \varphi_0 \tag{16}$$

$$l(\nu) = \eta(\nu)\frac{l_0}{1+e\cos(\nu)} \tag{17}$$

Suppose the following relation for the tether performance (Eq. \ref{equa16}), the Eq. \ref{equa14} with respect to the true anomaly ν takes the form:

$$\frac{\eta'(\nu)}{\eta(\nu)} = -\frac{3\sin\left(2(\omega\nu+\varphi_0)\right)}{4(\omega+1)(1+e\cos(\nu))} \tag{18}$$

The analytical integration function is difficult but substituting values for the variable ω and solving the equation with respect to the variable η, it is possible to obtain the following solutions (Table 1 and Figures \ref{fig2} - \ref{fig5}). Analytical solutions were found for this ω values previously chosen. The values for $\omega = \left(\pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{7}{4}, \pm\frac{9}{4}, \pm\frac{11}{4}, \pm\frac{13}{4}, \pm\frac{15}{4}\right)$ closed-form no solution. For $\omega = 0$ was examined in previous studies [12] where the relative equilibrium was studied.

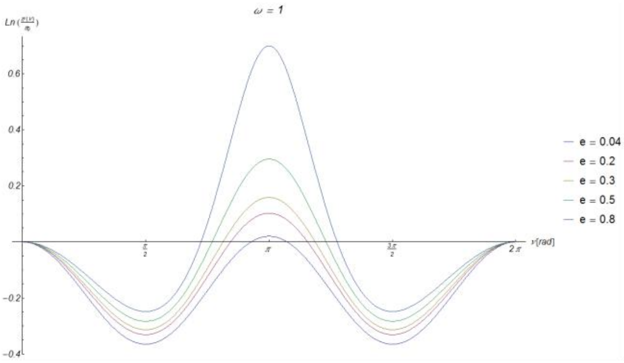

Fig.2: Tether length control for $\omega = 1$.

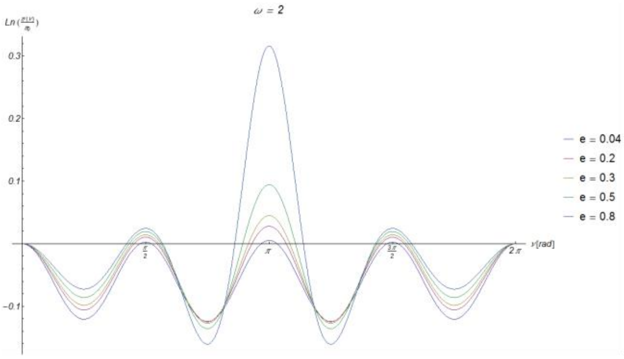

Fig.3: Tether length control for $\omega = 2$.

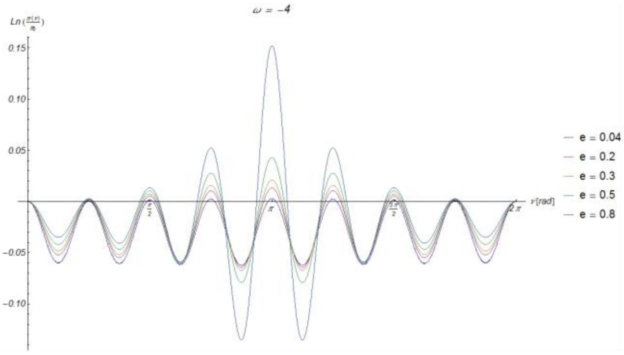

Figure 4 -Tether length control for $\omega = -4$

The control laws are periodic in true anomaly (ν). In Fig.2to Figure 4describe the change in tether length depending for different eccentricities,$e = (0.04, 0.2, 0.3, 0.5, 0.8)$. The formulation also guarantees uniform rotation for fractional values of ω, as seen in Figures6–9.

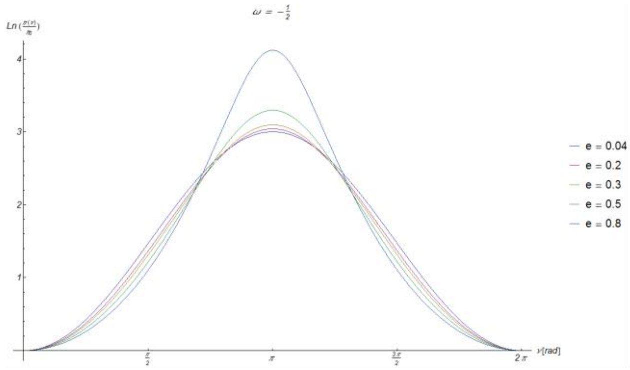

Fig.5: Tether length control for $\omega = -\frac{1}{2}$

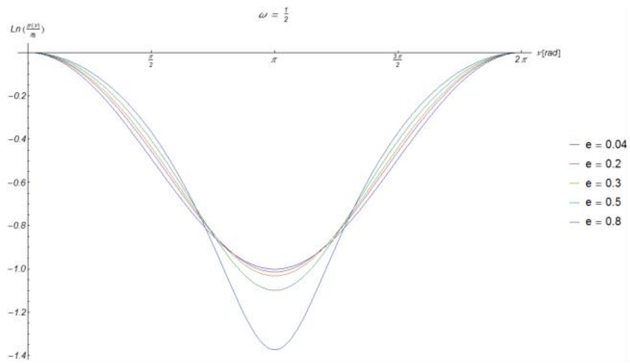

Fig.6: Tether length control for $\omega = \frac{1}{2}$

1.3. Uniform Rotations: $\varphi = \omega\nu + \varphi_0$

Substitute ($\varphi = \omega\nu + \varphi_0$) to Eq. (14)) and considering Eq. (17), it is possible to obtain the nonlinear differential equation that describes the motion of the tether.

A general analytical integration for all values of ω cannot be found but for some values ω the closed form solutions obtain. A numerical integration has been performed, where ω and φ_0 were substituted previously, with the objective of analyzing the behavior of the cable system. The logarithmic plots of $\ln\left(\frac{\eta}{\eta_0}\right)$ for a number of fractional angular velocities (Figures 6 - 9) was made. For the particular case when the value of ω is zero ($\omega = 0$) and φ_0 is constant, there are a uniform for $\varphi_0 = \frac{(k)\pi}{2}$, $k = \{0,1,2,3,4,\dots\}$.

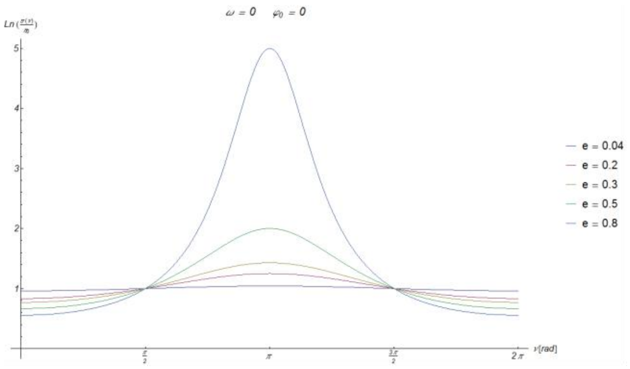

Fig.7: The control law for the tether length for $\omega = 0$ and $\varphi_0 = 0$.

An approximate solution for small eccentricities can be obtained using Taylor series in Eq. (17)and obtain applying the series of order 3, in the variable *e* obtain:

$$\frac{\eta'(\nu)}{\eta(\nu)} = \frac{3e^3\cos^3(\nu)\sin(2\nu\omega)}{4(\omega+1)} - \frac{3e^2\cos^2(\nu)\sin(2\nu\omega)}{4(\omega+1)} + \frac{3e\cos(\nu)\sin(2\nu\omega)}{4(\omega+1)} - \frac{3\sin(2\nu\omega)}{4(\omega+1)} \quad (19)$$

One obtains the expression that has analytical integration, however this procedure introduces impossibilities in the system and restricts the solutions to the variable ω, which after integration generates singularities for values: $\omega = \left(-\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}\right)$. Increasing the expansion terms for order 5, the singularities also increase, $\omega = \left(-\frac{5}{2}, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\right)$.

1.4. Stability Conditions

We study stability using to the Floquet Theory, let´s linearize the equation of motion in the vicinity of solution (Eq. (16)), it is possible to check the stability analyzing system behavior around small oscillations. Considering Eq. (19)andEq. (20)analyzing the neighborhood of the point $\varphi_0 = 0$, it follows:

$$\varphi = \omega\nu + \delta\varphi \quad (20)$$

Substituting Eq. (17)in Eq.(15)one obtains:

$$4(\omega + 1 + \delta\varphi')(1 + e\cos(\nu))\frac{\eta'(\nu)}{\eta(\nu)} + 3\sin\big(2(\omega\nu + \delta\varphi)\big) + 2(1 + e\cos(\nu))\delta\varphi'' = 0 \quad (21)$$

Replacing Eq. (18), concerning the $\eta(\nu)$ variable,

$$\delta\varphi''(1 + e\cos(\nu)) + 3\cos(2\omega\nu + \delta\varphi)\sin(\delta\varphi) - \frac{3\sin(\delta\varphi')}{2(\omega+1)} = 0 \quad (22)$$

This is the nonlinear equation of the perturbed motion \cite{ref16}. The linearized equation is

$$(1 + e\cos(\nu))\delta\varphi'' + \frac{3}{2}\left(2\cos(2\omega\nu)\,\delta\varphi - \frac{\sin(2\omega\nu)\,\delta\varphi'}{\omega+1}\right) = 0 \quad (23)$$

Applying the Floquet theory find the monodromy matrix (A) for this system, one is obtained numerically by integrating a periodic orbit (2π) and the variation equations (Eq. (23)).

A good numerical method to refine the closure of the orbit is essential to obtain an accurate monodromy matrix (A), which is obtained to analyze to stability with respect to small perturbations (δφ) of the orientation angle φ. It is possible to analyze the stability using the Floquet theory, since this equation is a differential equation of the second order. The linearized equation constricts this analysis to small variations of φ. The stability monodromy matrix (A) has some important properties, which are ([13] and [19]):

1. det(A)=1;
2. $\{\lambda_1 = \lambda; \lambda_2 = \lambda^{-1}\}$eigenvalues;
3. $Tr(A) = 2 + \sum_i \lambda_i$

The indicator of stability $2 - |Tr(A)|$ is given by $Tr(A)$ between 0 and 2 $(0 < 2 - |Tr(A)| \leq 2)$ \cite{ref13}, where $Tr(A)$ means Trace of the matrix A. Positive values between the described boundaries correspond to stable solutions (linear approximation), negative values correspond to instability and zero values correspond to critical cases. The conditions of stability for ω are demonstrated in Figures 8 -- 18, where the positive values of ω correspond to direct rotations (direction of the orbital motion).

Table 1 - Intervals of stability.

ω	e
-4	[0, 0.2956] U [0.9063, 0.9334]
-3.75	[0.7821, 0.8333]
-3.5	[0, 0.0966] U [0.6824, 0.7690]
-3.25	[0.6328, 0.7542]
-3	[0, 0.7895]
-2.75	[0, 0.6370]
-2.5	[0, 0.4906]

-2.25	[0, 0.1935]
-2	[0, 0.5453]
-1.75	No Solutions found
-1.5	No Solutions found
-1.25	No Solutions found
-1	-
-0.75	No Solutions found
-0.5	0
-0.25	[0.4521, 0.9999]
0	[0, 0.9999]
0.25	No Solutions found
0.5	No Solutions found
0.75	[0.0177, 0.0325]
1	[0.8789, 0.8805]
1.25	No Solutions found
1.5	[0.0, 0.1338]
1.75	[0.0, 0.2684]
2	[0, 0.5539] U [0.9778, 0.9789]
2.25	[0, 0.5820]
2.5	[0, 0.6451]
2.75	[0.6426, 0.7665]
3	[0, 0.0989] U [0.9666, 0.9859]
3.25	[0, 0.0890] U [0.7768, 0.8296]
3.5	[0, 0.2961] U [0.7656, 0.8173]
3.75	[0, 0.3412] U [0.8506, 0.8792]
4	[0, 0.4207]

The regions of stability with solutions (Figures 8–18 and Table 1) have been shown for a number of $\omega \in [-4,4]$. When $\omega = -1$ there is no solution and when $\omega = 1$ there is a small stable range (Table 1). The Table 1 shows the complete set of stable solutions for the monodromy matrix, as a function of the eccentricity and of the true anomaly.

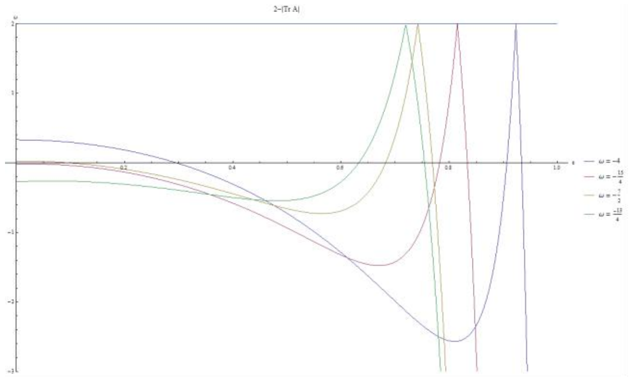

Fig.8: Stability region for $\omega = \left\{-4; -\frac{15}{4}; -\frac{7}{2}; -\frac{13}{4}\right\}$.

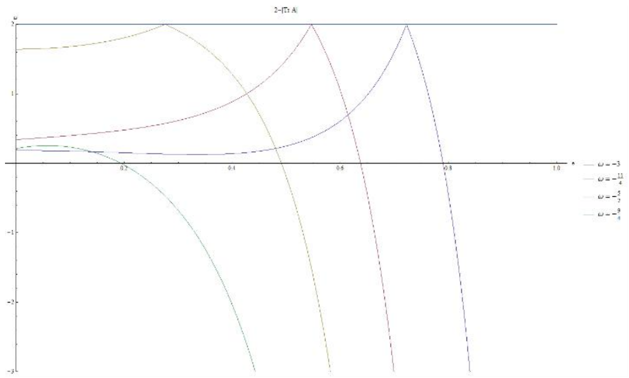

Fig.9: Stability curve for $\omega = \left\{-3; -\frac{11}{4}; -\frac{5}{2}; -\frac{9}{4}\right\}$

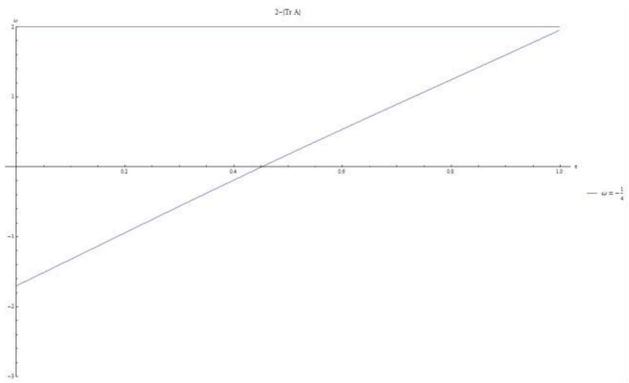

Fig.10: Stability curve for $\omega = \left\{-\frac{1}{4}\right\}$.

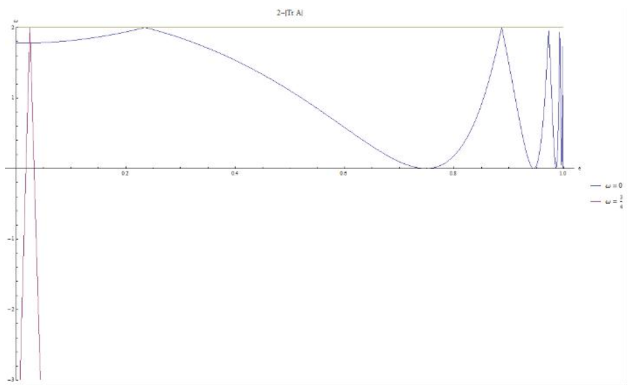

Fig.11: Stability curve for $\omega = \left\{0; \frac{3}{4}\right\}$.

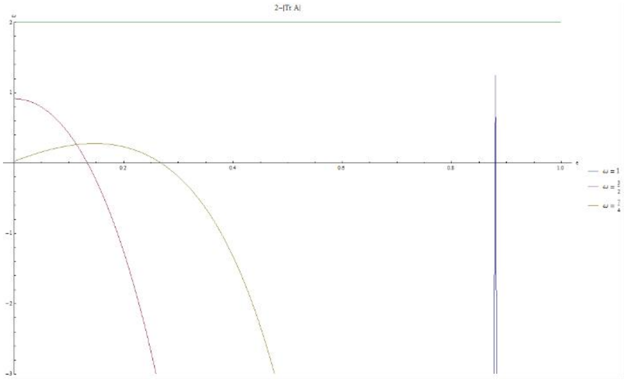

Fig.12: Stability curve for $\omega = \left\{1; \frac{3}{2}; \frac{7}{2}\right\}$.

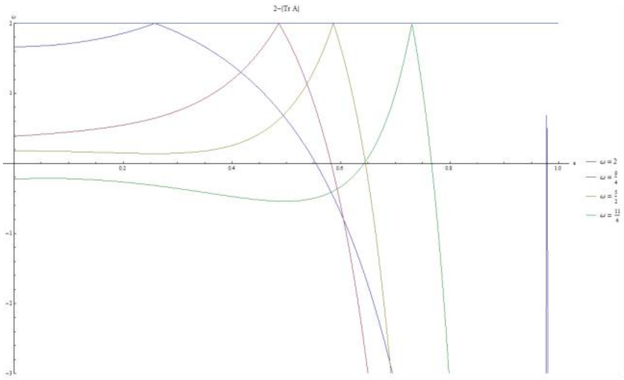

Fig.13: Stability curve for $\omega = \left\{2; \frac{9}{4}; \frac{5}{2}; \frac{11}{4}\right\}$

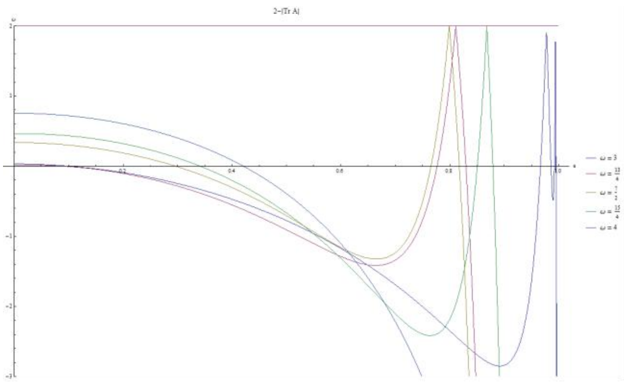

Fig.14: Stability curve for $\omega = \left\{3; \frac{13}{4}; \frac{7}{2}; \frac{15}{4}; 4\right\}$.

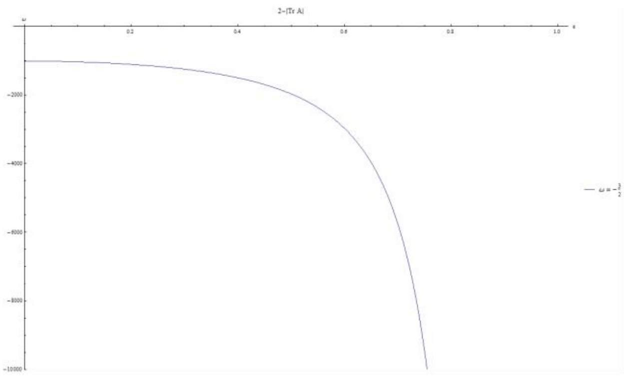

Fig.15: Stability curve for $\omega = \left\{-\frac{3}{2}\right\}$.

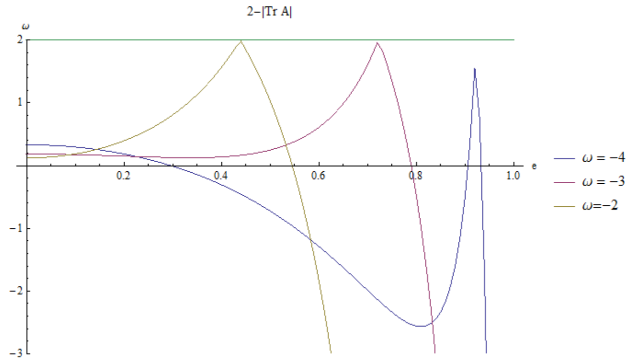

Fig.16: Stability curve for $\omega = \{-4;\ -3;\ -2\}$.

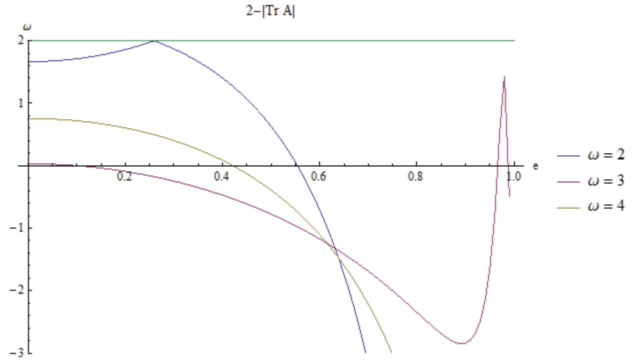

Fig.17: Stability curve for $\omega = \{2;\ 3;\ 4\}$.

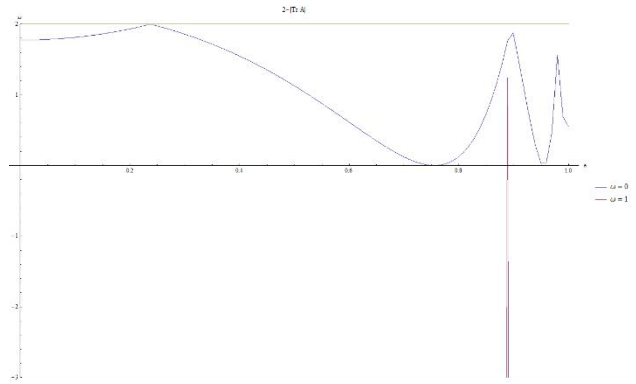

Fig.18: Stability curve for $\omega = \{\ 0;\ 1\}$.

The stability analysis suggests the system's viability. Some parameters for the system are shown in monodromy matrix.

III. FORCES IN TETHERS

The force on the tethers can be calculated using the relation of the forces involved in the problem, applied to the tether,

$$m_1 \ddot{r}_1 = -\mu_0 \frac{m_1}{|\vec{r}_1|^3} \vec{r}_1 + \vec{T} \quad (24)$$

The force on the tether for a system composed of: $m_1 = m_2 = 1000$ kg, $l_0 = 100$km, $p = 7000$km, the direction is tether length (l) and sense m_1 to m_2, and $T$is magnitude of the tether force, solving the Eq.(16) with respect to l, it is possible to show that the cable suffers a large variation in tension and that these values are periodic and, in some cases, increase with grows to the eccentricity (Figures 19 -- 32).

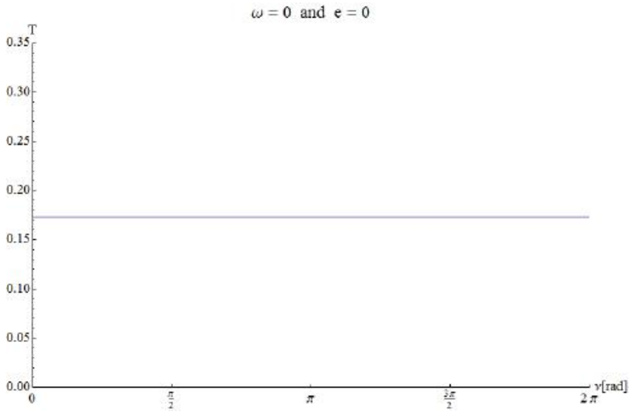

Fig.19: Tether Force T for $\omega = 0$ and e = 0.

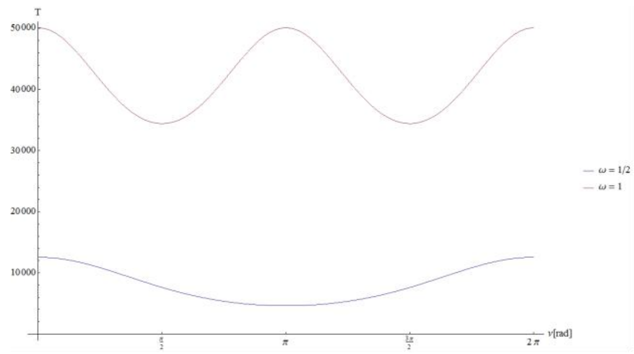

Fig.20: Tether Force T for $\omega = \left\{\frac{1}{2}, 1\right\}$ and e=0.

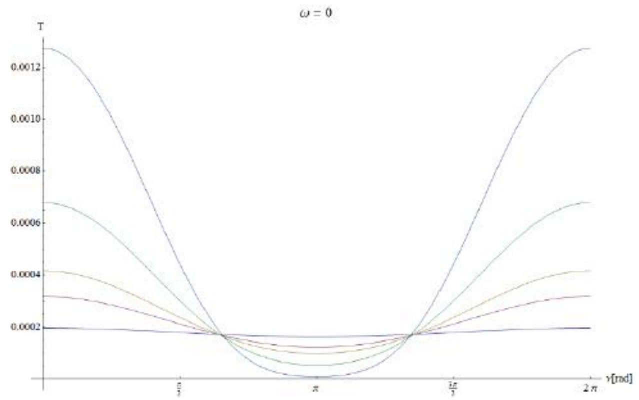

Fig.21: Tether Force T for $\omega = 0$.

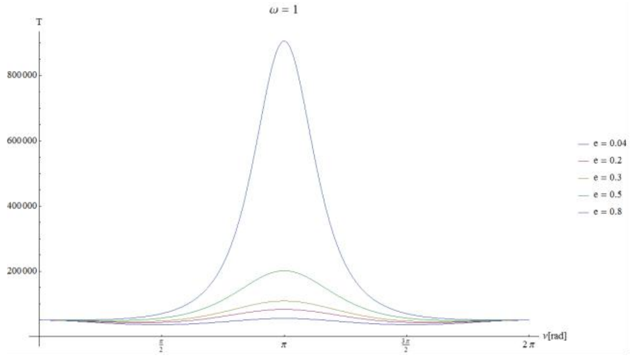

Fig.22: Tether Force T for $\omega = 1$.

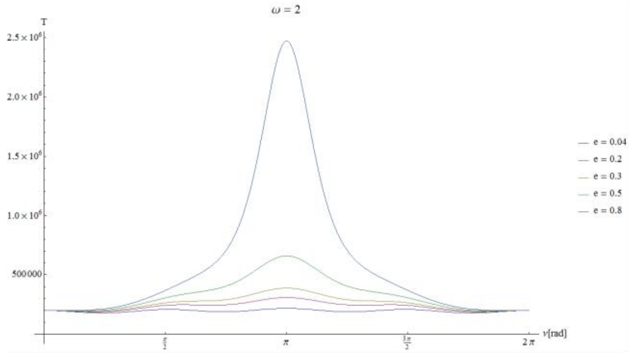

Fig.23: Tether Force T for $\omega = 2$.

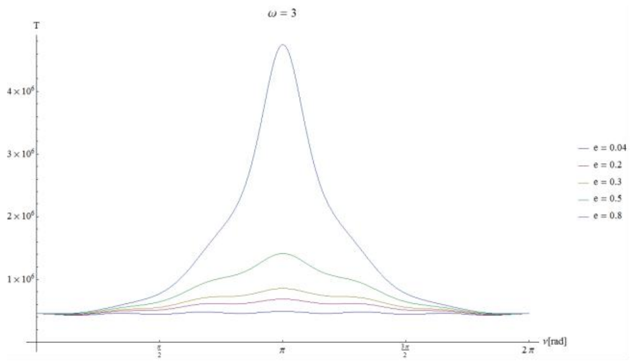

Fig.24: Tether Force T for $\omega = 3$.

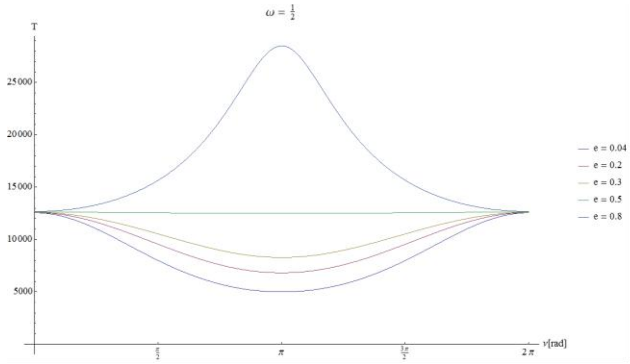

Fig.25: Tether Force T for $\omega = \frac{1}{2}$.

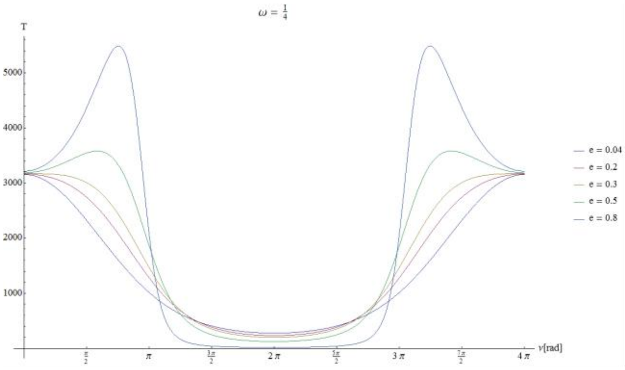

Fig.26: Tether Force T for $\omega = \frac{1}{4}$.

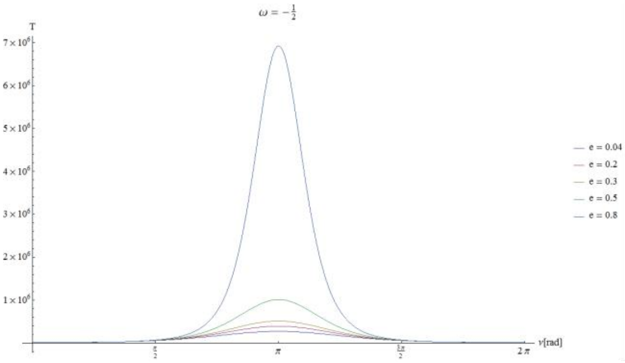

Fig.27: Tether Force T for $\omega = -\frac{1}{2}$.

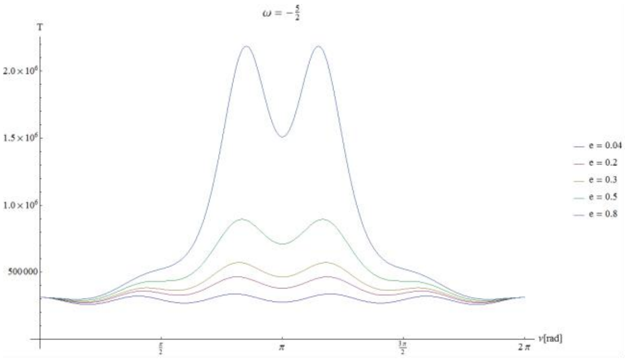

Fig.28: Tether Force T for $\omega = -\frac{5}{2}$.

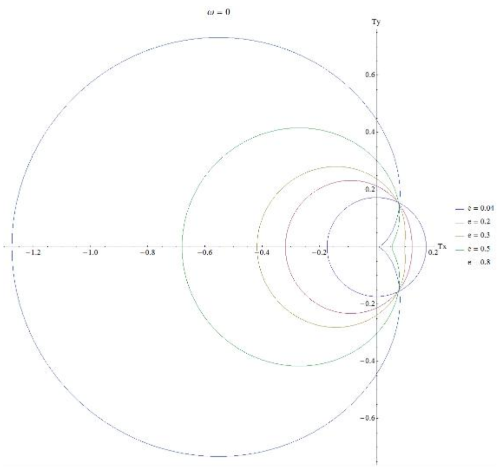

Fig.29: Tether Force $(\vec{T})$ with $\omega = 0$ for several eccentricities (e).

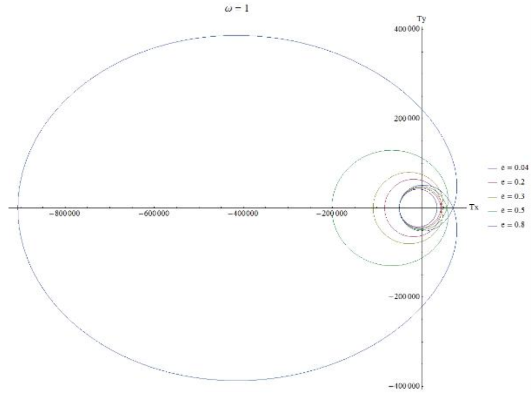

Fig.30: Tether Force $(\vec{T})$ with $\omega = 1$ for several eccentricities (e).

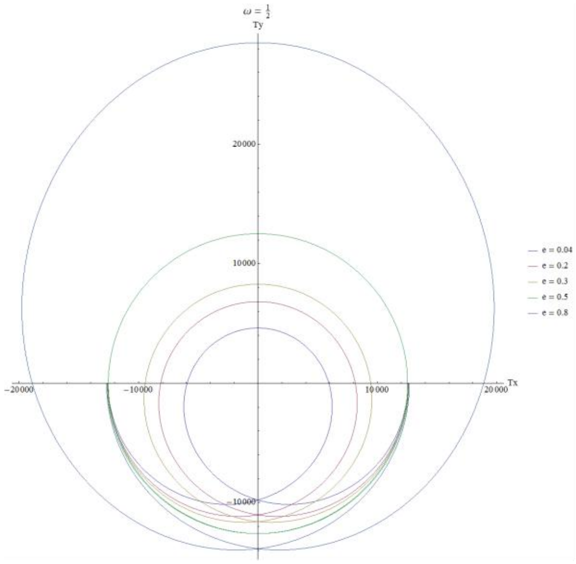

Fig.31: Tether Force $(\vec{T})$ with $\omega = \frac{1}{2}$ for several eccentricities (e).

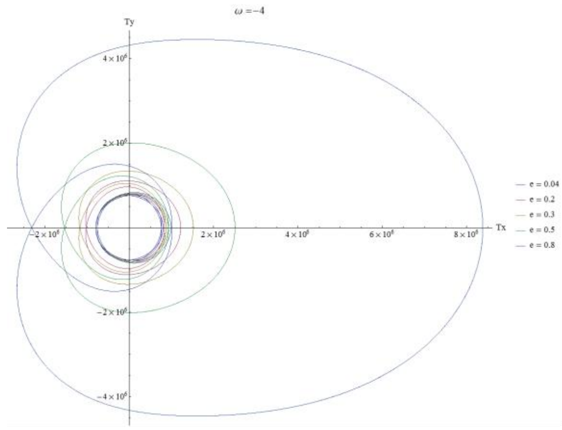

Fig.32: Tether Force $(\vec{T})$ with $\omega = -4$ for several eccentricities (e).

IV. CONCLUSION

The uniform rotations of a dumbbell and with several possibilities are considered in the present study, as well as the stability analysis and the viable control laws. In some cases are possible obtain solutions closes-form, in other cases only numerical solutions for the control of the tether systems are available. The necessary conditions of stability for uniform rotations were analyzed using the parameters ω and the eccentricity of the orbit, generating the control laws for the cable length.

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