EMD-IIT based High-Resolution Signal Reconstruction using LMS Noise Canceller

Sharath.M, Raghu.K

[1]Student, M.Tech (S.P), REVA ITM, Bangalore, India
[2]Asst. Professor, School of ECE, REVA University, Bangalore, India

***Abstract*—*High-resolution (HR) signal need to be reconstructed from the noisy Low-resolution (LR) signals that are measured naturally. The solution to this problem is that using EMD noise removal approach. This framework involves EMD iterative interval thresholding along with LMS algorithm which is applied to the available noisy low-resolution signal so that to obtain HR signals. Then the resulting signal is synthesized further to convert low-resolution signal to high-resolution signal. This method results in the reduction of normalized mean square error (NMSE) compared to the previous methods.*

***Keywords*— Empirical mode decomposition iterative interval thresholding (EMD-IIT), normalized mean square error (NMSE), HR signal, LR signal.**

I. INTRODUCTION

The great interest is present in the various fields of multirate statistical signal processing for the last few years. The problems such as sensor fusion, spectrum estimation are the few problems that come into existence from the researches carried out recently. The problem solution is provided by various researchers. The main focus of the study lies in high-resolution signal (HR) reconstruction. This is concentrated mainly in our study [1].

The signal recovery from the observed noisy data by preserving its important features is a challenging problem in signal processing [2]. There are various methods which are proposed for filtering additive non-stationary noise [3] but the linear filtering methods which are used to remove noise are not effective for the signal that are non-stationary and for the signal consisting of sharp edges. A non-linear filtering can be used instead of linear filtering. This kind of filtering is based on wavelet thresholding [3], [4]. Wavelet thresholding is based on predetermined threshold [4]. The basic idea of this is that signal dominates the noise and therefore signal differs from the noise by comparing with the threshold value. The disadvantage of wavelet approach is basis functions are fixed which does not match with real signals.

The HR reconstruction or the estimation from the LR signal uses maximum entropy (ME) principle and wiener filtering. ME principle uses maximum entropy interference engine (MEIE) for the power estimation of the HR signal from the autocorrelation of the low resolution (LR) signal which are obtained [6]. The major disadvantage of this method is high computational burden.

The Wiener filter theory involves estimation of wide sense stationary signal (WSS) from the low resolution (LR) measurements which are sampled at different rates [6]. Least square approach is proposed for the estimation of the signal from the two observations in which one observation is at full rate and low signal to noise ratio (SNR) whereas the other one is at low rate and high signal to noise ratio (SNR). The disadvantage of this principle is that it uses cross correlation between the HR signal and LR signal which uses second order statistics.

The proposed method in this paper, involves reconstruction of high resolution (HR) signal from noisy low resolution (LR) signal without usage of any correlations and also wavelet thresholding, instead uses EMD-IIT along with LMS in order to reduce NMSE and get back the estimate of original signal [1].

II. PROPOSED METHOD

The proposed work includes reconstruction of high resolution (HR) signal from its N set of low resolution signal ($x_i(n)$ where i=0,1,…. N-1) which are sampled at low rates and added with non-stationary noise which is nothing but noisy, time shifted version and down sampled version of the desired signal d(m). These kind of signal can be generated from the model used for testing purpose which is shown in the Figure 1. The down sampling factor, time shifting factor are selected on the basis of the particular signal of interest [7]. Thus, the ith LR signal is defined as $x_i(n)=S_i(m-i)$ where $m=L_i n$. The signal $x_i(n)$ are assumed as maximally decimated version of d(m) that is $L_i=L$, N=L.

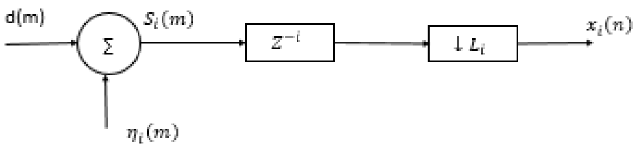

Fig.1: Model to generate LR noisy signal for testing purpose

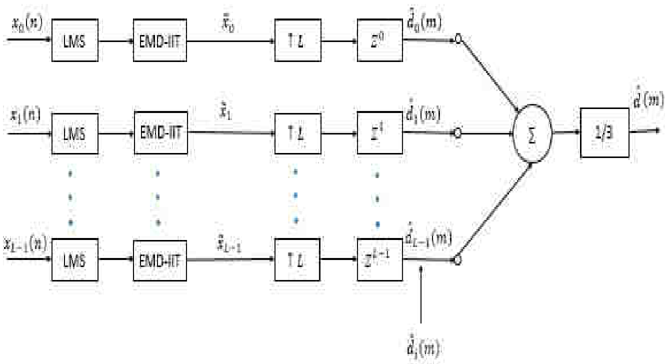

Fig.2: Proposed Block Diagram

Block diagram of the proposed method is shown in Figure 2. This method just wants to have a fixed value of L. The first step of the proposed method is least mean square (LMS) block which removes some amount of noise which is added in the form of non-stationary noise. The second block is the EMD-IIT which is a combination of Empirical mode decomposition (EMD) and iterative thresholding (EMD-IT) where EMD decomposes a signal into a series of intrinsic mode frequencies (IMF's) based on sifting process and EMD-IT is used as denoising approach which removes signal based on threshold approach. The next steps are up sampler and integrator block.

Least mean square (LMS): The noisy LR signals are the input for the block diagram. As we know that the signal is noisy, the removal of this noise needs to be done. This needs a block called least mean square (LMS). The LMS algorithm used for noise cancellation estimates the noise by repetitive iteration, which uses the length of the desired signal and subtracts the noise from the low resolution signal.

Empirical mode decomposition (EMD): The output signal of LMS is decomposed to its intrinsic mode frequencies (IMF'S). This is a kind of wavelet decomposition in which sub bands of the LR signal are the different frequency components of that signal [8]. In which each sub bands are called as IMF's, where each IMF replace the details of the LR signal at the certain scale of frequency [2]. There are 2 conditions to check whether the function is IMF they are 1) The number of zero crossing must be same or differ by one 2) the mean value of the envelope which are defined by local maxima and local minima are zero [2].

EMD algorithm: The algorithm has five major steps which are given here

Step 1): € is fixed and i=1;

Step 2): $r_{i-1}=x_i(t)$ [low resolution signal].

Step 3): extract the ith IMF.

A. $h_{i,j-1}(t)=r_{i-1}(t)$;
The local maxima/minima are extracted from result of step A.

B. $U_{i,j-1}(t)$ and $h_{i,j-1}(t)$ are computed as upper envelope and lower envelope respectively by applying cubic spline to the result obtained at step A.

C. The mean $\mu_{i,j-1}(t)$ is calculated as $[U_{i,j-1}(t)+h_{i,j-1}(t)]/2$.

D. $h_{i,j}(t)= h_{i,j-1}(t) - \mu_{i,j-1}(t)$ [j=j+1].

E. The stopping criterion is calculated as $\text{Std Dvn}(j)=\sum[(h_{i,j-1}(t)-)\,h_{i,j}(t))^2/(h_{i,j-1}(t))^2]$.

F. The steps from A to F are continued till StdDvn(j) < €, and then put $IMF_i(t) = h_{i,j}(t)$ which is ith IMF.

Step 4): Residual is $r_i(t) = r_{i,j-1}(t)-IMF_i(t)$.

Step 5): Step 3 is repeated with i=i+1 till the number of extrema in $r_i(t)$ < 2 here Std Dvn is a notation for standard deviation. By applying EMD algorithm the LR signal is decomposed as shown below.

$$x_j(n) = \sum_{k=1}^{K_j} c_j^{(k)}(n) + r^{(k)}(n) \qquad (1)$$

Empirical mode decomposition iterative thresholding (EMD-IT): This is the denoising approach that needs to be followed soon after EMD to remove the noise associated with the IMF's obtained from the EMD algorithm. The denoising approach has following procedure to be followed

$Z_i(k)$ be the ith zero crossing in the kth IMF and $\mathbf{Z_i(k)}$ be the signal in the interval given by $[Z_i(k), Z_{i+1}(k)]$. This instant can be said signal or noise by comparing the signal extrema given by $IMF_j^{(k)}(P_i^{(k)})$ where $P_i^{(k)}$ is the instant of extrema. If this absolute value of $P_i^{(k)}$ is greater than threshold value then it is termed as signal-dominant and that part of the signal is retained, else it is considered as noise and made as 0 [8].

$$\tilde{c}_j^{(k)}[Z_i^{(k)}] = \begin{cases} c_j^{(k)}[Z_i^{(k)}], |c_j^{(k)}[p_i^{(k)}]| > T_j^{(k)} \\ 0, \qquad |c_j^{(k)}[p_i^{(k)}]| \le T_j^{(k)} \end{cases} \quad (2)$$

This is the formula to replace the part of the IMF in the given interval and retain it to signal or make it as zero by comparing with the threshold value which is given by

$$T_j^{(k)} = c\sqrt{E_j^{(k)} 2\ln(M_j)} \quad (3)$$

$$E_j^{(k)} = \rho^{-k} E_i^{(1)}/\beta, \text{ k=2, 3 ...} \quad (4)$$

Where,

$$E_j^{(1)} = [\text{median}(|c_j^{(1)}[n]|)/0.6745]^2$$

The reconstructed formula of the estimated signal from IMF's is given by

$$\tilde{x}_j[n] = \sum_{k=M_1}^{M_2} \tilde{c}_j^{(k)}[n] + \sum_{k=M_2+1}^{K_j} c_j^{(k)}[n] + r^{K_j}[n] \quad (5)$$

EMD-IT is applied for all the noisy low resolution signal (LR) signal and all this signals are estimated through this algorithm.

Empirical mode decomposition iterative interval thresholding (EMD-IIT): This step is formed by the combination of EMD and EMD-IT. The steps followed in this block are explained below.

Step 1: Apply EMD to the jth noisy low resolution signal ($x_j(n)$) to obtain the first IMF given by $C_j^1(n)$.

Step 2: Partial reconstruction of first IMF is done through the formula $x_{j,p}(n) = x_j(n) - C_j^1(n)$.

Step 3: Apply EMD once again to the result of previous step ($x_{j,p}(n)$).

Step 4: The resulting IMF's from EMD are applied to the EMD-IIT, which results in the denoised version of IMF's from this a estimate of $x_j(n)$ is obtained as $\tilde{x}_j(n)$ which consists of the signal part only and doesn't consists of any noise.

Step 5: Apply step 1 to step 4 to obtain the denoised signals of all the LR signals.

Step 6: $\hat{x}(n)$ is obtained by averaging all the denoised LR signals obtained in the previous step.

Up sampler: The succeeding block after the EMD-IIT is up sampler which increases the code rate by the given interpolation factor (L) which was used in the model that was for testing purpose.

Integrator: This step acts as a inverse of delay block and it is followed by up sampler which is used for reconstruction of the input signal.

III. PROPOSED METHOD ALGORITHM

The algorithm for the proposed method is explained and given below:

Step 1: Apply the input signal to the model shown in figure 1, to obtain noisy signals based on L (L=3) which are $x_i(n)$.

Step 2: The noisy signals obtained in step 1 are applied to LMS to remove some extent of additive white Gaussian noise (AWGN).

Step 3: The results which are obtained in previous steps are given to EMD-IIT block, which results in decomposition of a signal into its intrinsic mode frequency (IMF's) from EMD and removal of noise in IMF's are done from EMD-IT. (Note that EMD-IIT is the combination of EMD and EMD-IT)

Step 4: The noise removed version of IMF's are combined together to get $d_{i(m)}$

Step 5: Step 2 to 4 are applied to all noisy Low resolution signal to obtain all the estimated signal (say L=3), therefore 3 estimated signals are obtained

Step 6: The 3 estimated signal obtained are averaged to form an estimate of the original signal.

IV. EXPERIMENTAL RESULTS

To illustrate the proposed method, the input is considered as the triangular wave which is having a SNR of 0db. Three noisy signals are generated which has SNR of 10db, 20db and 30db respectively.

In the proposed method LMS algorithm is used which has a step size of 0.01, interpolation and down sampler factor used is given by L=3. The values of ρ and β which are used in EMD-IT for denoising the IMF's are considered to be 2.01 and 0.719 respectively [8]. Multiplication constant C is 0.5 which is found by increasing the value in the interval [0.1,1.4] in which the best value was found to be 0.5 for which normalized mean square error (NMSE) is minimum, value of M_1 and M_2 are taken to be 2 and size of IMF-2 respectively. The formula in 6 is used to calculate NMSE.

$$\text{NMSE} = \sum_{n=0}^{M-1} mag(e(n)^2) / \sum_{n=0}^{M-1} mag(d(n)^2) \quad (6)$$

Where $e(n) = d(n) - \hat{x}(n)$

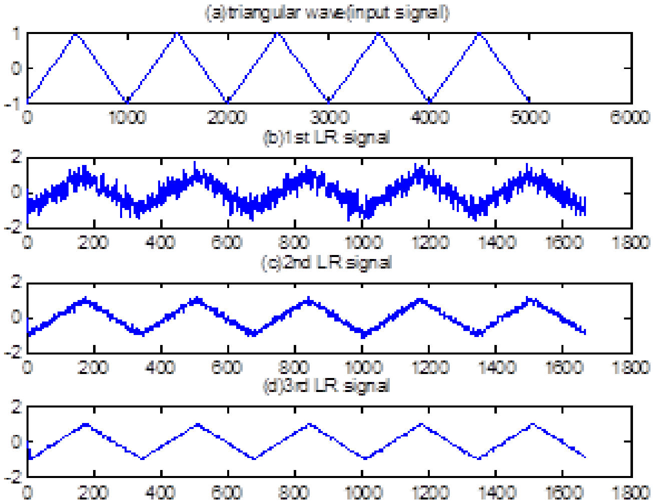

Fig.3: input and low resolution signal

Figure 3 shows the input signal and the low resolution signals formed by the model shown in figure 1. Figure 3a represents the input signal 3b, 3c and 3d represents the 1st low resolution signal (1st LR signal), 2nd low resolution signal (2nd LR signal) and 3rd low resolution signal (3rd LR signal) respectively.
The Figure 4a, 4b and 4c represents the LMS output of the 1st LR signal, 2nd LR signal and 3rd LR signal respectively. Figure 5a, 5b, 5c and 5d represents the first four IMF's of the first LR signal. Figure 6d represents the reconstructed the signal using the proposed method.

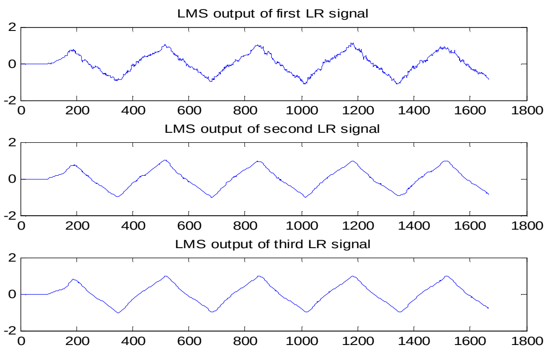

Fig.4: LMS outputs

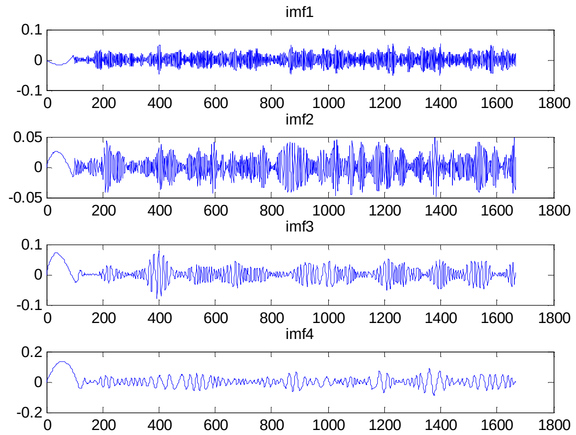

Fig.5: IMF's of 1st LR signal

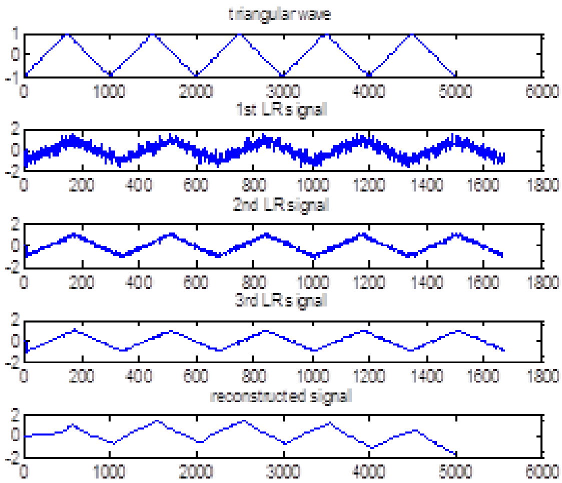

Fig.6: The LR signal and reconstructed signal using proposed method

Table 1: NMSE comparison for different methods

Methods (for HR reconstruction)	(Normalized mean square error)NMSE
EMD-IIT approach	0.4381
Proposed method	0.4132

V. CONCLUSION

The proposed method uses EMD-IIT, which doesn't include any linear filter instead this method uses EMD decomposes the input noisy LR signal into its IMF's and this is denoised by EMD-IT and this denoised IMF's are combined to form the estimated signal. EMD-IIT is used because the other filtering approach is not possible to apply for stationary signal.

LMS is also considered along with the proposed method so as to reduce the NMSE, also reduces the noise associated with the low resolution signal so that it improvises the reconstruction of the signal.

REFERENCES

[1] Adem Ukte, Aydin Kizilkaya and Mehmet Dogan Elbi "Statistical Multirate High-Resolution Signal Reconstruction Using The Empirical Mode Decomposition Based Denoising Approach," IEEE Int. Conf. on Applied electronics(AE), Sep 2014.
[2] Abdel-Ouahab Boudraa, "EMD-Based Signal Filtering," " IEEE Trans. On instrumentation and measurement, Dec 2007.
[3] D. L. Donoho, "De-noising by soft-thresholding," IEEE Trans. Inf. Theory, vol. 41, no. 3, pp. 613–627, May 1995.
[4] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage," Biometrica, vol. 81, pp. 425–455, 1994.
[5] S. Mallat and Z. Zhang, "Matching pursuits with time–frequency dictionaries," IEEE Trans. Signal Process., vol. 41, no. 12, pp. 3397–3415, Dec. 1993.
[6] R. Cristi, D. A. Koupatsiaris, and C. W. Therrien, "Multirate filtering and estimation: the multirate Wiener filter", in Proc. IEEE Thirty-Fourth Asilomar Conf. Signals, Syst., Comput., 2000, pp. 450–454.
[7] J. W. Scrofani, "Theory of multirate signal processing with application to signal and image reconstruction", Ph.D. thesis, Naval Postgraduate School, Sep. 2005.
[8] Y. Kopsinis and S. McLaughlin, "Development of EMDbased denoising methods inspired by wavelet thresholding," IEEE Trans. Signal Process., vol. 57, pp. 1351–1362, Apr. 2009.