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Water Wave Velocity Potential on Sloping Bottom in Water Wave Transformation Modeling

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***Abstract*— :** *In this research, we formulated the equation for water wave velocity potential on sloping bottoms. The resulting velocity potential equation for sloping bottoms closely mirrors that of flat bottoms, simplifying its application in sloping terrain scenarios. By exploring velocity potential on sloping bottoms, we derived conservation equations governing wave constant changes as waves transition from deep to shallow waters. These equations encompass the wave number conservation and energy conservation principles. Utilizing these conservation equations, we developed a comprehensive wave transformation model. The one-dimensional model focused on shoaling and breaking phenomena, while the two-dimensional model delved into refraction-diffraction, shoaling, and breaking. The model's framework allows for straightforward extensions, facilitating future advancements in the field.*

I. INTRODUCTION

Waves undergo significant transformations as they travel from deep waters to shallow coastal areas. These changes involve alterations in their fundamental properties, such as a decrease in wavelength and an increase in wave amplitude, eventually leading to the breaking of waves. Consequently, a velocity potential capable of accommodating these variations in wave constants becomes essential.

The velocity potential of water waves is typically determined by solving the Laplace equation. Dean (1994) utilized a flat bottom approach to formulate this potential, resulting in constant wave number and wave amplitude. However, this method obscures the detailed changes in wave constants. To address this limitation, researchers have explored the wave group concept, focusing on the conservation of energy within wave groups to model variations in wave number and wave height due to changes in water depth (Dean, 1994). Several wave transformation models have emerged from this concept, with the Mild Slope Equation (MSE) pioneered by Berkhoff (1972) being the most renowned. Subsequent researchers such as Booij (1983), Davies and Heathershaw (1984), and Porter (1995) have further developed and refined the MSE. Hsu, Lin, and Ou (2006) introduced the Complementary Mild Slope Equation (CMSE), while Lan, Yuan-Jyh, Hsu, Tai-Wen, Lin, Ta-Yuan, and Liang, Shin-Jye (2012) proposed a wave transformation model utilizing the Higher Order Mild-Slope Equation.

In our research, we delved into solving the Laplace equation within the context of a sloping bottom. This investigation enabled us to uncover the distinctive alterations in wave constants, specifically the conservation laws embedded within the solution of the Laplace equation at the sloping bottom. These fundamental conservation laws serve as the foundation upon which various wave transformation models can be constructed, such as shoaling breaking and diffraction refraction.

II. GENERAL SOLUTION OF LAPLACE'S EQUATION

The Laplace equation is,

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad ....(1)$$

$\phi(x,z,t)$ represents the potential velocity, where x represents the horizontal axis, z represents the vertical axis, and t represents time. The solution of this equation employs the separation variable method, as outlined by Dean (1994), which involves the application of periodic lateral boundary conditions. In the separation variable method, the velocity potential $\phi(x,z,t)$ is conceptualized as the product of three distinct functions, namely,

$$\phi(x,z,t)=X(x)Z(z)T(t)$$

$X(x)$ is a function of x only, $Z(z)$is a function of z only, and $T(t)$ is a function of t only. The properties of these functions are crucial to remember when formulating various equations using the velocity potential equation obtained through the separation variable method of the Laplace equation solution.

Dean (1994) obtained the general solution of the Laplace equation in the form of,

$$\phi(x,z,t)=(A\cos kx+B\sin kx)$$

$$(Ce^{kz}+De^{-kz})\sin\sigma t \qquad ...(2)$$

Where,

$X(x)=(A\cos kx+B\sin kx)$

$Z(z)=(Ce^{kz}+De^{-kz})$

$T(t)=\sin\sigma t$

k and σ are wave constants, where k represents the wave number $k=\frac{2\pi}{L}$, while L being the wavelength and σ represents the angular frequency $\sigma=\frac{2\pi}{T}$, and T being the wave period.

On the other hand, *A, B, C, and D* are constants of the solution that still need to be determined. To derive the values of these constants, the equation is manipulated at characteristic spatial points, where $\cos kx=\sin kx$, consequently, (2) can be expressed as,

$$\phi(x,z,t)=(A+B)\cos kx\ (Ce^{kz}+De^{-kz})\sin\sigma t$$

The solution constants will be formulated by applying the bottom kinematic boundary condition as follows.

$$w_{-h}=-u_{-h}\frac{dh}{dx}$$

w_{-h} bottom vertical water particle velocity at $z=-h$

u_{-h} bottom horizontal water particle velocity at $z=-h$

h water depth towards still water level.

$\frac{dh}{dx}$ Slope of the water, negative for waves propagating from deep water to shallow water..

Vertical water particle velocity is,

$$w(x,z,t)=-\frac{\partial\phi}{\partial z}$$
$$=-(A+B)k\cos kx\ (Ce^{kz}-De^{-kz})\sin\sigma t$$

$$w_{-h}=-(A+B)k\cos kx\ (Ce^{-kh}-De^{kh})\sin\sigma t$$

Horizontal water particle velocity is,

$$u(x,z,t)=-\frac{\partial\phi}{\partial x}$$
$$=(A+B)k\sin kx\ (Ce^{kz}+De^{-kz})\sin\sigma t$$

$$u_{-h}=(A+B)k\sin kx\ (Ce^{-kh}+De^{kh})\sin\sigma t$$

For $(A+B)\neq 0$ and at the characteristic point where $\cos kx=\sin kx$ and $\sin\sigma t\neq 0$, the bottom kinematic boundary condition equation becomes,

$$(Ce^{-kh}-De^{kh})=(Ce^{-kh}+De^{kh})\frac{dh}{dx} \qquad ....(3)$$

2.1 At the Flat Bottom

At the flat bottom, where $\frac{dh}{dx}=0$, (3) transforms to

$$(Ce^{-kh}-De^{kh})=0$$

Obtaining

$$C=De^{2kh}$$

Being substituted to (2)

$$\Phi(x,z,t)=(A+B)coskx(De^{2kh}e^{kz}+De^{-kz})$$
$$sin(\sigma t)$$

$$\Phi(x,z,t)=2(A+B)De^{kh}coskx$$
$$\frac{e^{k(h+z)}e^{kz}+e^{k(h-z)}}{2}sin(\sigma t)$$

$$\Phi(x,z,t)=2(A+B)De^{kh}coskx\cosh k(h+z)\,sin(\sigma t)$$

Defined as $A=2A$ and $B=2B$, thus

$$\Phi(x,z,t)=(A+B)De^{kh}coskx\cosh k(h+z)$$
$$sin(\sigma t)$$

Is defined by the wave constant,

$$G=(A+B)De^{kh}$$

$$\Phi(x,z,t)=G\ coskx\cosh k(h+z)\,sin(\sigma t) \qquad (4)$$

It is important to note that in G, there is an addition of two constants, namely $(A+B)$, where Hutahaean (2003a) demonstrated that $A = B$, thus G in equation (4) has a double value.

2.2 At the Sloping Bottom

Kinematic bottom boundary condition at the sloping bottom is expressed as,

$$(Ce^{-kh} - De^{kh}) = (Ce^{-kh} + De^{kh})\frac{dh}{dx}$$

$$Ce^{-kh}\left(1 - \frac{dh}{dx}\right) = De^{kh}\left(1 + \frac{dh}{dx}\right)$$

$$C = De^{2kh}\frac{1 + \frac{dh}{dx}}{1 - \frac{dh}{dx}}$$

Defined as : $\alpha = \frac{1+\frac{dh}{dx}}{1-\frac{dh}{dx}}$

$$C = D\,\alpha\, e^{2kh}$$

$$\Phi(x,z,t) = (A+B)coskx(D\,\alpha\, e^{2kh}e^{kz} + De^{-kz})\; sin(\sigma t)$$

$$\Phi(x,z,t) = 2(A+B)De^{kh}coskx \left(\frac{\alpha\, e^{k(h+z)} + e^{-k(h+z)}}{2}\right) sin(\sigma t)$$

$$\Phi(x,z,t) = 2(A+B)De^{kh}coskx\,\beta(z) sin(\sigma t)$$

Where : $\beta(z) = \frac{\alpha\, e^{k(h+z)} + e^{-k(h+z)}}{2}$

By defining the new constant G as stated in the previous section, the equation for the velocity potential on a sloping bottom is obtained as follows,

$$\Phi(x,z,t) = G\, coskx\,\beta(z) sin(\sigma t) \qquad \ldots\ldots(5)$$

Subsequently, a comparative analysis was conducted between the values of $\beta(z)$ and $\cosh k(h+z)$, for different bottom slope values represented by $\frac{dh}{dx}$, when $k(h+z) = \theta\pi$ where $\theta = 2.1$, and $\tanh\theta\pi = 1$, as indicated in Table (1). The use of $\tanh\theta\pi = 1$ is specific to deep water conditions, a concept that will be elaborated upon in section (3). Consequently, θ will be denoted as the deep water coefficient henceforth.

Table (1). The comparison between $\beta(\theta\pi) = \frac{\alpha e^{\theta\pi} + e^{-\theta\pi}}{2}$ and $\cosh\theta\pi$

$\frac{dh}{dx}$	α	$\beta(\theta\pi)$	$\cosh\theta\pi$	ε (%)
-0.01	0.98	359.315	366.573	1.98
-0.02	0.961	352.198	366.573	3.922
-0.03	0.942	345.22	366.573	5.825
-0.04	0.923	338.376	366.573	7.692
-0.05	0.905	331.662	366.573	9.524
-0.06	0.887	325.075	366.573	11.321
-0.07	0.869	318.611	366.573	13.084
-0.08	0.852	312.266	366.573	14.815
-0.09	0.835	306.039	366.573	16.514
-0.1	0.818	299.924	366.573	18.182

Note : $\varepsilon = \left|\frac{\beta(\theta\pi) - \cosh\theta\pi}{\cosh\theta\pi}\right| x100\%$

In Table (1), the disparity between the values of $\beta(\theta\pi) = \frac{\alpha e^{\theta\pi} + e^{-\theta\pi}}{2}$ and $\cosh\theta\pi$, where $\tanh\theta\pi = 1.$, is illustrated. It is evident that the difference increases with larger $\left|\frac{dh}{dx}\right|$ or steeper bed slope. However, it is common for the seafloor to have a gentle slope with a bed slope of less than 0.01. Consequently, it can be inferred that $\beta(\theta\pi) \approx \cosh\theta\pi$) or in other words, $\beta(\theta\pi)$ can be approximated by ccosh $\theta\pi$.

Table (2) Presents the comparison between $\frac{\beta_1(\theta\pi)}{\beta(\theta\pi)}$ and $\tanh\theta\pi$ at $\theta\pi = 2.1$.

$\frac{dh}{dx}$	α	$\frac{\beta_1(\theta\pi)}{\beta(\theta\pi)}$	$\tanh\theta\pi$
-0.01	0.98	1	1
-0.02	0.961	1	1
-0.03	0.942	1	1
-0.04	0.923	1	1
-0.05	0.905	1	1
-0.06	0.887	1	1
-0.07	0.869	1	1
-0.08	0.852	1	1
-0.09	0.835	1	1
-0.1	0.818	1	1

Subsequently, it is defined that

$$\beta_1(z) = \frac{\alpha\, e^{k(h+z)} - e^{-k(h+z)}}{2}$$

This equation is similar to $\sinh k(h+z)$, and a comparison is made between $\frac{\beta_1(\theta\pi)}{\beta(\theta\pi)}$ and $\tanh\theta\pi$ for the deep water coefficient $\theta = 2.1$, as shown in Table (2).

Table (2) illustrates that at $\theta = 2.1$, it is observed that $\frac{\beta_1(\theta\pi)}{\beta(\theta\pi)} = \tanh\theta\pi$. Based on this comparison, it can be concluded that the $\cosh k(h+z)$ function can be employed as a substitute for $\beta(z)$. Consequently, the velocity potential equation on a sloping bottom can be replaced with the velocity potential equation on a flat bottom, namely equation (4),

$$\phi(x,z,t) = G\cos kx \cosh k(h+z)\sin\sigma t \quad \ldots\ldots(4)$$

Where the complete equation is,

$$\phi(x,z,t) = \frac{G}{2}(\cos kx + \sin kx)\cosh k(h+z)\sin\sigma t \quad \ldots..(5)$$

III. EQUATIONS OF CONSERVATION.

3.1 Conservation Equation of Wave Number.

In solving the Laplace equation using the separation variable method, it is assumed that the velocity potential is a product of three functions. In equation (4), where $Z(z)$ represents a function of z only, it is defined as,

$$Z(z) = \cosh k(h+z)$$

Therefore, on a sloping bottom where changes in water depth h and wave number k occur, the following applies,

$$\frac{dZ(z)}{dx} = 0$$

$$\sinh k(h+z)\frac{dk(h+z)}{dx} = 0$$

Where

$$\frac{dk(h+z)}{dx} = 0$$

In this equation, it is essential to determine the appropriate value for z . Hutahaean (2022a, b), in formulating the wave amplitude function using the kinematic free surface boundary condition, determined that the suitable value for z is $z = \frac{A}{2}$. Therefore, the conservation equation of wave number becomes,

$$\frac{dk\left(h+\frac{A}{2}\right)}{dx} = 0 \quad \ldots\ldots.(6)$$

Consequently, the equation obtained is,

$$k\left(h+\frac{A}{2}\right) = constant$$

These constants apply throughout the entire water body, including deep waters where

$$\tanh k\left(h+\frac{A}{2}\right) = 1$$

For

$$k\left(h+\frac{A}{2}\right) = \theta\pi$$

The parameter θ will be henceforth referred to as the deep water coefficient. Hutahaean (2023), through the study of the breaker depth index $\frac{H_b}{h_b}$, H_b represents the breaker height and h_b represents the breaker depth, recommends the value $\theta = 2.1$.

Where small amplitude is assumed, (6) can be expressed is

$$\frac{dkh}{dx} = 0$$

For a wave moving from point x with water depth h_x to point $x+\delta x$ with water depth $h_{x+\delta x}$, where $h_{x+\delta x}$ is shallower than h_x, there exists a relationship.

$$k_{x+\delta x} = \frac{k_x h_x}{h_{x+\delta x}}$$

It can be observed that $k_{x+\delta x}$ is greater than k_x, or the wavelength $L_{x+\delta x}$ is smaller than L_x. This indicates a shortening of the wavelength when waves move from deeper water to shallower water. This means that in the wave number conservation equation, there is a phenomenon of shoaling, signifying a change in wavelength.

As a consequence of equation (6), for small amplitudes, the following applies,

$$\frac{d\tanh kh}{dx} = \frac{1}{cosh^2\, kh}\frac{dkh}{dx} = 0$$

Or

$$\tanh kh = constant$$

As known, the wave constants k, G and A are functions of water depth h, meaning they change with depth or are functions of h. Therefore, waves moving from deep water to shallow water will experience changes in these three wave constants. The water depth h is a function of the horizontal axis x, denoted as $h = h(x)$. Consequently, $k = k(x)$, $G = G(x)$ and $A = A(x)$. This implies the existence of values for $\frac{dk}{dx}$, $\frac{dG}{dx}$ and $\frac{dA}{dx}$. The governing equation controlling the changes in these wave constants is the continuity equation, where, with the variable velocity potential ϕ, this equation becomes the Laplace equation.

In deep water, $\tanh kh = 1$. Therefore, for all water depths, including shallow water, $\tanh kh = 1$. holds true. This condition is different from linear wave theory, where $\tanh kh = 1$ changes with varying water depth.

3.2 Conservation of Energy Equation

As it is known, the wave constants k, G and A change with depth or are functions of water depth h. Therefore, waves moving from deep water to shallow water will experience changes in these three wave constants. The water depth h is a function of the horizontal axis x, denoted as $h = h(x)$. Consequently, $k = k(x)$, $G = G(x)$ and $A = A(x)$ leading to the existence of values for $\frac{dk}{dx}$, $\frac{dG}{dx}$ dan $\frac{dA}{dx}$. The governing equation controlling the changes in these wave constants is the continuity equation, where, with the variable velocity potential ϕ this equation becomes the Laplace equation.

Equation (5) is substituted into (1), and when evaluated at the characteristic point, it yields,

$$\frac{d^2G}{dx^2} = 0 \quad \ldots\ldots\ldots\ldots(7)$$

If the velocity potential given in equation (4) is used, and considering equation (7),

$$G\frac{dk}{dx} + 2k\frac{dG}{dx} = 0 \quad \text{..........(8)}$$

Considering that G represents the rate of energy transfer per unit time, equation (8) can be referred to as the energy conservation equation.

IV. THE WAVE AMPLITUDE FUNCTION

By integrating the kinematic free surface boundary condition with respect to time, using the velocity potential given in equation (4), and evaluating it at characteristic points in space and time, Hutahaean (2022a,b) derived the water surface elevation equation as follows:

$$\eta(x,t) = A\cos kx\cos\sigma t$$

A is wave amplitude.

$$A = \frac{Gk}{2\sigma\gamma_2}\left(1-\frac{kA}{2}\right)\cosh\theta\pi \quad \text{........(9)}$$

This equation is referred to as the wave amplitude function. γ_2 is the coefficient in the time derivative of the Taylor series, due to the truncation of the series, to the first derivative only. Hutahaean (2022a) obtained $\gamma_2 = 1.4$. θ is the coefficient for deep water.

Taylor series for function $f = f(x,t)$

$$f(x+\delta x, t+\delta t) = f(x,t) + \delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x} + \frac{\delta t^2}{2!}\frac{\partial^2 f}{\partial t^2} + \delta t\,\delta x\frac{\partial^2 f}{\partial t\partial x} + \frac{\delta x^2}{2!}\frac{\partial^2 f}{\partial x^2} + \cdots.$$

When truncation is performed, only the first derivative is used, Hutahaean (2022a),

$$f(x+\delta x, t+\delta t) = f(x,t) + \gamma_2\delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x}$$

While for function $f = f(x,z,t)$

$$f(x+\delta x, x+\delta x, t+\delta t) = f(x,z,t) + \gamma_3\delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x} + +\delta z\frac{\partial f}{\partial z}$$

Equation (9) can be written as the equation for G as follows.

$$G = \frac{2\sigma\gamma_2 A}{k\left(1-\frac{kA}{2}\right)\cosh\theta\pi} \quad \text{.........(10)}$$

V. SHOALING-BREAKING MODEL

In this section, a one-dimensional wave transformation model is developed, including only shoaling and breaking.

Waves transitioning from deep water to shallower depths undergo alterations in the parameters G, k and A - a phenomenon known as shoaling. During shoaling, the wave number k increases, leading to a shorter wavelength, and the wave amplitude A rises until reaching a breaking point at a specific depth. Developing shoaling and breaking models becomes straightforward by employing two conservation equations and the wave amplitude function.

In the energy conservation equation, alterations in G interact with changes in k, while the wave number conservation equation shows a connection between variations in wave number k and changes in wave amplitude A due to alterations in depth h. Consequently, working with these conservation laws reveals an intricate interplay between the three wave constants: G, k and A

Equation (8) can be written as,

$$\frac{1}{G}\frac{dG}{dx} = -\frac{1}{2k}\frac{dk}{dx}$$

This equation is multiplied by dx and integrated, yielding:

$$\int_x^{x+\delta x}\frac{dG}{G} = -\frac{1}{2}\int_x^{x+\delta x}\frac{dk}{k}$$

$$\ln G_{x+\delta x} - \ln G_x = -\frac{1}{2}(\ln k_{x+\delta x} - \ln k_x)$$

Or

$$G_{x+\delta x} = e^{\ln G_x - \frac{1}{2}(\ln k_{x+\delta x} - \ln k_x)} \quad \text{........(11)}$$

This equation represents the relationship between the change in G concerning the change in k, for waves moving from point k, to point $x+\delta x$ where $h_{x+\delta x} < h_x$. From this equation, it is evident that knowing $x+\delta x$ where $h_{x+\delta x} < h_x$ is necessary to calculate $G_{x+\delta x}$. Therefore, what is needed is the equation describing the change in wave number k first.

Equation (9) is differentiated with respect to the horizontal axis x

$$\frac{dG}{dx} = \frac{2\sigma\gamma_2}{k\left(1-\frac{kA}{2}\right)\cosh\theta\pi}\frac{dA}{dx} - \frac{2\sigma\gamma_2 A}{k^2\left(1-\frac{kA}{2}\right)\cosh\theta\pi}\frac{dk}{dx}$$

In the differential, the term is zero $\frac{d\left(1-\frac{kA}{2}\right)}{dx}$ based on the wave number conservation equation. Considering (9), this equation can be written as,

$$\frac{dG}{dx} = \frac{G}{A}\frac{dA}{dx} - \frac{G}{k}\frac{dk}{dx}$$

Upon substitution into (8),

$$\frac{1}{2k}\frac{dk}{dx} = \frac{1}{A}\frac{dA}{dx} \quad \text{..................(12)}$$

The wave number conservation equation can be written as,

$$\frac{dA}{dx} = -\frac{2}{k}\left(h + \frac{A}{2}\right)\frac{dk}{dx} - 2\frac{dh}{dx}$$

Substituting into (12),

$$\frac{dk}{dx} = -\frac{2k}{\left(2h + \frac{3A}{2}\right)}\frac{dh}{dx} \qquad \text{.......(13)}$$

This equation is the change in wave number k equation that satisfies the energy conservation equation, wave number conservation equation, and kinematic free surface boundary condition, where (9) is derived from the kinematic free surface boundary condition equation

In a very small interval, small δx $k_{x+\delta x}$ can be assessed as,

$$k_{x+\delta x} = k_x + \delta x \frac{dk}{dx}$$

Furthermore, the change in wave amplitude A can be obtained by differentiating the wave amplitude function with respect to the horizontal axis x,

$$\frac{dA}{dx} = \frac{G}{4\sigma\gamma_2}\frac{dk}{dx}\left(1 - \frac{kA}{2}\right)cosh(\theta\pi) \qquad \text{.......(14)}$$

where $\frac{dk}{dx}$ is obtained from (13). In a very small interval δx, $A_{x+\delta x}$ can be calculated as follows,

$$A_{x+\delta x} = A_x + \delta x \frac{dA}{dx}$$

In the following section, an example of shoaling model results is presented for waves with a wave period $T = 7.15$ seconds and wave amplitude in deep water $\backslash A_0 = 1.2$ m meters, in waters with a bottom slope $\frac{dh}{dx} = -0.01$. The calculation parameters used are the deep water coefficient $\frac{dh}{dx} = -0.01$. The model results are presented in Figure 1.

The wave period is calculated based on the input wave amplitude (Hutahaean (2023a)).

$$T = \sqrt{\frac{8\,\pi^2\left(\gamma_2 + \frac{\gamma_3}{2}\right)^2 A}{g}}\ (sec) \qquad \text{......(15)}$$

where $\gamma_3 = 1.8$ (Hutahaean (2022a)).

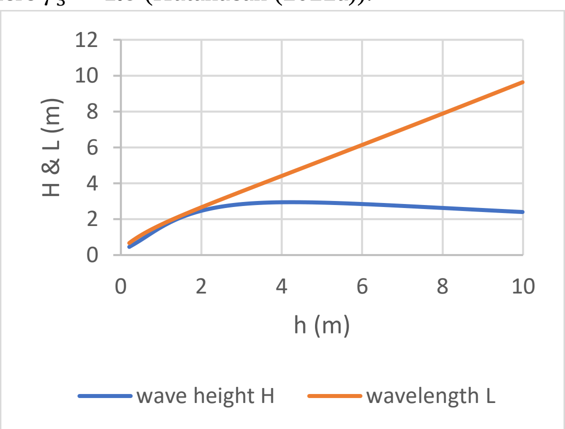

Fig (1) The outcomes of shoaling-breaking model.

The results of the shoaling and breaking model (Fig (1)) show that breaking occurs at a breaker depth $h_b = 4.258$ m, breaker height $\backslash h_b = 4.258$ m, and breaker length $\frac{H_b}{h_b} = 0.692$ dan $\frac{H_b}{L_b} = 0.637$.

Breaking occurs when $\frac{dA}{dx} = 0$.. Therefore, from (14), breaking occurs when $\left(1 - \frac{kA}{2}\right) = 0$,, and subsequently, there is a reduction in wave amplitude when $\frac{kA}{2} > 1$.

VI. REFRACTION – DIFFRACTION MODEL.

In this section, a comprehensive two-dimensional wave transformation model is formulated, encompassing refraction, diffraction, shoaling, and breaking phenomena. The diffraction considered here is solely due to bathymetric features and does not account for the dispersion of wave energy in a direction perpendicular to the wave's propagation.

The refraction-diffraction model was derived based on the shoaling-breaking equations elaborated in the preceding section. In the shoaling-breaking model, wave propagation occurs along the horizontal -x axis. However, in this section, the wave travels in the direction of the α, forming an angle α with the horizontal -x axis, as illustrated in Figure 2.

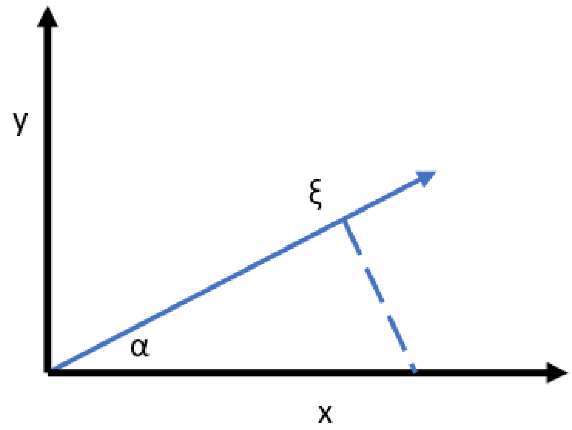

Fig (2). Coordinate system, the relationship between ξ and x.

The relationship between the ξ -axis and the x-axis is given by,

$$\xi(x) = x\cos\alpha$$

Where

$$\frac{d\xi}{dx} = \cos\alpha \qquad \text{.......(16)}$$

For the wave number k which is a function of ξ, denoted as $k = k(\xi(x))$,

$$\frac{dk}{dx} = \frac{dk}{d\xi}\frac{d\xi}{dx}$$

substituting x in (13) with ξ and then replacing it in $\frac{dk}{d\xi}$, while $\frac{d\xi}{dx}$ is substituted with (16).

$$\frac{dk}{dx} = -\frac{2k}{\left(2h + \frac{3A}{2}\right)}\frac{dh}{d\xi}\cos\alpha$$

Similarly, for the wave amplitude $A = A(\xi(x))$

$$\frac{dA}{dx} = \frac{dA}{d\xi}\cos\alpha$$

$$\frac{dA}{dx} = \frac{G}{4\sigma\gamma_2}\frac{dk}{d\xi}\left(1 - \frac{kA}{2}\right)cosh(\theta\pi)\ \cos\alpha$$

where $\frac{dk}{d\xi}$ is equation (16), replacing x to ξ.

The calculation involves changes in wave direction. The wave direction α is

$$\alpha = \operatorname{atan}\left(\frac{v}{u}\right) \quad \text{.......(17)}$$

$$\frac{\partial\alpha}{\partial x} = \frac{u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}}{u^2 + v^2} \quad \text{.........(18)}$$

Where u represents the particle velocity in the horizontal-x direction, and v represents the particle velocity in the horizontal- y direction. To calculate these velocities, a three-dimensional velocity potential is utilized.

$$\phi(x, y) = G\cosh k(h + z)$$
$$\cos k(x\cos\alpha + y\sin\alpha)\sin\sigma t$$

Where

$$u(x, y, z, t) = -\frac{\partial\phi}{\partial x}$$

$$v(x, y, z, t) = -\frac{\partial\phi}{\partial y}$$

4 The Results of Refraction-Diffraction Model.

In the following section, an example of the execution results of the refraction-diffraction model is presented for a coastal bathymetry configuration in the form of a bay (Fig (3)), with waves having a wave period $T = 7.15$ seconds and a wave amplitude in deep water $A_0 = 1.2$ meters. The incoming wave angle forms an angle $A_0 = 1.2$. The calculation parameters used include the deep water coefficient $\theta = 2.1$ and $\gamma_2 = 1.4$.

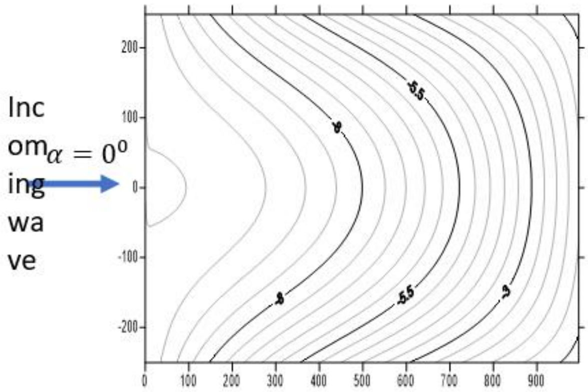

Fig (3). Coastal bathymetry with an incoming wave angle $\alpha = 0^0$

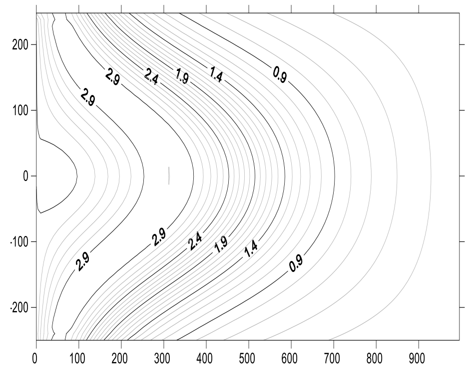

Fig (4). Wave height contours with an incoming wave angle $\alpha = 0^0$

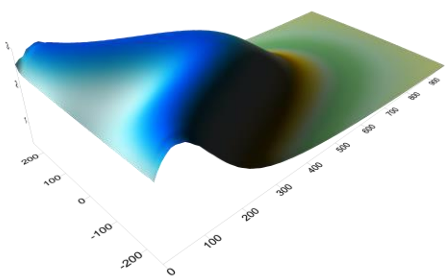

Fig (5). 3D wave height contours with an incoming wave angle $\alpha = 0^0$

In Figures (4) and (5), the spread of wave energy towards the bay's sides is evident, indicating the model's effective simulation of wave refraction-diffraction. Furthermore, the model accurately simulates shoaling and breaking phenomena.

Subsequently, the model results are showcased for the same wave with an incoming wave angle of $\alpha = 30^0$ (Fig (6)). The outcomes are displayed in Fig (7) and Fig (8), demonstrating the model's proficient simulation of refraction, shoaling, and breaking processes.

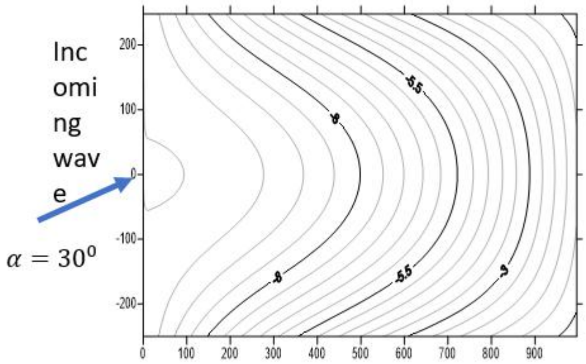

Fig (6). Contour bathymetry with an incoming wave angle $\alpha = 30^0$

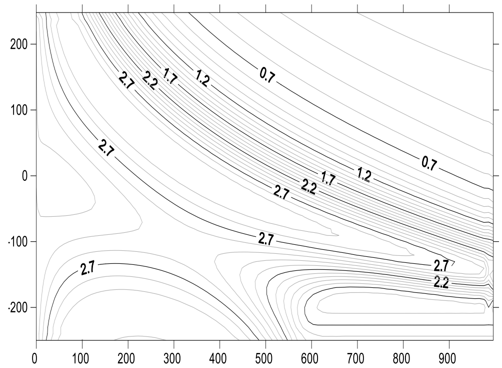

Fig (7). Wave height contours with an incoming wave angle $\alpha = 30^0$

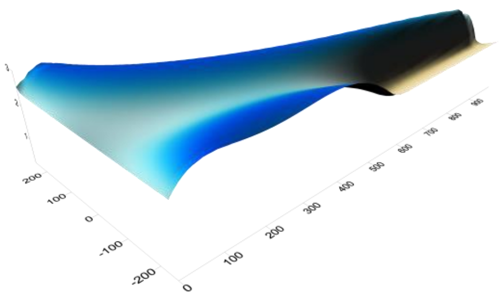

Fig (8). 3D wave height with an incoming wave angle $\alpha = 30^0$

In the subsequent analysis, the model is applied to a submerged island or sandbar (Fig (9)). Waves with a period of $T = 6.2$ seconds and an amplitude of $A_0 = 0.9$ m are utilized. The use of smaller waves is intentional; employing waves with larger amplitudes, such as $A_0 = 1.2$ meters, would lead to the island being submerged. Consequently, the wave height contours would flatten, devoid of any visible variation.

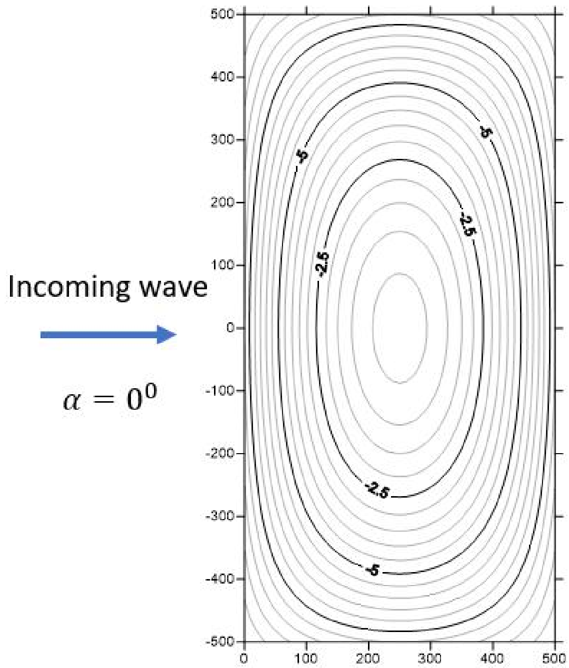

Fig (10) Submerged island bathymetry

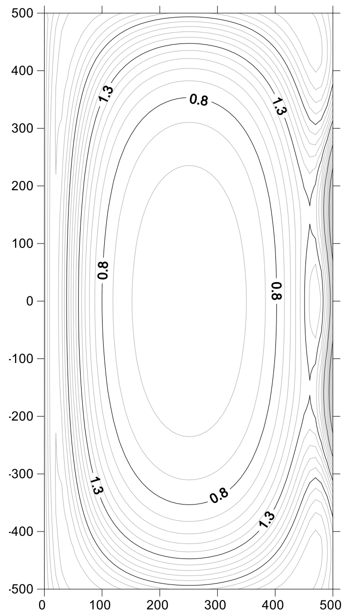

Fig (11) Wave height contours

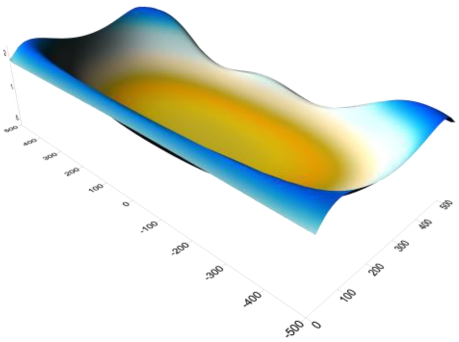

Fig (12) 3D wave height

The model execution results on the submerged island (Fig (11) and Fig (12)) also demonstrate that the model can simulate the phenomena of refraction-diffraction, shoaling, and breaking effectively.

VII. CONCLUSIONS

By solving the Laplace equation along the sloping bottom, conservation equations governing water waves are derived, including the wave number conservation equation and the energy conservation equation. These equations regulate variations in wave constants as waves transition from deeper to shallower waters.

Utilizing these conservation equations, formulating the equation for alterations in water wave constants as waves progress from deep to shallow waters becomes straightforward.

Alterations in wave constants are interconnected, with changes in wave amplitude influencing wave length and vice versa. Consequently, modifications in these wave constants must be addressed simultaneously.

The outcomes from the 1-D shoaling breaking model and 2-D refraction-diffraction model simulations reveal the model's ability to effectively replicate shoaling-breaking and refraction-diffraction phenomena influenced by bathymetry. A necessary advancement involves incorporating the diffraction phenomenon into the 2-D model, accounting for the spread of wave energy perpendicular to the wave's direction.

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