Design Templates for Some Fractional Order Control Systems

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***Abstract*—*Time domain characteristics of first and second order systems are well known. But the same simplicity and explicitness do not exist for low order fractional order systems (FOSs). Considering the step response, the templates are developed for designing the behavior of simple FOSs with a 2-term denominator polynomial (one is unity and the other involves fractional power). Although the explicit relations between design parameters and the performance parameters such as time constant, rise time, overshoot, settling time for fractional order control systems (FOCSs) do not exist and can't be obtainable as in the ordinary integer order control systems, the obtained templates in this paper can be used for designing low order FOCSs. Hence, the drawback of non-existence of similar explicit formulas for FOCSs is eliminated by using these templates.*

***Keywords*— Control system, fractional order, rise time, step response, time constant.**

I. INTRODUCTION

FOSs have founded many applications in the last two decades and a great deal of literature has appeared for analyzing and designing these systems [1-7]. Especially fractional order proportional integral derivative (FOPID) controllers have appeared very frequently in control system design [8-10].

Focusing on some very recent literature, for example [11] proposes an adaptive FOPI control method based on enhanced virtual reference feedback tuning to meet high precision and speed requirements for controlling flexible swing arm system in the light-emitting diode (LED) packaging industry. E. Cokmez et. all have obtained and visualized stability regions based on specified gain and phase margins for a FOPI controller to control integrating processes with time delay [12]. J. R. Nayak and B. Shaw have shown how to enhance the performance of the automatic generation control by adopting cascade proportional derivative (PD) - FOPID controller in a two-area mutually connected thermal power plant with generation rate constraint; group hunting search algorithm is adopted to explore the gain parameters of the controllers [13]. In [14], PI controller design is performed by using optimization for FOSs; first, controller parameters for a stable control are calculated by using the stability boundary locus method and then optimization is used to provide the best control. In [15], a new robust FOPID controller to stabilize a perturbed nonlinear chaotic system on one of its unstable fixed points is proposed based on the PID actions using the bifurcation diagram. In [16], fractional-order discrete synchronization of a new fourth-order memristor chaotic oscillator and the dynamic properties of the fractional-order discrete system are investigated; a new method for synchronizing is proposed and validated.

In spite of the existence of a great deal of publications about FOSs some of which have just been mentioned above, most of the present analysis and design techniques deal with sophisticated and rather special applications [17-24]. Although the step response characteristics such as rise time, settling time, delay time, overshoot and some others are well known by explicit formulas for simple integer order systems [25], such formulas are not available for FOSs. And a compact publication yielding the relations between the design parameters and the step response characteristics of even simple FOSs are not yet present. The purpose of this paper is to fulfill this vacancy and to supply some design tools for simple order FOSs.

The paper is organized as follows; in Section 2, basic definitions of time domain characteristics of first and second integer order systems are given. Section 3 introduces the FOSs that is studied and the investigation of its step responses depending on the fractional power. Section 4 gives and discusses the templates that can be used for the design of low order FOSs. Finally, Section 5 finishes with conclusions.

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II. FIRST AND SECOND INTEGER ORDER SYSTEMS

Let the first order system transfer function H_1 be

$$H_1(s) = \frac{1}{p_1 s + 1}. \tag{1}$$

It is assumed that $p_1 \geq 0$ for stability. Since the study is confined to step response characteristics, it is easily obtained by applying the unit step input $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t \geq 0$, and compute the step response $y(t)$ as

$$y(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-t/p_1}, & t \geq 0 \end{cases}. \tag{2}$$

The variation of the step response is shown in Fig. 1. It is an increasing exponential starting from 0 at $t = 0$, and rising to the steady-state value of 1 as $lim\ t \to \infty$. The following time domain characteristics are defined for a response of the type shown in Fig. 1.

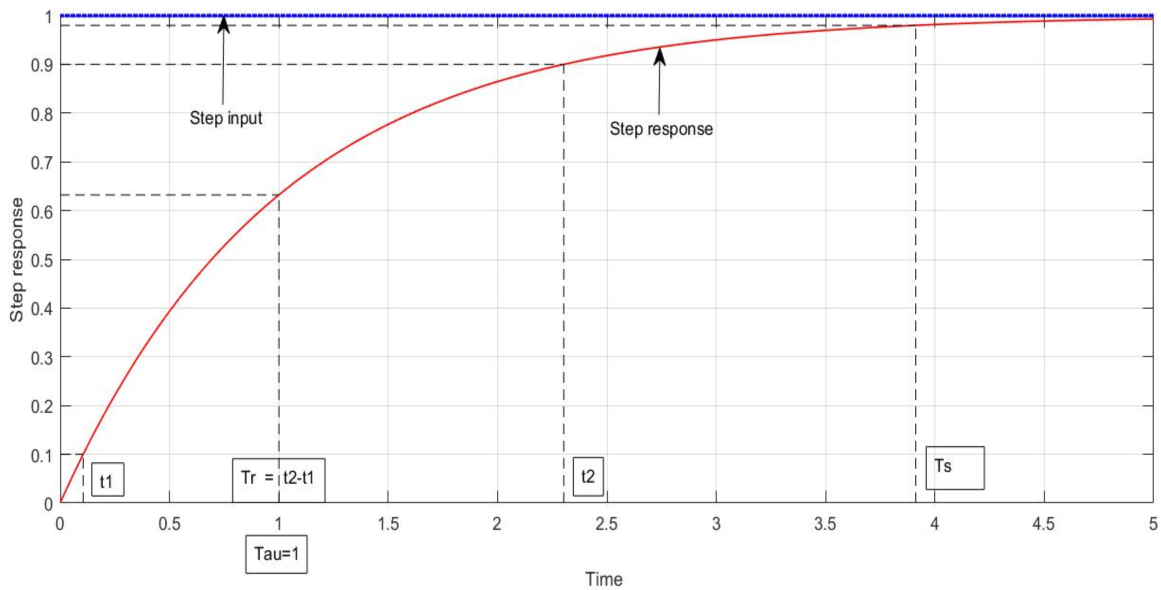

Fig.1: Step response of a first order system and some important characteristics.

Time constant τ: It is the time required for the response to reach $1 - (1/e) = 0.632121 \cong 63\ \%$ of its final value. For this exponential, from Eq. (2) it is true that

$$\tau = p_1. \tag{3a}$$

Rise time T_r: It is the time required for the response reach from 10 % to 90 % of its final value.

$$T_r = t_2 - t_1 = 2.302585\tau - 0.105360\tau = \tau ln9$$
$$= 2.197225\tau \cong 2\tau. \tag{3b}$$

Settling time T_S: It is the time required for the response to stay around its final value with an error less than 2 %.

$$T_s = \tau ln50 = 3.912023\tau \cong 4\tau. \tag{3c}$$

Note that all the time characteristics depend on only the coefficient $\tau = p_1$ in Eq. (1). So, p_1 is chosen according to satisfy all the specifications on τ, T_r, T_s. Note also that for $p_1 = 0$, the system is a unity gain system which yields $y(t) = u(t)$; that is all the characteristic times τ, T_r, T_s are zero and no delay occurs at the response.

In summary, the following properties of time domain characteristics are valid: i) The response increases exponentially to its steady state value without any oscillations; ii) Rise time and settling time are some multiples of time constant $\tau = p_1$.

It is well known that a time domain normalization with respect to τ_e corresponds to replacing t by t/τ_e which also corresponds to frequency domain normalization by writing s instead of $s\tau_e$. In Eq. (1), if we replace s by s/p_1 where $\tau_e = p_1$ is the time constant, then we have

$$H_1(s) = \frac{1}{s + 1}, \tag{4}$$

which has normalized time constant $\tau = 1$.

As the reference transfer function for a second order system

$$H_2(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{5}$$

is considered [25]; obviously dc gain is equal to 1; ω_n is undamped natural frequency and $\xi \geq 0$ is the damping ratio. For $\xi > 1$ which corresponds to overdamped case, Eq. (5) yields the step response

$$y(t) = 1 - \frac{1}{\tau_2 - \tau_1}\left(\tau_2 e^{-\frac{t}{\tau_2}} - \tau_1 e^{-\frac{t}{\tau_1}}\right), \tag{6a}$$

which starts from 0 and rises to 1 monotonically as shown in Fig. 2 (zeta=2.0). In this expression, there are two time constants τ_1 and τ_2 so that

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$$\tau_1 = -\frac{1}{s_1} = \frac{1}{\omega_n\left(\xi + \sqrt{\xi^2 - 1}\right)}, \qquad \tau_2 = -\frac{1}{s_2} = \frac{1}{\omega_n\left(\xi - \sqrt{\xi^2 - 1}\right)}, \qquad (6b)$$

where s_1 and s_2 are the poles of transfer function. It can be shown by using (6b) that

$$\tau_e = \tau_1 + \tau_2 = 2\xi\sqrt{\tau_1\tau_2} = \frac{\xi}{\pi}T_n = \frac{2\xi}{\omega_n}, \qquad (6c)$$

where $T_n = 2\zeta/\omega_n$ is the period of sustained oscillations of the undamped ($\xi = 0$) system.

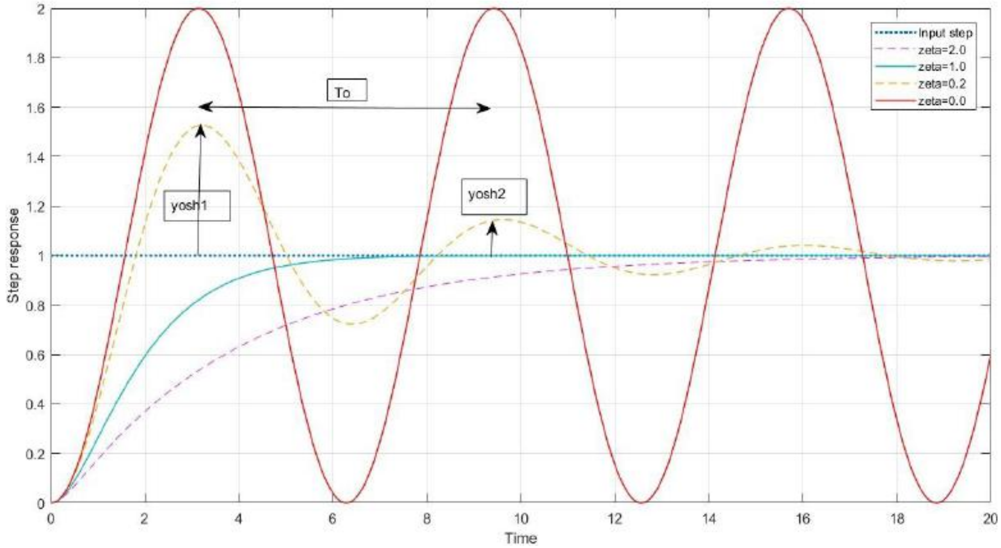

Fig.2: Step response of the second order system in Eqs. (6a), (6b) for values 2, 1, 0.2, 0 of damping ratio ξ; $\omega_n = 1$.

For $\xi = 1$, the system in Eq. (5) is said to be critically damped; in this case there is only one time constant which is $\tau = 1/\omega_n = T_n/2\pi$. The step response is found as

$$y(t) = \mathcal{L}^{-1}\left\{\frac{{\omega_n}^2}{s^2 + 2\omega_n s + {\omega_n}^2}\frac{1}{s}.\right\} = 1 - e^{-\omega_n t}(\omega_n + 1), \qquad (7)$$

which increases monotonically from 0 to the steady-state value 1 as shown in Fig. 2 (zeta=1.0).

For $0 < \xi < 1$, the system is said to be underdamped; for $\xi = 0.2$ the step response is shown in Fig. 2 (zeta=0.2). It is seen that the response is stable and approaches to the reference value 1 in a damped oscillatory manner. It is a routine process to show that the response is given by

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}}sin\left(\sqrt{1 - \xi^2}\omega_n t + sin^{-1}\sqrt{1 - \xi^2}\right). \qquad (8)$$

Finally, for $\xi = 0$, the system is undamped and the step response is

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\frac{{\omega_n}^2}{s^2 + {\omega_n}^2}\right\} = 1 - cos\omega_n t, \qquad (9)$$

which represents sustained oscillations as shown in Fig. 2 (zeta=0) with undamped natural frequency $\omega_n = 1$ and undamped oscillation period $T_n = 2\pi$.

For the step responses corresponding to underdamped case two new time characteristics are defined. The oscillation period from Eq. (8) is

$$T_o = \frac{2\pi}{\sqrt{1 - \xi^2}\omega_n} = \frac{\pi}{\xi\sqrt{1 - \xi^2}}\tau_e. \qquad (10a)$$

The settling time is approximately obtained from Eq. (8) as by equating the coefficient of sin function to $1 - 0.98 = 0.02$. The result is

$$T_s = \frac{1}{\xi\omega_n}ln\frac{50}{\sqrt{1 - \xi^2}} = \frac{\tau_e}{2\xi^2}ln\frac{50}{\sqrt{1 - \xi^2}}. \qquad (10b)$$

Another time which is important is T_{max} when the first peak occurs in the response. From Eq. (8) T_{max} and $y(T_{max}) = y_{max}$ are found to be

$$T_{max} = \frac{\pi}{\omega_n\sqrt{1 - \xi^2}} = \frac{\pi}{2\xi\sqrt{1 - \xi^2}}\tau_e, \qquad (11a)$$

$$y_{max} = 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1 - \xi^2}}. \qquad (11b)$$

The overshoot y_{osh} and percent overshoot ($POSH$) are defined by

$$POSH = \frac{y_{osh} = y_{max} - 1}{y_{ref}}100 = \frac{100\,e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1 - \xi^2}}. \qquad (11c)$$

For the succeeding peaks, Eqs. (12a), (12b), (12c) are modified by replacing π by $(2k-1)\pi$ where k represents the peak numbers; for the first peak $k=1$.

The reduction ratio (RR) is defined as the ratio of successive overshoots, and from Eq. (11c)

$$RR = e^{-\frac{2\xi\pi}{\sqrt{1-\xi^2}}}. \tag{11d}$$

For the undamped system ($\xi = 0$), Eq. (9) yields the following time domain characteristics:

$$\tau = 1 + \frac{cos^{-1}\left(\frac{1}{e}\right)}{\omega_n}, \tag{12a}$$

$$T_r = \frac{cos^{-1}(0.1) - cos^{-1}(0.9)}{\omega_n}, \tag{12b}$$

$$T_{max} = \frac{\pi}{\omega_n}, \quad T_s = \infty, \quad T_o = \frac{2\pi}{\omega_n}, \qquad (12c, d, e)$$

$$y_{max} = 2, \quad y_{osh} = 1, \qquad (12f, g)$$

$$POSH = 100, \quad RR = 1. \qquad (12h, i)$$

For $0 \le \xi < \infty$, time domain characteristics $\tau, T_r, T_{max}, T_o, T_s$ and the overshoot y_{osh} are plotted against $\xi = \tau_e/2$.; The results are shown in Fig. 3.

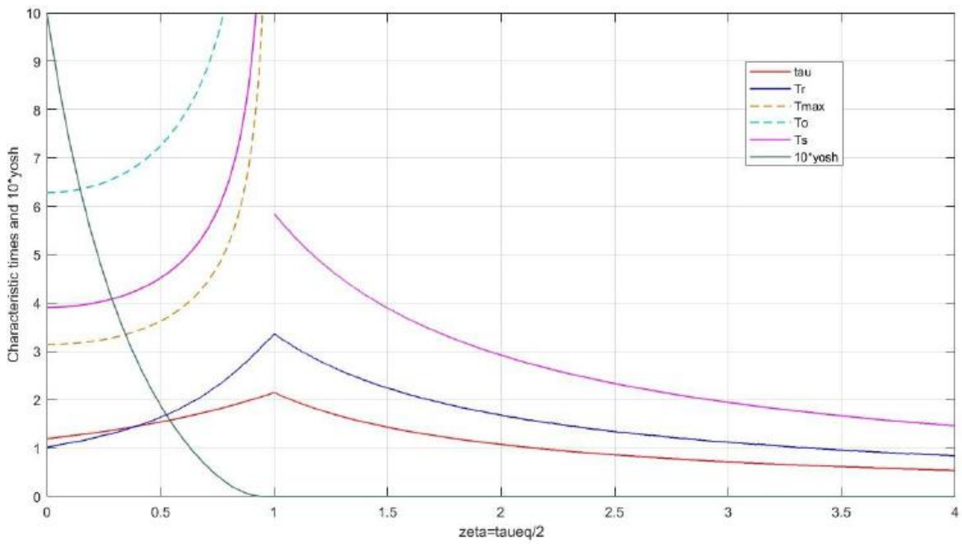

Fig.3: Variation of time domain characteristics against the damping ratio for $\xi \in [0,4]$.

III. INVESTIGATION OF 2-TERM FRACTIONAL DENOMINATOR CHARACTERISTICS

Consider the following fractional order transfer function with a constant numerator and 2-term fractional denominator:

$$H(s) = \frac{a}{bs^\alpha + c}. \tag{13a}$$

Assuming dc gain (a/c) to be 1, letting $b/c = p_1$, and normalizing with $\tau_e = p_1^{\frac{1}{\alpha}} = 1$ we result with

$$H(s) = \frac{1}{s^\alpha + 1}. \tag{13b}$$

We have the following observations on the step response of the fractional transfer function in Eq. (13b); see Fig. 4 for these observations:

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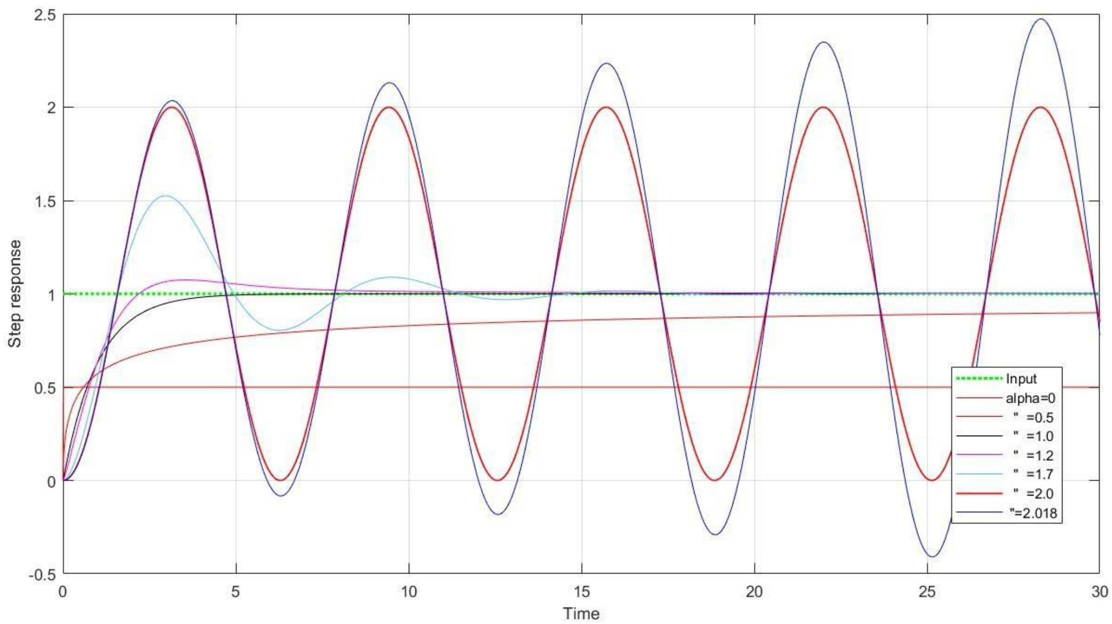

Fig.4: Step response of FOS in Eq. (14b) for different values of α.

1) It is obvious that dc gain of this system is 1, which results from Eq. (13b) by inserting $s = 0$ and assuming $\alpha \neq 0$. Therefore, all the step responses for stable cases will tend to 1 as $lim\, t \to \infty$; ($\alpha = 0.5, 1.2, 1.7$).
2) For $\alpha = 0$, the transfer function will be equal to a constant gain of $1/2$, hence the step response is 0.5 ($\alpha = 0$).
3) For $\alpha = 1$, the transfer function is equal to a first order integer type transfer function, hence, the step response is an increasing exponential with a time constant $\tau = 1$ ($\alpha = 1$, see also Fig. 1).
4) For $\alpha = 2$, the fractional system in Eq. (13b) is equivalent to the second order integer type system in (5) with $\omega_n = 1, \xi = 0$; hence, the step response is sustained oscillation ($\alpha = 2$, see also Fig. 2).
5) For $\alpha > 2$, the system is not stable, and the step response increases exponentially (and oscillatory)-like manner ($\alpha = 2.018$).
6) For $1 < \alpha < 2$, the system is stable, and it has step responses ($\alpha = 1.2, \alpha = 1.7$). The first of these responses ($\alpha = 1.2$) is a decaying curve after an overshoot; and the second ($\alpha = 1.7$) is an oscillatory-like motion with exponentially-like decaying.
7) For $0 < \alpha < 1$, step response is a stable exponentially-like increasing behavior ($\alpha = 0.5$).

We note that those responses for $\alpha = 2.018, 1.7, 0.5$, resemble to those of a second order (for $\alpha = 2.018, 1.7$) and of a first order ($\alpha = 0.5$) integer order systems. But the explicit formulas as in Eqs. (3,10,11,12) between the system parameter α and the step response characteristics do not exist for the considered FOSs. So, in the following section, instead of using explicit formulas, some templates are obtained to be used for designing FOSs.

IV. DEPENDENCE OF STEP RESPONSE CHARACTERISTICS ON α

In this section the dependence curves (templates) of step response characteristics, namely, duration of first oscillation period (T_o), time constant τ, rise time T_r, and settling time T_s vs $\alpha \in [0.01, 1.99]$, percent overshoot ($POSH$) vs $\beta = 2 - \alpha \in [0.01, 1.99]$ are obtained by simulations. Simulations are carried for 30 $s$in steps of $\Delta\alpha = 0.01$ by subprograms of FOMCON toolbox [7] integrated with MATLAB R2017 [7].

Fig. 5 shows the variation of the duration of the first oscillation against α. Numerical data show that the first peak occurs for $\alpha = 1.01$ and it is equal to 1.0014. Then, until $\alpha = 1.34$ second peak does not appear; more elaborate numerical analysis show that, the second maximum starts exhibiting for the first time for $\alpha = 1.3396$ for which the first and second overshoots are $0.63695221, 0.011386081$, respectively; but for $\alpha = 1.34$ following the first peak of value 1.1640, the second peak of value 1.0114 occurs. This means period of the first oscillation is defined for $\alpha \geq 1.3396$. Since there are no peaks (maximums) until $\alpha = 1.01$, the graph is started from $\alpha = 0.8$, though numerical data is obtained for all $\alpha \in [0.01, 1.99]$.

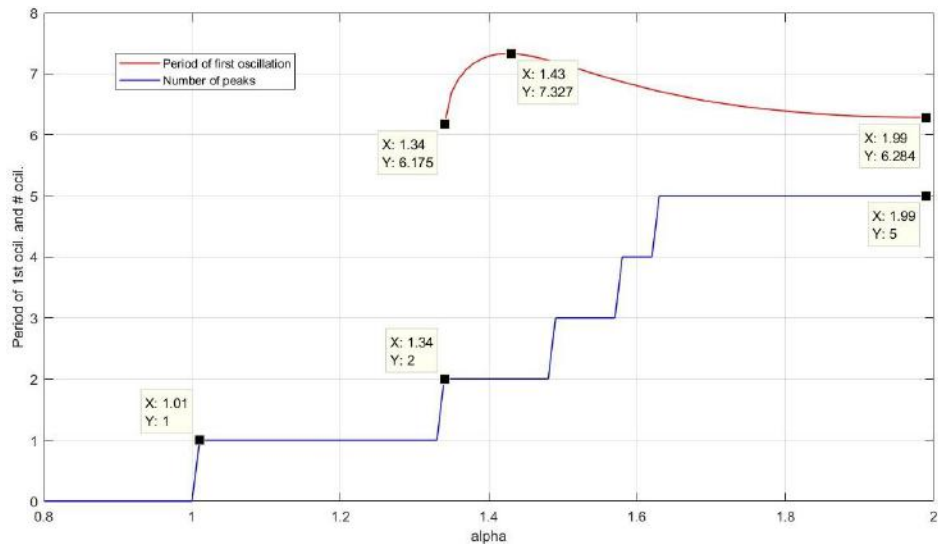

Fig.5: Duration of first oscillation against α.

Fig. 6 shows plots of time constant τ (Tau), rise time T_r, and settling time T_s. Time constant plot Tau starts from $\alpha = 0.14$, because, for smaller values of α,τ is larger than 30 s so that the response can't reach the critical value 0.632121 until 30 s; similar arguments are true for the rise time T_r which starts at $\alpha = 0.51$, and for settling time T_s which starts at $\alpha = 0.78$. Tau is obviously decreasing with increasing α, and it becomes 1.1915 for $\alpha = 1.99$. T_r also decreases with increasing $\alpha$and it changes from 28.0350 at $\alpha = 1.51$ to 1.0220 at $\alpha = 1.99$. For $\alpha = 1.70$, τ is almost equal to T_r ($\tau = 1.0965, T_r = 1.0970$); for $\alpha \in [0.01, 1.70]$, $\tau < T_r$; and for $\alpha \in (1.70, 1.99]$, $\tau > T_r$. Settling time plot starts from $T_s = 27.33$ for $\alpha = 0.78$ and ends at $T_s = 29.82$ for $\alpha = 1.99$. T_s decreases until $\alpha = 1.07$ and reaches to its minimum value $T_s = 2.8345$ at $\alpha = 1.07$, then it jumps up to $T_s = 5.1525$ at $\alpha = 1.08$. The plot terminates at $\alpha = 1.99$ with $T_s = 29.8215$. The irregular shape of increase of T_s for $\alpha \in [1.07, 1.99]$ is due to the dependence of T_s on discrete change of oscillations remaining in the limit$[1, \mp 0.02]$. Contrary to monotonic decrease of τ and T_s, it is true that (disregarding the irregular changes mentioned) T_s decreases monotonically for $\alpha \in [0.01, 1.07]$ and it increases for $\alpha \in [1.07, 1.99]$.

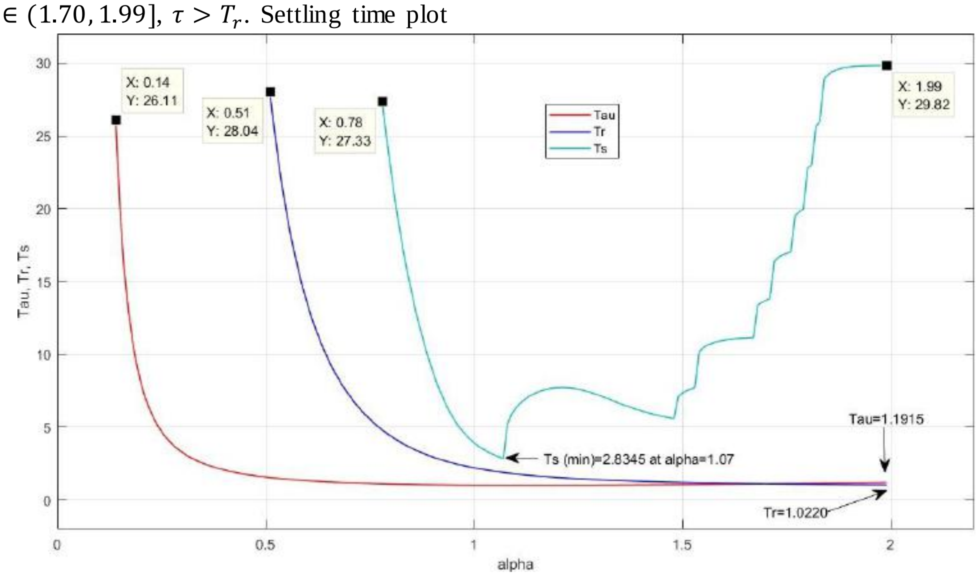

Fig.6: Time constant τ, rise time T_r, and settling time T_s vs α.

Fig. 7 shows the comparison of the variation of overshoots with $\alpha \in [1.01, 1.99]$ ($\beta = 2 - \alpha, \beta \in [0.01, 0.99]$) and with the damping ratio ξ of a second order system. This plot is useful for finding the fractional order α and damping ratio

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ξ for a given overshoot. For example, to achieve an overshoot of 60 %; $2 - \alpha = 0.24 \rightarrow \alpha = 1.76$ and $\xi = 0.16$ are appropriate for FOS and for a second order system, respectively.

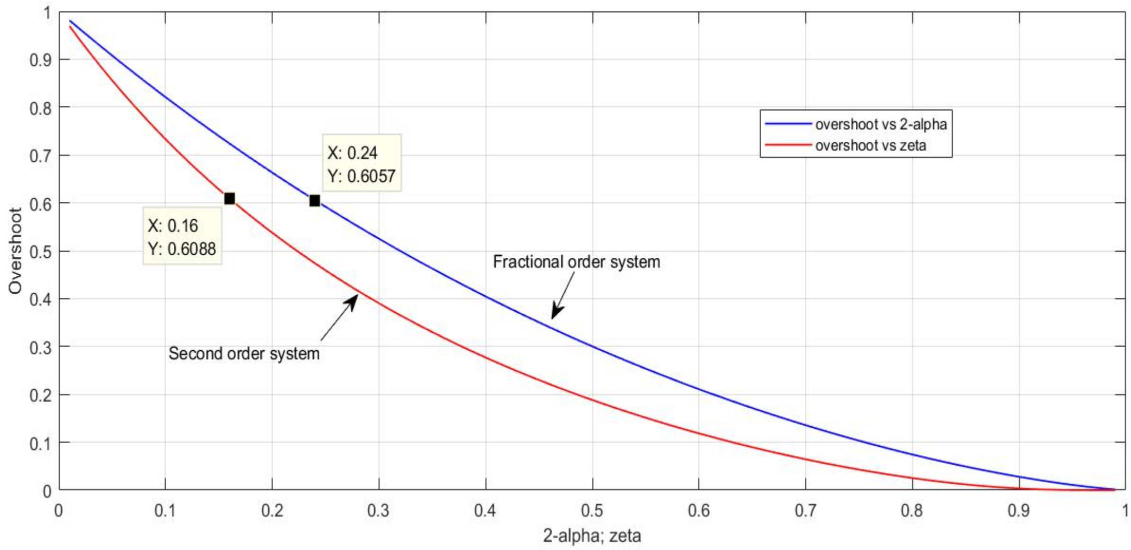

Fig.7: Overshoots versus $\beta = 2 - \alpha$ (for fractional) and ξ (for 2nd order) systems.

Fig. 8 better illustrates the relations between the overshoot and directly α (not $\beta = 2 - \alpha$) for fractional system, and ξ for a second order system.

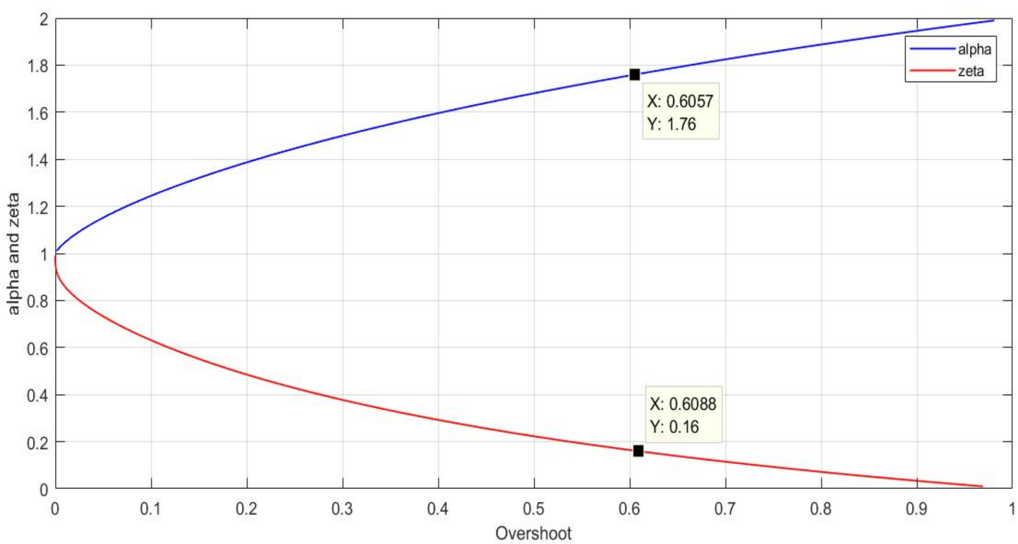

Fig.8: Variation of α for fractional order, and ξ for second order systems with the overshoot.

Fig. 9 shows the values $\beta = 2 - \alpha$ and ξ against the rise time. It is obvious that for a rise time of 4.371, β and ξ have the same values of 1.2. This means for $\xi = 1.2$ and for $\alpha = 2 - \beta = 2 - 1.2 = 0.8$ the rise times are equal to 4.371.

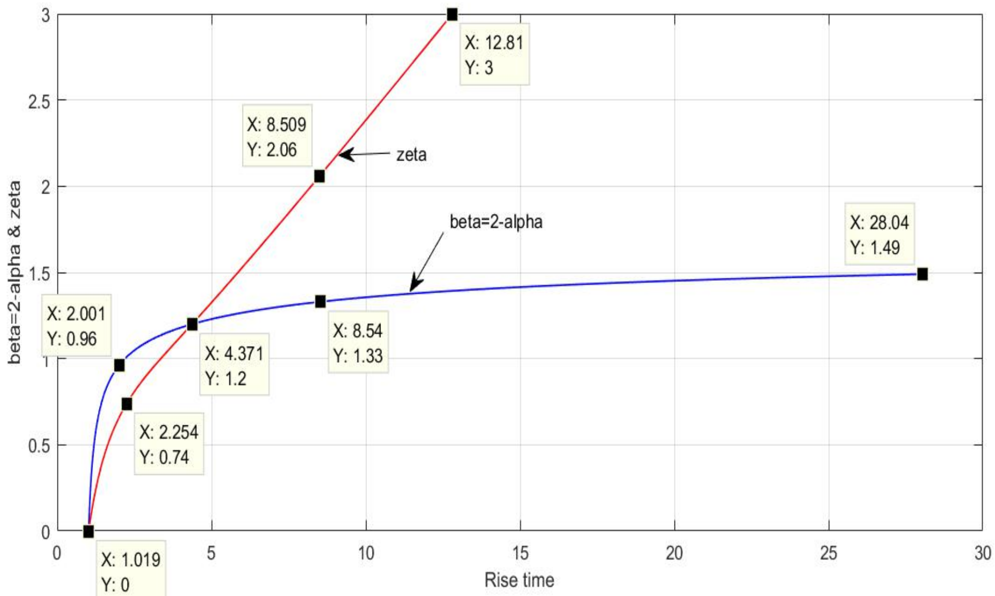

Fig. 9: $\beta = 2 - \alpha$ *versus rise time and* ξ *versus rise time.*

V. CONCLUSIONS

Time domain characteristics of the FOS with a 2-term denominator polynomial involving a single fractional power is investigated in this presentation. Dependence of important step response characteristics, namely rise time, settling time, delay time, overshoot, and oscillation period on the fractional order α are derived, and the results are presented in graphical forms that can be used as templates for design purposes. The study is conducted comparatively by considering integer order systems of 1st and 2nd order types. It is shown that the same simplicity and explicitness present for second order systems do not exist between the transfer function parameters and the step response characteristics for low order fractional systems. The results bring light for designing simple FOCSs, thus a vacancy has been fulfilled by this work.

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