International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-8, Issue-12; Dec, 2021
Journal Home Page Available: https://ijaers.com/
Article DOI: https://dx.doi.org/10.22161/ijaers.812.21

The harmonic and melodic connection numbers involving the mutual inclusions among the generic groups of notes arbitrarily emitted

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Received: 29 Oct 2021,

Received in revised form: 02 Dec 2021,

Accepted: 09 Dec 2021,

Available online: 16 Dec 2021

***Keywords*— Chords, Harmonic substitutions, Harmonization, Scales, Unconventional Notation.**

***Abstract*—** *The present paper is aimed to provide specific details and new results related to the music network described in the previous study by the author. While the first paper served as an introduction and extensive analysis of the model in all its potential applications, the present study is aimed at providing supplementary data to quantitatively support the previous one. The music concepts described in the present paper concern the evaluation of all the potential mutual inclusions and connections among chords, scales, and more in general any generic group of notes arbitrarily played by each musician. These concepts characterize the specific algorithm purposely developed and implemented in an interactive software application tool that can be employed for performing the melodic and harmonic analyses of any kind of tune. Geometric music graphs that take into account these groups are automatically designed by the software in order to trace the harmonic and melodic zones touched by each music composition. These geometric graphs are constituted by the connections (that can also represent mutual inclusions) among the generic groups of notes. The primary focus of the present paper is on the quantitative results and data characterizing the inner structure related to these graphs.*

I. INTRODUCTION

The music network and the related concept can be chosen as the suitable model to describe the mutual relationships and inclusions among generic groups of notes (and related frequencies). Notes, scales and chords provide the melodic and harmonic resources to which each musician is inevitably addressed for the composition process. Each of these elements is characterized by its specific sound. Any generic group of notes (constituted by 1 up to 12 notes) can be considered pertaining to one or more scales and chords that are the melodic and harmonic subgroups in which the set of 12 notes is organized. Studies related to this research field are those of [1, 2, 3, 4, 5, 6, 7]. It is of interest to highlight the studies of Cataldo [8, 9, 10] who developed an innovative transformations method of harmonic progression by employing advanced harmonic substitutions in order to introduce a new formalism related to Music Algebra incorporating concepts derived by [11, 12, 13, 14, 15, 16, 17]. The present study and the related methodological approach fit into this line of research. The present study is the natural completion of the author's previous study [18] in which the method related to the graphical view of the music network and the related nodes has been introduced and extensively described. In some terms, scales and chords can be thought as the result of an iterative process through “trials and errors” aimed to identify the melodic and harmonic

subgroups in which the set of 12 notes is organized. As the defined scales and chords are employed, they instantaneously reduce the number of attempts, minimizing the number of "trials and errors". As explained in detail in the previous study [18], the specific algorithm implemented in the calculation program allows to trace the connections among groups of notes arbitrarily played in real-time (or typed on a pc-keyboard) by the musician and mapping harmonic and melodic movements on 2D music network graphs that can also be considered as 3D music solid network graphs. Each node represents a specific musical scale or a specific chord, more in general any generic group of notes arbitrarily played. The system is not dependent on the instrument played by the musician, representing a synthesized geometric view (divisible into different sub-networks) of all possible interconnections and mutual inclusions among different generic groups of notes and the related frequencies arbitrarily considered by each musician (the shape and extension of the entire music network depend on the number of generic groups of notes considered by the musician, thus on the subjective degree of knowledge of the musician who will enrich his graphic-musical network in the learning process). Each chord is considered as generated by specific degrees of different scales as well as an autonomous entity (a generic group of notes) and completely unrelated to one (or more) specific scale. The generic groups of one up to twelve notes are interconnected by following a parsimonious voice-leading approach as explained in detail in [18]. Each scale is considered as a container of chords since each scale is like a big chord that can be subdivided into its singular components that are the chords. Even if the scales did not exist and only notes and generic groups of notes (i.e. chords or equivalently the related arpeggios of three, four, maximum five notes) were assumed to exist autonomously, it is possible to reasonably arrange them and reconstruct the harmonizations and also the fingerings of many different scales (that share those specific chords and related arpeggios) in any case, thus employing a relatively low number of elements. Chords and scales constituted by the same or a different number of notes can be harmonically and melodically interconnected. To this aim, the *harmonic connection degree number*, the *real harmonic connection degree number* and the *melodic connection degree number*, whose definitions are reported in [18], have been suitably introduced and quantitatively reported in the present paper. The connections among chords and scales are represented by the connections lines whose colors are those proper of the connected scales and chords that in turn are represented in a condensed/synthetic manner through the use of different symbols (markers) and colors as described and shown in [18]. Chords and scales constituted by a different number of notes can also be mutually included (i.e. each chord and each scale can be included in other chords and other scales having a higher number of notes) as reported and shown in the following sections of the present paper. The focus of the present paper is on the following points related to the software in which the method is implemented:

- Different input settings of the software can be set by the user musician in order to choose the environment of scales and chords to be considered and handled by the process. Different examples have been reported in the present paper, included some preliminary analyses related to the input setting scales defined by [7].

- Detailed lists reporting the *melodic connection degree number* related to each scale or chord are provided. The *melodic connection degree number* univocally represents the melodic connection's level between different scales.

- The harmonization starting from each degree of each considered scale or chord is automatically computed by the software and provided in the present paper. The main chords constituted by three and four notes defined and reported in [18] are primarily considered in the present paper.

- Detailed lists reporting the *harmonic connection degree number*, and the *real harmonic connection degree number* related to a specific scale or a specific chord are provided (as examples). As reported in [18], for the determination of the *harmonic connection degree number* the main chords of three and four notes can be employed, while all the *basic chords* are strictly needed to determine the *real harmonic connection degree number*. A *basic chord* is a chord type constituted at least by two and a maximum of three notes. The *basic chords* are not explicitly presented but only used and reported in the text as background information. The *real harmonic connection degree number* univocally represents the real harmonic connection's level between different scales. From these analyses, it is also possible to underline all the chords which do not allow a harmonic connection among the scales chosen by the user. These chords are not in common between scales since they are specific and proper of a specific scale besides of course the *12 notes chromatic scale.*

- Since each scale and each chord can be included by other bigger scales and chords if all the notes of the smaller scale and smaller chord are in common with the bigger ones, the list of the scales and chords included by each scale type and each chord type is provided by the software and reported in the present paper. Furthermore,

the list of the scales and chords in which each scale type and each chord type is included is provided.

- Scales and chords which include a single or a generic group of single notes chosen/played by the user are provided by the software and reported in the lists.

- Scales and chords which include a single or a generic group of chords chosen/played by the user are provided by the software and reported in the lists.

- Chords (and also scales) that are equal but have different names can be suitably employed for defining new perspectives in the music network graphs application, specifically for what concerns the inversion substitutions in case of chords and rotations of the main scale in case of scales.

- New perspectives for what concerns music improvisation can be experimented employing the geometrical-based approach that constitutes the music network graphs. In particular, it can be demonstrated how following the trajectories traced by the parsimonious voice-leading approach to connect generic groups of notes (scales and chords) can provide a human ear-friendly response.

- Further examples of the graphic and time-dependent network constituted by the considered chords and scales which are melodically connected since similar in terms of common notes have been provided by means of some scales related to the *Modes of limited transposition* [6].

II. INTERVAL NOTATION AND GENERAL CONVENTIONS

The definition of the notes and intervals is primarily needed in order to define the different music scale and chord types and the related structures reported in detail in [18]. In particular, it is important to introduce the *12 notes chromatic scale* from which all the other scales and chords are originated and included. This scale is comprehensive of all the 12 notes of the equal temperament system; its structure is based on a series of 12 semitones (all its constituting notes are separated from each other by a minor second interval). Since this scale includes all the 12 notes, it is not characterized by any specific starting note (which is in some cases known also as the tonic of the scale and denoted by number 1, as the first degree of the scale). Each of the 12 notes can be equivalently considered and assumed as the starting note (first degree) of the scale. As a consequence, this scale is not capable of generating any *mode* (except the one denoted by the proper structure of the scale itself) since the intervals structure remains the same regardless of the starting note chosen. It has to be specified that a *mode* is a rotation of the same considered scale obtained by assuming each of their constituting note as the first degree of the scale (the scale is the same, the *modes* related to the scale are obtained by considering all the possible rotations of that scale which assumes a different first-degree note for each generated *mode*). The same concept applies for the chord *inversions* that are characterized by having a constituting chord note, which is different from the fundamental, placed as the bass note. Each chord *inversion* is constituted by the same notes of the original chord despite a different name. Each chord *inversion* of a chord can be of the same family related to the original chord or not. It has to be noted that all the music intervals can be defined according to the chosen starting note of the *12 notes chromatic scale*. For instance, it is possible to choose the *C* note as the starting note (denoted by number 1) of the chromatic scale thus referring all its constitutive intervals to *C* (the chosen starting note could be any note different from *C* while the related interval names remain the same). The structure of the *12 notes chromatic scale* in terms of intervals has been proposed in Table 1. All the intervals are referred to the chosen starting note (1, in this case, is the *C* note):

Table.1: Structure of the 12 notes chromatic scale in terms of interval related to the chosen starting note (1)

Notes names of the *12 notes chromatic scale* (as referred to the *C* starting note):
C=(B#), Db=C#, D, D#=Eb, E=(Fb), F=(E#), F#=Gb, G, G#=Ab, A=(Bbb), Bb=A#, B=(Cb)
Constitutive intervals of the *12 notes chromatic scale* referred to the chosen starting note (1):
1 or 8 or (7#), 2b or (1#), 2, 2# or 3b, 3 or (4b), 4 or (3#), 4# or 5b, 5, 5# or 6b, 6 or (7bb), 7b or (6#), 7 or (8b)

The notes' names of the *12 notes chromatic scale* are reported in the first line of Table 1, while its constitutive intervals are reported in the second line. The "*or*" symbol denotes the possible alternatives that can be used in order to achieve a diesis # and flat *b* parsimonious use. The less-used notes names and related intervals (referred to 1) are reported inside the brackets "*()*". These last names are conventionally existent and used only in some specific cases. The intervals are identified by numbers (the number 1 identifies the starting note, the so-called tonic or first-degree of the scale, or the fundamental of the chord) and their names are reported in Nomenclature. The scale types (and the related *modes*) considered in the present study and the related structures in terms of intervals related to their first degree are those reported in [18] and [7]. Some

properties related to the transposition within an octave of each considered scale are summarized in Table 2.

Table.2: Some properties related to each scale type considered and reported by [18] and [7].

Scale type	***Is the transposition within an octave possible?***	***Number of scales and related first degrees within an octave***	***Number of modes inner to the scale within an octave***
Major scale (ionian mode) ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*7*
Melodic minor scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*7*
Harmonic minor scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*7*
6 notes blues scale ([18])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*6*
Whole-tone scale ([18], [7])	*Yes*	*2; C, C#*	*1*
Half-step/whole step diminished scale ([18], [7])	*Yes*	*1; C*	*2*
Whole step/half-step diminished scale ([18], [7])	*Yes*	*2; C, D*	*2*
Augmented half-step minor-third scale ([18])	*Yes*	*2; C, D*	*2*
Augmented minor-third half-step scale ([18])	*Yes*	*2; C, D*	*2*
Messiaen mode #3 (Rotation 1) ([18], [7])	*Yes*	*4; C, C#, D, Eb*	*3*
12 notes chromatic ([18])	*No*	*1; C*	*1*
Harmonic major scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*7*
Bebop major scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*8*
Harmonic minor scale+6 notes blues scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*9*
Bebop dominant scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*8*
Melodic minor scale+Dorian scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*8*
Melodic minor scale+ Harmonic minor scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*8*
Melodic minor scale+4# ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*8*
Melodic minor scale+Mixolydian scale ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*9*
Minor pentatonic scale+3+6 ([18], [7])	*Yes*	*12; C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B*	*7*

Many other typologies of scales exist. For the sake of brevity, they are not reported in the present study. The scales incorporate, therefore generate different types of chords. The main chords of three and four notes defined in [18] are primarily considered in the present study. The *basic chords* are defined in [18] and reported in the text as background information.

2.1 SCALES HARMONIZATIONS AND MUTUAL INCLUSIONS

The harmonizations of the scales presented in [18] are reported in Table 3 by means of the main chords constituted by three and four notes defined in [18]. The *C* note is chosen as the first degree of each scale in order to have a common reference to distinguish and highlight the differences among the scales in terms of structure. For the sake of brevity, the harmonizations of the scale types specifically considered by [7] have not been reported.

Table.3: Scales harmonizations by means of the main chords of three and four notes defined in [18].

***C Ionian**:C D E F G A B*
Cmaj Cadd9 Cadd4 C6 Cmaj7 Dm Dm/add9 Dm/add4 Dm6 Dm7 Em Em/add9b Em/add4 Em7 F5b Fmaj Fadd9 Fadd4# Fmaj7/5b F6 Fmaj7 Gmaj Gadd9 Gadd4 G6 G7 Am Am/add9 Am/add4 Am7 Bdim Bdim/add4 Bm7/5b
***C Melodic minor**:C D Eb F G A B*
Cm Cm/add9 Cm/add4 Cm6 Cm/maj7 Dm Dm/add9b Dm/add4 Dm6 Dm7 Eb5b Eb5# Ebmaj7/5b Ebmaj7/5# F5b Fmaj Fadd9 Fadd4# F7/5b F6 F7 Gmaj G5# Gadd9 Gadd4 G7 G7/5# Adim Adim/add4 Am7/5b Bdim B5b B5# B5b/add9b Bm7/5b B7/5b B7/5#
***C Harmonic minor**: C D Eb F G Ab B*
Cm Cm/add9 Cm/add4 Cm/maj7 Cm/maj7/5# Ddim Ddim/add4 Ddim7 Dm7/5b Eb5# Ebmaj7/5# Fdim Fm Fm/add9 Fm/add4# Fdim7 Fm7/5b Fm6 Fm7 Gmaj G5# Gadd9b Gadd4 G7 G7/5# Abdim Abm Ab5b Abmaj Abm/add4b Abm/add4# Abdim7 Abm/maj7/5b Abm6 Abm/maj7 Abadd4# Abmaj7/5b Ab6 Abmaj7 Bdim B5b B5# B5b/add9b Bdim7
***C 6 notes blues**: C Eb F F# G Bb*
Cdim Cm Cdim/add4 Cm/add4 Cm/add4# Cm7/5b Cm7 Ebm Ebmaj Ebm/add9 Ebadd9 Ebm/add4b Ebm6 Eb6 F#5b F#5b/add9b F#maj7/5b Gm/maj7/5#
***C Whole-tone**: C D E F# G# Bb*
C5b C5# C7/5b C7/5# ; the same harmonization order applies for the tones related to D E F# G# Bb
***C Half-step/whole step diminished**:*
C Db Eb E Gb G A Bb
Cdim Cm C5b Cmaj Cm/add9b C5b/add9b Cadd9b Cm/add4b Cm/add4# Cdim7 Cm7/5b Cm6 Cm7 Cadd4# C7/5b C6 C7 Dbdim Dbdim/add4 Dbdim7 Dbm/maj7/5b Dbm/maj7/5# ; the same harmonization order applies for the tones related to Eb E Gb G A Bb
***C Whole step/half-step diminished**:* *C D Eb F Gb G# A B*
Cdim Cdim/add4 Cdim7 Cm/maj7/5b Cm/maj7/5# Ddim Dm D5b Dmaj Dm/add9b D5b/add9b Dadd9b Dm/add4b Dm/add4# Ddim7 Dm7/5b Dm6 Dm7 Dadd4# D7/5b D6 D7 ; the same harmonization order applies for the tones related to Eb F Gb G# A B
***C Augmented half-step minor-third**: C Db E F G# A*
C5# Dbm Dbmaj Db5# Dbm/add4b Dbm/maj7 Dbm/maj7/5# Dbmaj7 Dbmaj7/5# ; the same harmonization order applies for the tones related to E F G# A
***C Augmented minor-third half-step**: C Eb E G G# B*
Cm Cmaj C5# Cm/add4b Cm/maj7 Cm/maj7/5# Cmaj7 Cmaj7/5# Eb5# ; the same harmonization order applies for the tones related to E G G# B
***C Messiaen mode #3 (Rotation 1)**:* *C Db D E F F# G# A Bb*
C5b C5# C5b/add9b C5b/add4 C7/5b C7/5# Dbm Dbmaj Db5# Dbm/add9b Dbadd9b Dbm/add4b Dbm/add4 Dbm6 Dbm/maj7 Dbm/maj7/5# Dbadd4 Db6 Dbmaj7 Dbmaj7/5# Ddim Dm D5b Dmaj D5# Dm/add9 Dadd9 Dm/add4b Dm/add4# Dm7/5b Dm/maj7/5b Dm7 Dm/maj7 Dm/maj7/5# Dadd4# D7/5b Dmaj7/5b D7 Dmaj7 D7/5# Dmaj7/5# ; the same harmonization order applies for the tones related to E F F# G# A Bb
***C 12 notes chromatic**: C Db D Eb E F Gb G G# A Bb B*
Cdim Cm C5b Cmaj C5# Cm/add9b C5b/add9b Cadd9b Cm/add9 Cadd9 Cm/add4b Cdim/add4 Cm/add4 Cm/add4# Cdim7 Cm7/5b Cm/maj7/5b Cm6 Cm7 Cm/maj7 Cm/maj7/5# C5b/add4 Cadd4 Cadd4# C7/5b Cmaj7/5b C6 C7 Cmaj7 C7/5# Cmaj7/5# ; the same harmonization order applies for the tones related to Db D Eb E F Gb G G# A Bb B

It has to be specified that the scales and chords considered in the present Section are all included in the *12 notes chromatic scale*. Focusing on the scales, there are no further mutual inclusions among the other scales considered (*12 notes chromatic scale* excluded). The *minor pentatonic* scale is considered as an extra-scale

among those primarily considered in the present study. The structure related to the *minor pentatonic* scale in terms of intervals related to its first degree is *1 3b 4 5 7b*. For instance, the *C minor pentatonic* scale is constituted by the notes: *C Eb F G Bb*. The *C minor pentatonic* scale generates the *Eb6/9* chord on its third degree (its third degree is denoted by the name *Eb major pentatonic* scale or the *first pentatonic mode* that corresponds to the *Eb6/9* chord arpeggio). When the *minor pentatonic (*also known as the *fifth pentatonic mode)* scale is considered, mutual inclusions among the scales considered in the present Section turn out. It has to be specified that in general, among the general properties explained in detail in [18], when mutual inclusions between two different sets A and B (each constituted by a certain number of elements, i.e. A can have the same or a different number of elements of B) turn out, the following relationships are satisfied:

A+B=A if B already included in A (1)

A+B=B if A is already included in B (2)

(A+B)-B=E if A and B share some or have all the elements in common (3)

(A+B)-A=F if A and B share some or have all the elements in common (4)

A-B=D if A and B share some or have all the elements in common (5)

(A-B)-B=D-B if A and B share some or have all the elements in common (6)

(A-B)-A=D-A if A and B share some or have all the elements in common (7)

(A-B)-B=A-B if B already included in A (8)

(B-A)-A=B-A if A already included in B (9)

These mutual inclusions among the scales are reported in Table 4. The *C* note is chosen as the first degree of each scale.

Table.4: Mutual inclusions among the scales considered in the present Section when the minor pentatonic scale is introduced.

Scale type	***Scales included by the scale type***	***Scales in which the scale type is included***
C Ionian	*D minor pentatonic, E minor pentatonic, A minor pentatonic*	*C 12 notes chromatic*
C Melodic minor	*D minor pentatonic*	*C 12 notes chromatic*
C Harmonic minor	*None*	*C 12 notes chromatic*
C 6 notes blues	*C minor pentatonic*	*C 12 notes chromatic*
C Whole-tone	*None*	*C 12 notes chromatic, C Messiaen mode #3 (Rotation 1), D Messiaen mode #3 (Rotation 1)*
C Half-step/whole step diminished	*None*	*C 12 notes chromatic*
C Whole step/half-step diminished	*None*	*C 12 notes chromatic*
C Augmented half-step minor-third	*None*	*C 12 notes chromatic, C Messiaen mode #3 (Rotation 1), Eb Messiaen mode #3 (Rotation 1)*
C Augmented minor-third half-step	*None*	*C 12 notes chromatic, D Messiaen mode #3 (Rotation 1), Eb Messiaen mode #3 (Rotation 1)*
C Messiaen mode #3 (Rotation 1)	*C Whole-tone, C Augmented half-step minor-third, D Augmented minor-third half-step*	*C 12 notes chromatic*
C minor pentatonic	*None*	*C 12 notes chromatic, C 6 notes blues, Bb Melodic minor, Eb Ionian, Ab Ionian, Bb Ionian*
C 12 notes chromatic	*All*	*None*

The same investigation about the mutual inclusions among different sets of notes has been made for all the main chords constituted by three and four notes considered in the present study. This investigation is reported in Table 5. Please notice how the investigation reported in Table 4 does not involve the chords, as well as the one in Table 5 does not involve scales despite chords and scales can be considered as generic groups of notes, thus legitimately directly compared and, if similar [18], interconnected (also mutually included very often). The *C* note is chosen as the fundamental of each chord.

Table.5: Mutual inclusions among the main chords of three and four notes considered in the present study.

Chord type	***Chords included by the chord type***	***Chords in which the chord type is included***
Cdim	*None*	*Badd9b, Cdimadd4, Cmadd4#, Cdim7, Cm7/5b, Ab7*
Cm	*None*	*Cmaddb9, Cmadd9, Cmadd4b, Cmadd4, Cmadd4#, Am7/5b, Cm7, Cm/maj7, Abmaj7*
C5b	*None*	*C5badd9b, F#m7/5b, C5badd4, Cadd4#, C7/5b, Cmaj7/5b, Ab7/5#*
Cmaj	*None*	*Cadd9b, Cadd9, Cmadd4b, Am7, Cadd4, Cadd4#, C7, Cmaj7, Abmaj7/5#*
C5#=E5#=G#5#	*None*	*C#m/maj7, Fm/maj7, Am/maj7, C7/5#, E7/5#, G#7/5#, Cmaj7/5#, Emaj7/5#, G#maj7/5#*
Cm/add9b	*Cm*	*None*
C5b/add9b	*C5b*	*None*
Cm/add9	*Cm*	*None*
Cadd9	*Cmaj*	*None*
Cdim/add4	*Cdim*	*None*
Cm/add4	*Cm*	*None*
Cm/add4#	*Cdim, Cm*	*None*
Cdim7=Ebdim7= =Gbdim7= Adim7	*Cdim, Ebdim, F#dim, Adim*	*None*
Cm7/5b = Ebm6	*Cdim, Ebm, F#5b*	*None*
Cm/maj7/5b=Badd9b	*Cdim, Bmaj*	*None*
Cm7 = Eb6	*Cm, Ebmaj*	*None*
Cm/maj7	*Cm, Eb5#*	*None*
Cm/maj7/5#=Abm/add4b	*Abm, Abmaj*	*None*
C5b/add4	*C5b*	*None*
Cadd4	*Cmaj*	*None*
Cadd4#	*C5b, Cmaj*	*None*
C7/5b = Gb7/5b	*C5b, Gb5b*	*None*
Cmaj7/5b	*C5b*	*None*
C7	*Edim, Cmaj*	*None*
Cmaj7	*Em, Cmaj*	*None*
C7/5#	*E5b, C5#*	*None*
Cmaj7/5#	*Emaj, C5#*	*None*

It has to be specified that the *minor pentatonic* scale is considered in some of the following investigations (like the one reported in Table 6). In Table 6 some examples of scales that can be employed over chords are reported according to the property that has been found out in [18], which is reinforced here:

- scales (more in general generic groups of notes) that do not generate/include the target chord can, in general, be employed over the target chord together with some of the scales in which they are included into only if the containing (bigger) scales generate/include the target chord.

Table.6: Some examples of scales that can be employed over chords.

Target chord	***Scales that can be employed over the chord given***
F6/9/add4#	*C 12 notes chromatic **then all**; C Ionian, C Melodic minor **then** D minor pentatonic, E minor pentatonic, A minor pentatonic; C Harmonic minor*
Dm7/6/add4	*C 12 notes chromatic **then all**; C Ionian, C Melodic minor **then** D minor pentatonic, E minor pentatonic, A minor pentatonic*
G7	*C 12 notes chromatic **then all**; C Ionian, C Melodic minor, D Melodic minor **then** D minor pentatonic; E minor pentatonic; A minor pentatonic; C Harmonic minor; D Whole step/half-step diminished; C# Messiaen mode #3 (Rotation 1) **then** C# Whole-tone, D Augmented half-step minor-third, D Augmented minor-third half-step*
Eb6/9	*C 12 notes chromatic **then all**; Eb Ionian, Ab Ionian, Bb Ionian, Bb Melodic minor, C 6 notes blues **then** C minor pentatonic, D minor pentatonic, F minor pentatonic, G minor pentatonic, Bb minor pentatonic*
Abmaj7	*C 12 notes chromatic **then all**; Eb Ionian, Ab Ionian **then** C minor pentatonic, F minor pentatonic, G minor pentatonic, Bb minor pentatonic; C Harmonic minor; C Augmented minor-third half-step; D Messiaen mode #3 (Rotation 1) **then** C Whole-tone, D Augmented half-step minor-third; Eb Messiaen mode #3 (Rotation 1) **then** C# Whole-tone, C Augmented half-step minor-third*
Bm7/5b	*C 12 notes chromatic **then all**; C Ionian, C Melodic minor, D Melodic minor, B 6 notes blues **then** B minor pentatonic, D minor pentatonic, E minor pentatonic, A minor pentatonic; A Harmonic minor; F# Harmonic minor; C Whole step/half-step diminished; C# Messiaen mode #3 (Rotation 1) **then** C# Whole-tone, D Augmented half-step minor-third, D Augmented minor-third half-step*
Dm7	*C 12 notes chromatic **then all**; C Ionian, F Ionian, Bb Ionian, C Melodic minor, D 6 notes blues **then** C minor pentatonic, D minor pentatonic, E minor pentatonic, G minor pentatonic, A minor pentatonic; A Harmonic minor; C Whole step/half-step diminished; C Messiaen mode #3 (Rotation 1) **then** C Whole-tone, C Augmented half-step minor-third, D Augmented minor-third half-step*
Dm	*C 12 notes chromatic **then all**; C Ionian, F Ionian, Bb Ionian, C Melodic minor, D Melodic minor, D 6 notes blues, B 6 notes blues **then** C minor pentatonic, D minor pentatonic, E minor pentatonic, G minor pentatonic, A minor pentatonic, B minor pentatonic; D Harmonic minor; F# Harmonic minor; A Harmonic minor; C Whole step/half-step diminished; C Messiaen mode #3 (Rotation 1) **then** C Whole-tone, C Augmented half-step minor-third, D Augmented minor-third half-step; C# Messiaen mode #3 (Rotation 1) **then** C# Whole-tone, D Augmented half-step minor-third, D Augmented minor-third half-step; D Augmented minor-third half-step*
Cm/maj7	*C 12 notes chromatic **then all**; C Melodic minor **then** D minor pentatonic; C Harmonic minor; E Harmonic minor; D Messiaen mode #3 (Rotation 1) **then** C Whole-tone, D Augmented half-step minor-third, C Augmented minor-third half-step; Eb Messiaen mode #3 (Rotation 1) **then** C# Whole-tone, C Augmented half-step minor-third, C Augmented minor-third half-step; C Augmented minor-third half-step*

Some of the chord types shown in Table 6 not necessarily are the main chords of three and four notes primarily considered in the present study (some of them can be considered as the superposition of two or more main chords of three and four notes). From Table 6 inspection, it is straightforward to notice and underline the following general property, as a consequence of the previously highlighted property (already introduced in [18] and applied in Table 6):

- A scale (or chord arpeggio) can be employed over the chord given (target chord) if directly generates the chord given or if that scale (or chord arpeggio) is included by a bigger scale (one or more) that generates the chord given. In this last case the scale (or chord arpeggio, one or more), being included by a bigger scale (one or more), not necessarily has to directly generate the chord given (the target chord might be generated by the bigger containing scale and not by the included one). Therefore, in this last case, the included scale (or chord arpeggio, one or more), not directly generating the target chord, has a weaker harmonic and melodic connection with the chord given than in the case in which a scale (or chord arpeggio, one or more), even if included by another one (one or more), directly generates the chord given thus also melody patterns more harmonically and melodically connected to the chord given.

It is interesting to notice how the bolded "***then***" in Table 6 emphasizes particular cases related to the application of this property, according to what is defined in [18] and here reinforced. According to this property, it is of interest how "***then***" highlights the particular usage ("indirect usage") of the scales included by the *Messiaen mode #3 (Rotation 1)* over chords not properly generated by these scales as well as the usage ("indirect usage") of the *B minor pentatonic* scale over the *Bm7/5b* and *Dm* chords since both directly generated by the *B 6 notes blues* scale that includes the *B minor pentatonic* scale. The usage of "***then all***" is symptomatic of how the *12 notes chromatic scale* can generate any chord and scale type, i.e. all the potential melody patterns that can be sustained by all the chord types which are all generated and included in the *12 notes chromatic scale.* It has to be specified that the meaning related to the term "indirect usage" expressed in this study is different and more general than the one usually found in literature. Conventionally, the "indirect usage" term refers to the use of *modes* of the same generating scale (rotations of the same scale) over chords (or melody patterns, as will be explained by the following Table 7) not properly related to these *modes* since generated by other degrees of the same main scale (for instance the use of the *D Melodic minor*, *C Melodic minor*, *C Ionian* scales over the *G7* chord generated by its fourth and fifth degrees respectively, or the use of the *Eb Ionian, C Harmonic minor* scales over the *Abmaj7* chord generated by its fourth and sixth degrees respectively, as already reported in Table 6). As it is evident, the term "indirect usage" expressed in this study (and highlighted by "***then***" and "***then all***" in Tables 6 and 7) denotes a more general meaning that includes the conventional one, representing its extremization.

As seen, besides the mutual inclusions among generic groups of notes (scales and chords) the method allows finding out all the different scales (among those considered by the system since chosen by the user-musician) that can be potentially employed over the same chord type and, more in general, on the same chord progression (see the examples reported in Section 5 of the present paper).

The method allows also detecting all the scales (among those considered by the system and chosen by the user-musician) that can generate a given melody pattern potentially sustained by different chords generated by the same scales (that can be characterized by mutual inclusions) as shown by the examples reported in Table 7.

Table.7: Examples of chords generated by different potential scales at the same time that can sustain the same melody pattern.

Melody pattern	***Scales that include the given melody pattern***	***Chords that potentially can sustain the given melody pattern***
C, Eb, D	*C 12 notes chromatic* ***then all****; Eb Ionian, Bb Ionian, C Melodic minor, Eb Melodic minor, A 6 notes blues* ***then*** *C minor pentatonic, D minor pentatonic, F minor pentatonic, G minor pentatonic, A minor pentatonic; C Harmonic minor; G Harmonic minor; C Whole step/half-step diminished; D Messiaen mode #3 (Rotation 1)* ***then*** *C Whole-tone, D Augmented half-step minor-third, C Augmented minor-third half-step*	*All the chords included/generated by the scales that include the given melody pattern*

C, E, D	*C 12 notes chromatic **then all**; C Ionian, F Ionian, G Ionian, F Melodic minor, G Melodic minor, A Melodic minor, A 6 notes blues **then** B minor pentatonic, D minor pentatonic, E minor pentatonic, G minor pentatonic, A minor pentatonic; A Harmonic minor; C Whole-tone; C Messiaen mode #3 (Rotation 1) **then** C Augmented half-step minor-third, D Augmented minor-third half-step; D Messiaen mode #3 (Rotation 1) **then** D Augmented half-step minor-third, C Augmented minor-third half-step*	*All the chords included/generated by the scales that include the given melody pattern*

From Table 7 inspection, it is straightforward to notice and underline the following general property, specularly to what asserted so far:

- A chord (or a group of chords) can sustain a given melody pattern if the chord includes the given melody pattern or if the chord is included in a scale that directly generates the given melody pattern, as well as, a chord (or a group of chords) can sustain a given melody pattern if the chord is included by a bigger chord (one or more) or a bigger scale (one or more) that generates the given melody pattern (besides including also other chords and scales that not necessarily generate the given melody pattern). In this last case the chord (one or more), being included by a bigger chord or a bigger scale (one or more), not necessarily has a direct link with the given melody pattern (the given melody pattern might be generated by the bigger containing chord or scale and not by the included one). Therefore, in this last case, the included chord (one or more), not directly generating the given melody pattern, has a weaker harmonic and melodic connection with the given melody pattern than in the case in which a chord (one or more) or a scale (one or more), even if included by a bigger chord (one or more) or a bigger scale (one or more), directly generates sustaining chords more harmonically and melodically connected to the given melody pattern.

It is interesting to notice that the bolded "***then***" in Table 7 emphasizes particular cases related to the application of this property. According to this property, it is of interest how "***then***" highlights the particular usage ("indirect usage") of scales generating chords that can sustain the melody pattern even if the melody pattern is not properly generated by these scales and chords. This is because these scales and chords are included in the bigger containing scales and chords that generate the given melody pattern (besides including also other chords and scales that do not necessarily generate the given melody pattern). Once again, the usage of "***then all***" demonstrates how the *12 notes chromatic scale* can generate any chord type as well as all the potential melody patterns that can be sustained by all the chord types which are all generated and included in the *12 notes chromatic scale.* The two last properties reported in the present Section can be unified in a general form by considering, as it is, a target chord like a given melody pattern as well as the scales that can be employed over the target chord like the chords that potentially can sustain the given melody pattern.

For the sake of completeness, the chord harmonization (starting from each "degree" of the chord) related to the main chords constituted by three and four notes is reported in Table 8. The fundamental (the first degree of each chord) that has been chosen to present each chord is the *C* note, as a common reference.

Table.8: Chords harmonizations related to the main chords of three and four notes defined in [18].

C7:*C E G Bb*
Cmaj C7 Edim
Cm*:C Eb G*
Cm
C5b*:C E Gb*
C5b
Cmaj*:C E G*
Cmaj
C5#*: C E G#*
C5# E5# G#5#
Cadd9b*:C Db E G*
Cmaj Cadd9b Dbdim Dbm/maj7/5b
Cadd9*:C Db E G*
Cmaj Cadd9
Cmadd4b*:C Eb E G*
Cm Cmaj Cmadd4b Em/maj7/5#
Cm7*:C Eb G Bb*

Cm Cm7 Ebmaj Eb6
***Cadd4**:C E F G*
Cmaj Cadd4
***Cadd4#**:C E F# G*
C5b Cmaj Cadd4#
***C7/5b**:C E Gb Bb*
C5b C7/5b Gb5b Gb7/5b
***C6**:C E G A*
Cmaj C6 Am Am7
***Cmaj7**:C E G B*
Cmaj Cmaj7 Em
***C7/5#**: C E G# Bb*
C5# C7/5# E5b E5# G#5#
***Cdim**:C Eb Gb*
Cdim
***Cm/add9b**:C Db Eb G*
Cm Cmaddb9
***C5b/add9b**:C Db E Gb*
C5b C5badd9b
***Cm/add9**:C D Eb G*
Cm Cmadd9
***Cdim/add4**:C Eb F Gb*
Cdim Cdimadd4
***Cm/add4**:C Eb F G*
Cm Cmadd4
***Cm/add4#**:C Eb F# G*
Cdim Cm Cmadd4#
***Cdim7**:C Eb Gb A*
Cdim Cdim7 Ebdim Ebdim7 Gbdim Gbdim7 Adim Adim7
***Cm7/5b**:C Eb Gb Bb*
Cdim Cm7/5b Ebm Ebm6 Gb5b
***Cm/maj7/5b**:C Eb Gb B*
Cdim Cm/maj7/5b Bmaj Badd9b
***Cm6**:C Eb G A*
Cm Cm6 Eb5b Adim Am7/5b
***Cm/maj7**:C Eb G B*
Cm Cm/maj7 Eb5# G5# B5#
***Cm/maj7/5#**: C Eb G# B*
Cm/maj7/5# G#m G#maj G#madd4b
***C5b/add4**:C E F Gb*
C5b C5badd4
***Cmaj7/5b**:C E Gb B*
C5b Cmaj7/5b
***Cmaj7/5#**: C E G# B*
C5# Cmaj7/5# Emaj E5# G#5#

III. THE REAL HARMONIC CONNECTION DEGREE NUMBER

In the present Section, the *harmonic connection degree number* and the *real harmonic connection degree number* between generic groups of notes introduced in [18] are presented in detail. It has to remember that the *harmonic connection degree number* between generic groups of notes expresses the number of common chords (also known as pivot chords) between scales and/or chords (generic groups of notes). The *real harmonic connection degree number* can be obtained by taking into account all the existent *basic chords* constituted at least by two and a maximum of three notes. The *basic chords* are not explicitly presented but only used and reported in the text as background information (for the sake of brevity). Each *basic chord* contains the minimum group of notes needed to derive all the other chords and in general groups of notes by combining (superposing) different *basic chords* (all the generic groups of notes constituted by four or more notes can be considered as the superposition of two or more *basic chords*). The *real harmonic connection degree number* expresses the number of common *basic chords* between scales and/or chords (generic groups of notes), thus univocally representing the real harmonic connection's level between different generic groups of notes.

The *harmonic connection degree numbers* related to the *C major (ionian)* scale are reported in Table 9 by considering the main chords constituted by three and four notes as pivot chords among the scales types presented in the previous Section. It has to be stressed that the general algorithm implemented in the software tool can manage whatever type of scale having any tonic. For the sake of brevity, only the *harmonic connection degree numbers* related to the *C major (ionian)* scale are reported in Table 9.

Table.9: Harmonic connection degree numbers related to the C major (ionian) scale.

Harmonically compared scales (generic groups of notes)	***Harmonic connection degree number***	***Common chords between the compared scales (pivot chords)***
C Ionian - C 12 notes chromatic	33	*Am Am7 Am/add4 Am/add9 Bdim Bdim/add4 Bm7/5b C6 Cadd4 Cadd9 Cmaj Cmaj7 Dm Dm6 Dm7 Dm/add4 Dm/add9 Em Em7 Em/add4 Em/add9b F5b F6 Fadd4# Fadd9 Fmaj Fmaj7 Fmaj7/5b G6 G7 Gadd4 Gadd9 Gmaj*
C Ionian - D Melodic minor	17	*Bdim Bdim/add4 Bm7/5b Dm Dm6 Dm/add4 Dm/add9 Em Em7 Em/add4 Em/add9b F5b Fmaj7/5b G6 G7 Gadd9 Gmaj*
C Ionian - Eb Messiaen mode #3 (Rotation 1)	16	*Am Am7 Am/add9 C6 Cadd4 Cmaj Cmaj7 Em Em/add4 Em/add9b F5b Fadd4# Fadd9 Fmaj Fmaj7 Fmaj7/5b*
C Ionian - A Harmonic minor	16	*Am Am/add4 Am/add9 Bdim Bdim/add4 Bm7/5b Dm Dm6 Dm7 Dm/add9 F5b F6 Fadd4# Fmaj Fmaj7 Fmaj7/5b*
C Ionian - G Ionian	15	*Am Am7 Am/add4 Am/add9 C6 Cadd9 Cmaj Cmaj7 Em Em7 Em/add4 G6 Gadd4 Gadd9 Gmaj*
C Ionian - F Ionian	15	*Am Am7 Am/add4 C6 Cadd4 Cadd9 Cmaj Dm Dm7 Dm/add4 Dm/add9 F6 Fadd9 Fmaj Fmaj7*
C Ionian - C Melodic minor	15	*Bdim Bm7/5b Dm Dm6 Dm7 Dm/add4 F5b F6 Fadd4# Fadd9 Fmaj G7 Gadd4 Gadd9 Gmaj*
C Ionian - C Whole step/half-step diminished	9	*Bdim Bm7/5b Dm Dm6 Dm7 F5b F6 Fadd4# Fmaj*
C Ionian - C# Messiaen mode #3 (Rotation 1)	9	*Bdim Bm7/5b Dm Dm6 Dm/add4 F5b G7 Gadd9 Gmaj*
C Ionian - E Harmonic minor	8	*Am Am7 Am/add9 C6 Cmaj Cmaj7 Em Em/add4*
C Ionian - D Messiaen mode #3 (Rotation 1)	8	*Cadd9 Cmaj Cmaj7 Em Em7 G6 Gadd4 Gmaj*
C Ionian - D Whole step/half-step diminished	8	*Bdim Bdim/add4 Em Em7 Em/add9b G6 G7 Gmaj*
C Ionian - C Messiaen mode #3 (Rotation 1)	8	*Am Am/add4 Dm Dm7 Dm/add9 F6 Fmaj Fmaj7*
C Ionian - B 6 notes blues	8	*Bdim Bdim/add4 Bm7/5b Dm Dm6 Dm/add9 F5b Fmaj7/5b*
C Ionian - G Melodic minor	6	*Am Am7 Am/add4 C6 Cadd9 Cmaj*
C Ionian - E 6 notes blues	6	*Em Em7 Em/add4 G6 Gadd9 Gmaj*
C Ionian - D Ionian	6	*Em Em7 Em/add4 G6 Gadd9 Gmaj*
C Ionian - D 6 notes blues	6	*Dm Dm7 Dm/add4 F6 Fadd9 Fmaj*
C Ionian - A 6 notes blues	6	*Am Am7 Am/add4 C6 Cadd9 Cmaj*
C Ionian - Bb Ionian	6	*Dm Dm7 Dm/add4 F6 Fadd9 Fmaj*
C Ionian - F# Harmonic minor	5	*Bdim Bm7/5b Dm Dm6 F5b*
C Ionian - C Harmonic minor	4	*Bdim G7 Gadd4 Gmaj*
C Ionian - C Half-step/whole step diminished	4	*Am Am7 C6 Cmaj*
C Ionian - B Harmonic minor	4	*Em Em7 G6 Gmaj*

C Ionian - F Melodic minor	3	*Cadd4 Cadd9 Cmaj*
C Ionian - D Harmonic minor	3	*Dm Dm/add4 Dm/add9*
C Ionian - C Augmented minor-third half-step	3	*Cmaj Cmaj7 Em*
C Ionian - C Augmented half-step minor-third	3	*Am Fmaj Fmaj7*
C Ionian - A Melodic minor	3	*Am Am/add4 Am/add9*
C Ionian - F Harmonic minor	2	*Cadd4 Cmaj*
C Ionian - F# 6 notes blues	2	*Am Am/add9*
C Ionian - E Melodic minor	2	*Em Em/add4*
C Ionian - Bb Melodic minor	2	*Fadd9 Fmaj*
C Ionian - Ab Harmonic minor	1	*Em*
C Ionian - F# Melodic minor	1	*F5b*
C Ionian - D Augmented minor-third half-step	1	*Dm*
C Ionian - D Augmented half-step minor-third	1	*Gmaj*
C Ionian - Eb Harmonic minor	1	*Bdim*
C Ionian - C# Harmonic minor	1	*Am*
C Ionian - C# Whole-tone	1	*F5b*
C Ionian - C# 6 notes blues	1	*Em*
C Ionian - Bb Harmonic minor	1	*Fmaj*

It has to be noted that the *harmonic connection degree number* expresses the harmonic similarity level among generic groups of notes. The harmonically compared scales are classified from the highest to the lowest *harmonic connection degree number*. The *C Ionian* scale (as well as any other considered scale, except for the *12 notes chromatic* scale that is harmonically linked with all the groups of notes) has a harmonic link with some (not all) of the other scales when the above main chords of three and four notes are considered. It has to be noted that among the scale types considered in the present Section and chord types constituted by three and four notes, the *5b/add4* chord type is specific and proper of the *Messiaen mode #3 (Rotation 1)* scale besides of course the *12 notes chromatic scale*. The only harmonic connection allowed by the *5b/add4* chord type is between the *Messiaen mode #3 (Rotation 1)* and the *12 notes chromatic scale* since these are the only scales that directly generate the *5b/add4* chord type. Among all the main chords constituted by three and four notes, the *harmonic connection degree numbers* related to the *C major* chord have been reported in Table 10 as an example, even if the general algorithm implemented in the software tool can handle whatever chord type having any fundamental.

Table.10: Harmonic connection degree numbers related to the C major chord.

Harmonically compared chords (generic groups of notes)	***Harmonic connection degree number***	***Common chords (pivot chords) between the compared chords***
Cmaj - Abmaj7/5#	1	*Cmaj*
Cmaj - Em/maj7/5#	1	*Cmaj*
Cmaj - Cmaj7	1	*Cmaj*
Cmaj - Cmadd4b	1	*Cmaj*
Cmaj - Cadd9b	1	*Cmaj*
Cmaj - Cadd9	1	*Cmaj*
Cmaj - Cadd4#	1	*Cmaj*
Cmaj - Cadd4	1	*Cmaj*

Cmaj - C7	1	*Cmaj*
Cmaj – C6	1	*Cmaj*
Cmaj - C#m/maj7/5b	1	*Cmaj*
Cmaj - Am7	1	*Cmaj*

Similar to what was noticed in Section 2.1, the investigation reported in Table 9 involves the harmonic connections between scales, as well as the one in Table 10 involves the harmonic connections between chords, despite chords and scales can be considered as generic groups of notes, thus legitimately directly compared. From Table 10 inspection it is evident how the *Cmaj* chord is the pivot chord that determines a harmonic link among the *Cmaj* chord family and 12 of the 31 different chord families constituted by three and four notes presented in [18] and primarily considered in the present study. The *real harmonic connection degree numbers* are reported in Table 11 (for the sake of brevity only those related to the *C major/ionian* scale).

Table.11: Real harmonic connection degree numbers related to the C major (ionian) scale.

Harmonically compared scales	***Real harmonic connection degree number***
C Ionian - C 12 notes chromatic	147
C Ionian - G Ionian	90
C Ionian - F Ionian	90
C Ionian - D Melodic minor	90
C Ionian - Eb Messiaen mode #3 (Rotation 1)	90
C Ionian - C Melodic minor	90
C Ionian - A Harmonic minor	90
C Ionian - G Melodic minor	50
C Ionian - F Melodic minor	50
C Ionian - E Harmonic minor	50
C Ionian - E 6 notes blues	50
C Ionian - D Messiaen mode #3 (Rotation 1)	50
C Ionian - D Harmonic minor	50
C Ionian - D Ionian	50
C Ionian - D Whole step/half-step diminished	50
C Ionian - D 6 notes blues	50
C Ionian - C Messiaen mode #3 (Rotation 1)	50
C Ionian - C Harmonic minor	50
C Ionian - C Whole step/half-step diminished	50
C Ionian - C# Messiaen mode #3 (Rotation 1)	50
C Ionian - B 6 notes blues	50
C Ionian - A Melodic minor	50
C Ionian - A 6 notes blues	50
C Ionian - Bb Ionian	50
C Ionian - G Harmonic minor	24
C Ionian - G 6 notes blues	24
C Ionian - F Harmonic minor	24
C Ionian - F# Harmonic minor	24
C Ionian - F# 6 notes blues	24
C Ionian - E Melodic minor	24
C Ionian – Eb Ionian	24
C Ionian - C Half-step/whole step diminished	24
C Ionian - C Augmented minor-third half-step	24
C Ionian - C Augmented half-step minor-third	24
C Ionian - C# Whole-tone	24
C Ionian - B Harmonic minor	24
C Ionian - A Ionian	24
C Ionian - Bb Melodic minor	24

C Ionian - Ab Melodic minor	9
C Ionian - Ab Harmonic minor	9
C Ionian - Ab Ionian	9
C Ionian - F 6 notes blues	9
C Ionian - F# Melodic minor	9
C Ionian - E Ionian	9
C Ionian - D Augmented minor-third half-step	9
C Ionian - D Augmented half-step minor-third	9
C Ionian - Eb Melodic minor	9
C Ionian - Eb Harmonic minor	9
C Ionian - C Whole-tone	9
C Ionian - C 6 notes blues	9
C Ionian - C# Harmonic minor	9
C Ionian - C# 6 notes blues	9
C Ionian - B Melodic minor	9
C Ionian - Bb Harmonic minor	9
C Ionian - Ab 6 notes blues	2
C Ionian – F# Ionian	2
C Ionian - C# Melodic minor	2
C Ionian - C# Ionian	2
C Ionian - B Ionian	2
C Ionian - Bb 6 notes blues	2

Comparing Tables 11 and 9 it is possible to notice that the *real harmonic connection degree number* and the *harmonic connection degree number* do not coincide. These can coincide if the *basic chords* are exclusively taken into account in the harmonic comparison investigation. Only the *real harmonic connection degree number* is needed to quantitatively and synthetically represent the real harmonic connection's level between different scales. As already introduced in [18], the classification related to the compared scales from the highest to the lowest *real harmonic connection degree number* reported in Table 11 would coincide with the one denoted by the *harmonic connection degree number* reported in Table 9 only if all the *basic chords* were taken into account and included in both the harmonic comparison investigation analyses. Since all the existent *basic chords* have been considered in the analysis reported in Table 11, the *C Ionian* scale presents a harmonic link with all the other scales considered in this Section except with the *Eb 6 notes blues* scale that (among the scales considered in this Section) is the harmonically (and firstly melodically) farthest scale from the *C Ionian* scale since these scales have only one common note (the *A* note from the comparison at the same *k* number of notes). Furthermore, all the *basic chords* are favorably employed allowing a harmonic connection among the scales considered in this Section. Then, there is no *basic chord* univocally proper of a specific scale besides of course the *12 notes chromatic scale*. The *real harmonic connection degree numbers* among the scales primarily considered in the present study demonstrate that the *12 notes chromatic, Harmonic minor, Melodic minor, Messiaen mode #3 (Rotation 1), Whole step/half-step diminished, Half-step/whole step diminished* scale families have a harmonic link with all the other scales through the *basic chords.* The same does not happen for the *Ionian, 6 notes blues, Whole-tone, Augmented minor-third half-step, Augmented half-step minor-third* scale families since each *Ionian* scale is not harmonically linked with the *6 blues scale* built on a minor third ahead as well as the *C Whole-tone* scale is not harmonically linked with the *C# Whole-tone* scale as well as the *C Augmented minor-third half-step* scale is not harmonically linked with the *D Augmented minor-third half-step* scale as well as the *C Augmented half-step minor-third* scale is not harmonically linked with the *D Augmented half-step minor-third* scale. The scales that originally have a low number of notes tend to have a low *real harmonic connection degree number* by their nature, despite their high melodic connection level with the bigger scales to which they relate to. These scales that are constituted by a low number of notes are often included and similar to the bigger ones. This concept leads to the *melodic connection degree number*.

IV. THE MELODIC CONNECTION DEGREE NUMBER

In the present Section, the *melodic connection degree number* related to each scale type (among those primarily considered in the present study) is presented in detail. The *melodic connection degree number* has been introduced in

[18] and expresses the number of scales and/or chords (generic groups of notes) that are similar to each scale or chord (generic group of notes). It has to be stressed that chords (being little scales that can contain other chords as well as being generated by bigger scales) can be harmonically and melodically compared and also directly compared with scales. In general, the present method allows to melodically compare all the chords and scales, thus detecting the *melodic connection degree number* related to each chord or scale. The *melodic connection degree number* represents the melodic level of connection among scales and/or chords in absolute terms. A scale or chord characterized by a high *melodic connection degree number* has a high corresponding number of connected scales [18] and/or chords. Being the chords generated/included into the scales, chords are unavoidably similar thus connected to the scales from which are generated. All the scales and chords considered in the present study are included (thus similar and connected) in the *12 notes chromatic scale*. Tables 4 and 5 reported in the present study show the mutual inclusions (thus melodic connections) among the scales and chords primarily considered in the present study. As described in [18], the algorithm of the present method compares the scales at the same *k* number of notes (if two compared scales are originally characterized by a different *k*, the additional uncommon notes between the two scales have not to be taken into consideration in the comparison process). To be declared similar, two scales of *k* notes must have at least (*k-1*) common notes. It has to be specified that even if the *modes* of a scale and chord *inversions* would satisfy the already stated mathematical similarity condition, these are not considered in the melodically-based similarity comparison. This is because the *modes* of a scale and chord *inversions* result as rotations of the same considered scale and chord respectively assuming only a different first-degree note (*modes* of a scale and *inversions* of a chord are constituted by the same notes of the original scale and chord respectively despite having only a different name). Taking the concept to extremes, it is possible to highlight that, according to the present parsimonious model algorithm, every single note is similar thus formally connected to the other ones, as reported in Fig.1.

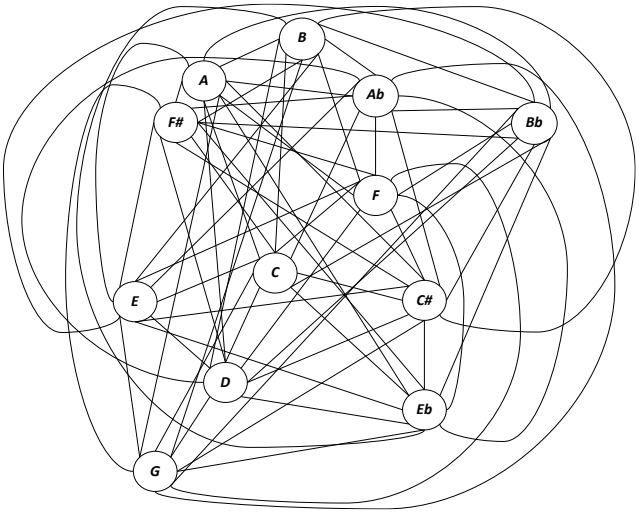

Fig. 1: Melodic connection degree numbers among all the single notes constituting the music network.

It has to be stressed that the general algorithm implemented in the software tool can manage whatever type of scale having any tonic. For the sake of brevity, only the *melodic connection degree numbers* related to each scale type primarily considered in the present study having the *C* note as the first degree have been reported in Table 12.

Table.12: Melodic connection degree numbers related to each scale type having the C note as the first-degree of the scale.

Scale	***Melodic connection degree number***	***Connected/similar scales***
C Ionian	11	*F Ionian* *G Ionian* *C Melodic minor* *D Melodic minor* *A Harmonic minor* *D 6 notes blues* *E 6 notes blues* *A 6 notes blues* *B 6 notes blues* *Eb Messiaen mode #3 (Rotation 1)*

		C 12 notes chromatic
C Melodic minor	10	*C Ionian* *Bb Ionian* *C Harmonic minor* *D 6 notes blues* *A 6 notes blues* *C# Whole-tone* *C Whole step/half-step diminished* *C# Messiaen mode #3 (Rotation 1)* *Eb Messiaen mode #3 (Rotation 1)* *C 12 notes chromatic*
C Harmonic minor	9	*Eb Ionian* *C Melodic minor* *D 6 notes blues* *F 6 notes blues* *C Whole step/half-step diminished* *C Augmented minor-third half-step* *D Messiaen mode #3 (Rotation 1)* *Eb Messiaen mode #3 (Rotation 1)* *C 12 notes chromatic*
C 6 notes blues	12	*C# Ionian* *Eb Ionian* *Ab Ionian* *Bb Ionian* *Eb Melodic minor* *Bb Melodic minor* *G Harmonic minor* *Bb Harmonic minor* *C Half-step/whole step diminished* *C# Messiaen mode #3 (Rotation 1)* *D Messiaen mode #3 (Rotation 1)* *C 12 notes chromatic*
C Whole-tone	9	*C# Melodic minor* *Eb Melodic minor* *F Melodic minor* *G Melodic minor* *A Melodic minor* *B Melodic minor* *C Messiaen mode #3 (Rotation 1)* *D Messiaen mode #3 (Rotation 1)*

		C 12 notes chromatic
C Half-step/whole step diminished	13	*C# Melodic minor* *E Melodic minor* *G Melodic minor* *Bb Melodic minor* *C# Harmonic minor* *E Harmonic minor* *G Harmonic minor* *Bb Harmonic minor* *C 6 notes blues* *Eb 6 notes blues* *F# 6 notes blues* *A 6 notes blues* *C 12 notes chromatic*
C Whole step/half-step diminished	13	*C Melodic minor* *Eb Melodic minor* *F# Melodic minor* *A Melodic minor* *C Harmonic minor* *Eb Harmonic minor* *F# Harmonic minor* *A Harmonic minor* *D 6 notes blues* *F 6 notes blues* *Ab 6 notes blues* *B 6 notes blues* *C 12 notes chromatic*
C Augmented half-step minor-third	6	*C# Harmonic minor* *F Harmonic minor* *A Harmonic minor* *C Messiaen mode #3 (Rotation 1)* *Eb Messiaen mode #3 (Rotation 1)* *C 12 notes chromatic*
C Augmented minor-third half-step	6	*C Harmonic minor* *E Harmonic minor* *Ab Harmonic minor* *D Messiaen mode #3 (Rotation 1)* *Eb Messiaen mode #3 (Rotation 1)* *C 12 notes chromatic*
C Messiaen	25	*C# Ionian*

mode #3 (Rotation 1)		*F Ionian* *A Ionian* *C# Melodic minor* *Eb Melodic minor* *F Melodic minor* *G Melodic minor* *A Melodic minor* *B Melodic minor* *C# Harmonic minor* *D Harmonic minor* *F Harmonic minor* *F# Harmonic minor* *A Harmonic minor* *Bb Harmonic minor* *D 6 notes blues* *Eb 6 notes blues* *F# 6 notes blues* *G 6 notes blues* *Bb 6 notes blues* *B 6 notes blues* *C Whole-tone* *C Augmented half-step minor-third* *D Augmented minor-third half-step* *C 12 notes chromatic*
C 12 notes chromatic	61	*C Ionian* *C# Ionian* *D Ionian* *Eb Ionian* *E Ionian* *F Ionian* *F# Ionian* *G Ionian* *Ab Ionian* *A Ionian* *Bb Ionian* *B Ionian* *C Melodic minor* *C# Melodic minor* *D Melodic minor* *Eb Melodic minor*

		E Melodic minor *F Melodic minor* *F# Melodic minor* *G Melodic minor* *Ab Melodic minor* *A Melodic minor* *Bb Melodic minor* *B Melodic minor* *C Harmonic minor* *C# Harmonic minor* *D Harmonic minor* *Eb Harmonic minor* *E Harmonic minor* *F Harmonic minor* *F# Harmonic minor* *G Harmonic minor* *Ab Harmonic minor* *A Harmonic minor* *Bb Harmonic minor* *B Harmonic minor* *C 6 notes blues* *C# 6 notes blues* *D 6 notes blues* *Eb 6 notes blues* *E 6 notes blues* *F 6 notes blues* *F# 6 notes blues* *G 6 notes blues* *Ab 6 notes blues* *A 6 notes blues* *Bb 6 notes blues* *B 6 notes blues* *C Whole-tone* *C# Whole-tone* *C Half-step/whole step diminished* *D Whole step/half-step diminished* *C Whole step/half-step diminished* *C Augmented half-step minor-third* *D Augmented minor-third half-step* *D Augmented half-step minor-third*

		C Augmented minor-third half-step *C Messiaen mode #3 (Rotation 1)* *C# Messiaen mode #3 (Rotation 1)* *D Messiaen mode #3 (Rotation 1)* *Eb Messiaen mode #3 (Rotation 1)*

Tables 11 and 12 demonstrate how the scales that originally have a lower number of notes tend to have a lower *real harmonic connection degree number* by their nature, despite their high melodic connection level (*melodic connection degree number*) with the bigger scales to which they relate to. Each *melodic connection degree number* involves scales connected at the same melodic level to each scale. Each *melodic connection degree number* does not necessarily imply the same harmonic level of connection that instead is represented by the *real harmonic connection degree number* among the same scales. This aspect can be seen by observing and comparing the numbers and the related involved scales reported in Table 11 and Table 12 (for the sake of brevity only those related to the *C major/ionian* scale). Table 12 expresses that the *C major (ionian)* scale is melodically connected to the *D 6 notes blues*, *E 6 notes blues*, *A 6 notes blues*, *B 6 notes blues* scales, as well as the *F Ionian, G Ionian, C Melodic minor, D Melodic minor, A Harmonic minor, Eb Messiaen mode #3 (Rotation 1)* scales despite Table 11 reports that the *C major (ionian)* scale has a lower *real harmonic connection degree number* related to the *6 notes blues* scales (and a higher *real harmonic connection degree number* related to the *Ionian, Melodic minor, Harmonic minor, Messiaen mode #3 (Rotation 1)* scales). Viceversa, the same *real harmonic connection degree number*, involving scales connected at the same harmonic level, necessarily implies the same melodic level of connection that is represented by the *melodic connection degree number* among the same scales. This aspect can be seen by observing and comparing the numbers and the related involved scales reported in Table 11 and Table 12. In most of the cases involving scales having the same number of notes, the scales connected at the same melodic level to each scale (for example those connected to the *C major/ionian* scale) necessarily provide the same corresponding harmonic level of connection represented by the *real harmonic connection degree number* (those related to the *C major/ionian* scale). More in general, scales having the same number of notes and melodically similar are also necessarily harmonically linked at the same level (according to the number of notes and consequently of *basic chords* generated by each considered scale). Among the scales of 7 notes melodically connected to the *C major (ionian)* scale (Table 12), the *F Ionian, G Ionian, C Melodic minor, D Melodic minor, A Harmonic minor* scales provide the same corresponding high value of the *real harmonic connection degree number* (Table 11). Among all the main chords constituted by three and four notes, the *melodic connection degree numbers* related to each chord type of three notes having the *C* note as the fundamental have been reported in Table 13 as an example, even if the general algorithm implemented in the software tool can handle whatever chord type having any fundamental.

Table.13: Melodic connection degree numbers related to each chord type of three notes and having the C note as the fundamental of the chord.

Chord	***Melodic connection degree number***	***Connected/similar chords***
Cmaj	9	***C#dim*** ***Edim*** ***Cm*** ***Em*** ***Am*** ***C5b*** ***C5#*** ***E5#*** ***Ab5#***
Cm	9	***Cdim*** ***Adim*** ***Eb5b*** ***Cmaj*** ***Ebmaj*** ***Abmaj*** ***Eb5#*** ***G5#*** ***B5#***
C5#	9	***C#m*** ***Fm*** ***Am***

		C5b
		E5b
		Ab5b
		Cmaj
		Emaj
		Abmaj
C5b	8	***Cdim***
		F#dim
		Am
		F#5b
		Cmaj
		C5#
		E5#
		Ab5#
Cdim	9	***Ebdim***
		F#dim
		Adim
		Cm
		Ebm
		C5b
		F#5b
		Abmaj
		Bmaj

It is interesting to notice that some melodic similarity connections among the chords of the major (*maj*) and minor (*m*) families reported in Table 13 represent also some examples of *diatonic substitution* type. This kind of harmonic substitution involves the similar chords included/generated by the same scale and conventionally takes place when the tonal scales of seven notes (*major/ionian, melodic minor, harmonic minor, harmonic major, double harmonic* and other scales) and all the existent bigger scales which contain them are considered. On the other hand, it has to be noted that Table 13 includes all the chords similar to each chord type of three notes having the *C* note as the fundamental in more general terms (in some cases unrelated to a specific scale, accordingly to the *melodic connection degree number* general definition). Similarly to what was noticed in Section 2.1 and Section 3 of the present paper, the investigation reported in Table 12 involves the melodic connections among scales, as well as the one in Table 13 involves the melodic connections among chords, despite chords and scales can be considered as generic groups of notes, thus legitimately directly compared. It has to remember that the computed *melodic connection degree number* related to each scale allows to express and build the network of the music connections among chords and scales introduced and described in detail in [18]. This network is constituted by superposed geometric graphs of connections (that can also represent mutual inclusions) among the generic groups of notes.

V. THE FLOW OF SOUND PATHWAYS GENERATED BY THE GROUPS OF NOTES ARBITRARILY EMITTED

In the present Section, harmonic analysis examples of some chord progressions reported in [18] have been provided (namely Case#2, Case#3). In the present study, these analyses include also some scales (not all) related to the *Modes of limited transposition* [6]. The analyses have been carried out in terms of the graphic and time-dependent network constituted by the considered chords and scales which are melodically connected. In particular, the *Harmonic minor, Melodic minor, Ionian, Half-step/whole step diminished, Whole step/half-step diminished, Messiaen mode #3 (Rotation 1)* scale families have been considered in the analyses. It has to be noted that these scale families are considered also in [7] and that the *Messiaen mode #3 (Rotation 1)* scale family includes the *Whole-tone* (having the same first-degree note), *Augmented half-step minor-third* (having the same first-degree note), *Augmented minor-third half-step* (having the first-degree note one tone ahead) scale families as previously reported in Table 4. The graph that expresses the *melodic connection degree numbers* among scales reported in Fig. 2 has been chosen to represent the music transient process related to the chord progression of each Case. This choice confers a good representation clarity as well as provides a better description and justification in graphical terms of some of the harmonic and melodic passages related to the chord progressions reported in [18] and in the present paper.

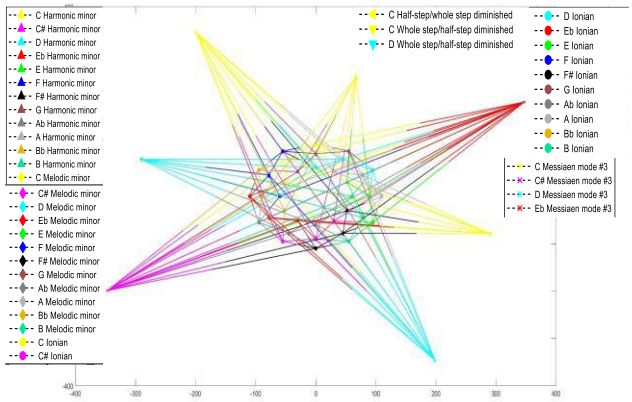

Fig. 2: Harmonic minor, Melodic minor, Major/Ionian, Half-step/whole step diminished, Whole step/half-step diminished and Messiaen mode #3 (Rotation 1) scale families.

Case#2 is characterized by the chord progression reported in Table 14:

Table.14: Chord progression related to Case#2

Cmaj7	*Cmaj7*	*Fm7*	*Bb7*
Cmaj7	*Cmaj7*	*Bbm7*	*Eb7*
Abmaj7	*Abmaj7*	*Am7*	*D7*
Dm7	*G7*	*Cmaj7 Eb7*	*Abmaj7G7/5#*

It is possible to graphically represent the chord progression related to Case#2 by adopting different *N* values. In particular, the value related to *N=2* has been employed for the first six bars (Figs. 3, 4), the value related to *N=4* has been employed from the 7th to 10th bar (Fig. 5), the value related to *N=2* has been employed from the 11th to 12th bar (Fig. 6), the value related to *N=3* has been employed from the 13th to 15th bar (Fig. 7), the value related to *N=2* has been employed from the 15th to 16th bar (Figs. 8 and 9). The whole music-transient process related to the chord progression denoted by Case#2 is summarized by Fig. 10 for the different *N* values.

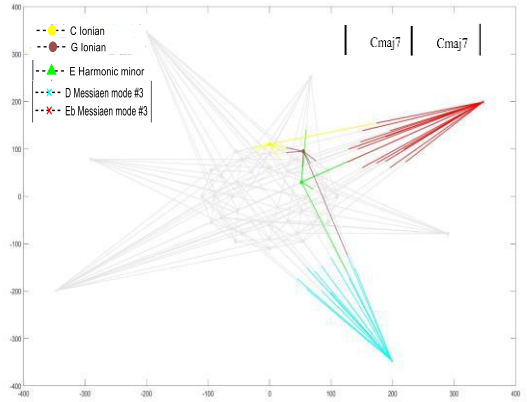

Fig. 3: Case#2 related to N=2: 1st, 2nd, 5th and 6th bars.

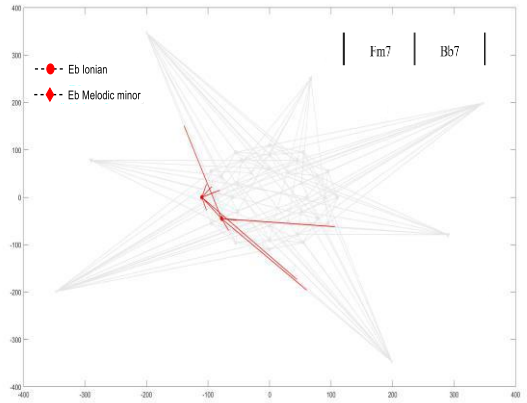

Fig. 4: Case#2 related to N=2: 3rd and 4th bars.

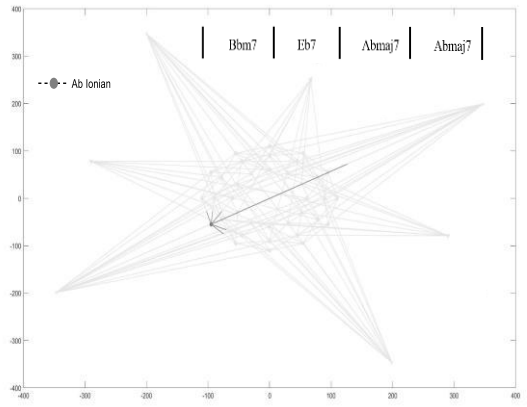

Fig. 5: Case#2 related to N=4: 7th, 8th, 9th and 10th bars.

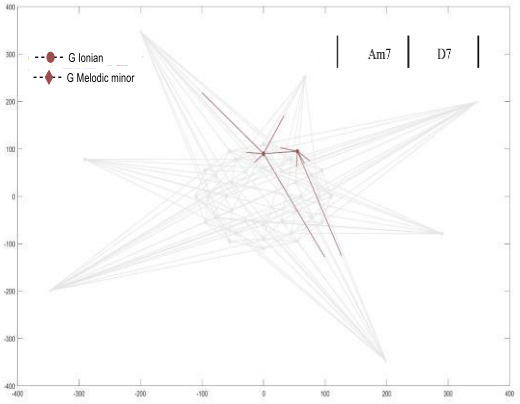

Fig. 6: Case#2 related to N=2: 11th and 12th bars.

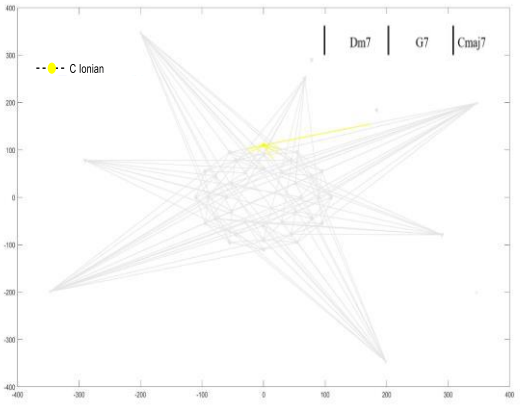

Fig. 7: Case#2 related to N=3: 13th, 14th and 15th bars.

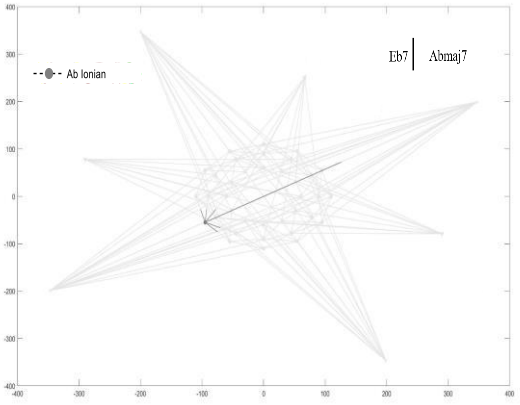

Fig. 8: Case#2 related to N=2: 15th and 16th bars.

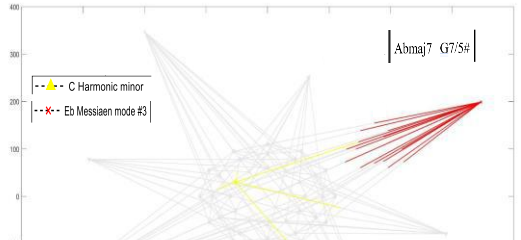

Fig. 9: Case#2 related to N=2: 16th bar.

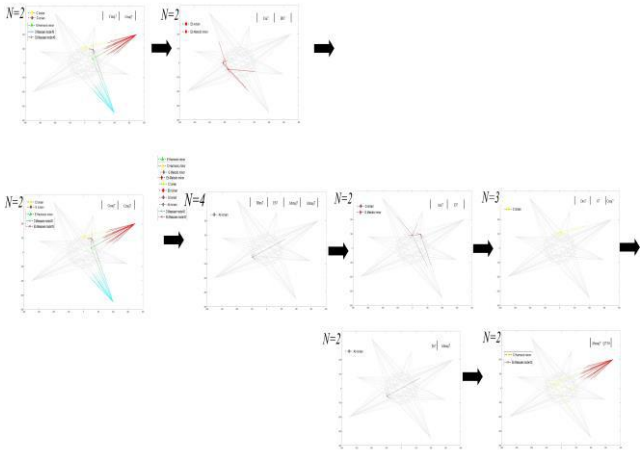

Fig. 10: The flow of sound pathways through the music network related to the chord progression denoted by Case#2 for different N values.

Case#3 is characterized by the chord progression reported in Table 15:

Table.15: Chord progression related to Case#3

Ebmaj7	*Ebmaj7*	*Ebm7*	*Ab7*
Abmaj7	*Abmaj7*	*Abm7*	*Db7*
Ebmaj7	*F#m7 B7*	*Fm7*	*Bb7*
Ebmaj7	*F#m7 B7*	*Fm7*	*Bb7*

It is possible to graphically represent the chord progression related to Case#3 by adopting different *N* values. In particular, the value related to *N=2* has been employed for the first eight bars (Figs. 11 to 14), the value related to *N=1* has been employed for the 9th and 13th bar (Fig. 15), the value related to *N=2* from the 10th to 12th bar and from the 14th to 16th bar (Figs. 16 and 17). The whole music-transient process related to the chord progression

denoted by Case#3 is summarized by Fig. 18 for the different *N* values.

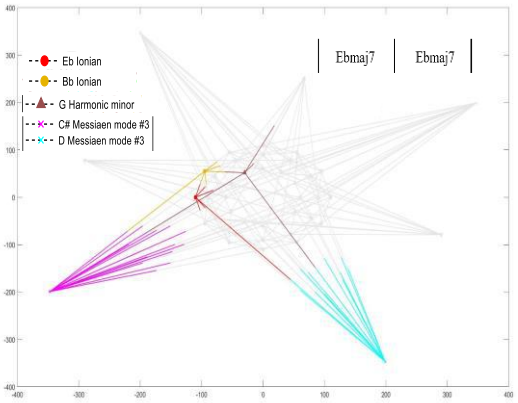

Fig. 11: Case#3 related to N=2: 1st and 2nd bars.

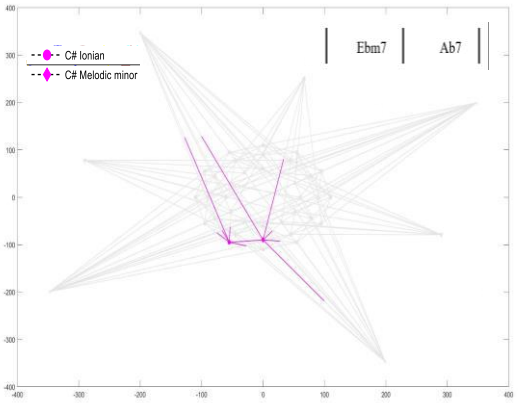

Fig. 12: Case#3 related to N=2: 3rd and 4th bars.

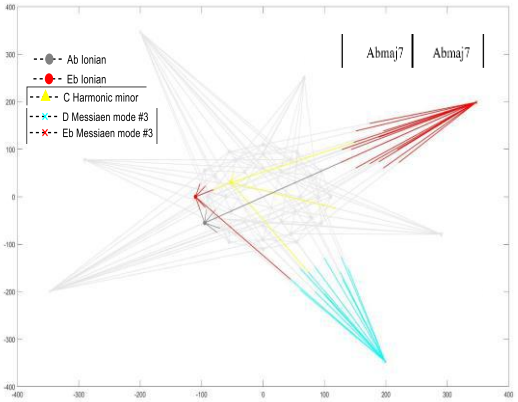

Fig. 13: Case#3 related to N=2: 5th and 6th bars.

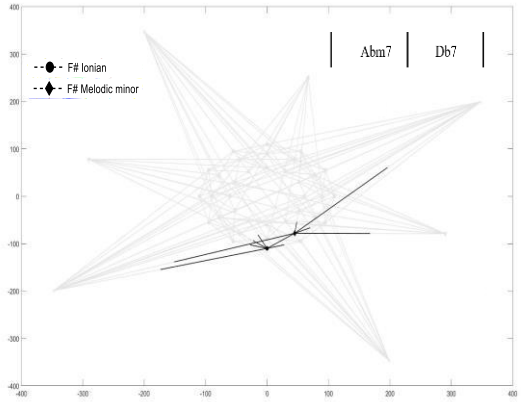

Fig. 14: Case#3 related to N=2: 7th and 8th bars.

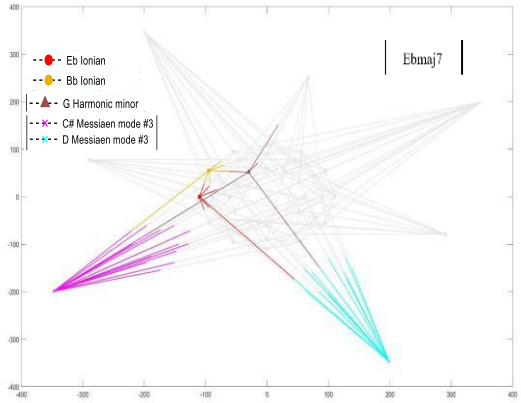

Fig. 15: Case#3 related to N=1: 9th and 13th bars.

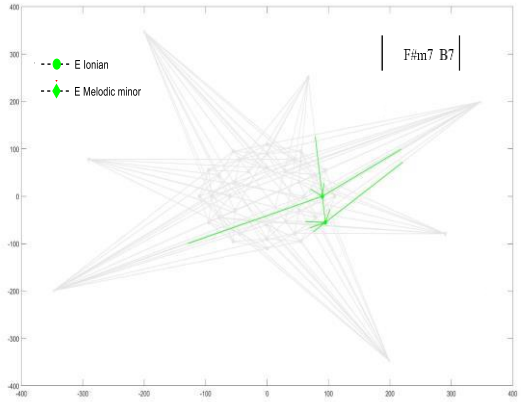

Fig. 16: Case#3 related to N=2: 10th and 14th bars.

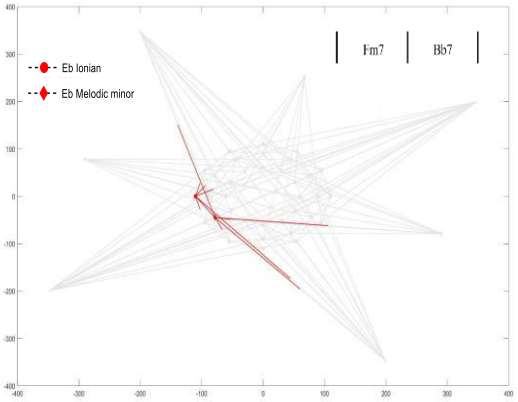

Fig. 17: Case#3 related to N=2: 11th, 12th, 15th and 16th bars.

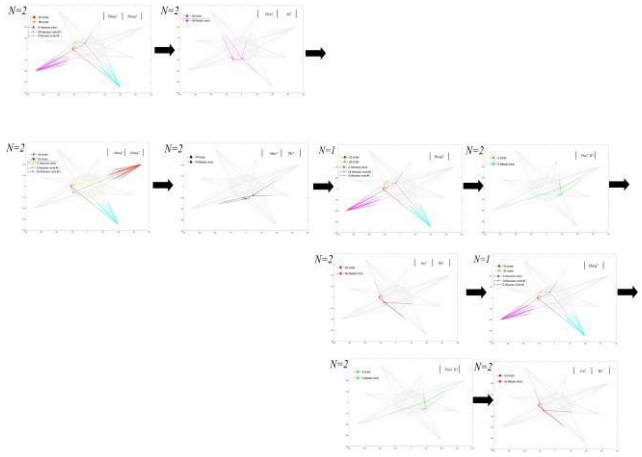

Fig. 18: The flow of sound pathways through the music network related to the chord progression denoted by Case#3 for different N values.

VI. FURTHER APPLICATIONS AND FUTURE PERSPECTIVES RELATED TO THE MUSIC NETWORK GRAPHS

By observing these graphs it is interesting to notice how new perspectives for what concerns music improvisation can be experimented employing the geometrical-based approach that constitutes the music network graphs. In particular, it can be demonstrated how following the trajectories traced by the parsimonious voice-leading approach to connect generic groups of notes (scales and chords) can provide a human ear-friendly response. These trajectories are the graphical expression of the *melodic connection degree number*. The next example is aimed to demonstrate that, among all potential applications related to the present method, it is possible to generate a melody pattern (or a chord progression pattern) following a specific elemental geometric feature/pattern of connected generic groups of notes, as reported in Fig. 19. This elemental geometric pattern is constituted by five nodes (five groups of notes that are scale families in this case). The elemental geometric pattern considered in the graph can be followed iteratively touching these scale families in this specific order exploiting exclusively the common notes between two adjacent nodes (among all the potential orders and scale families that can be considered): *C Harmonic minor*, *F 6 notes blues*, *Ab Major (Ionian)*, *Bb Melodic minor*, *Bb Harmonic minor*, *F Harmonic minor*, *etc*. Therefore the following elemental melody pattern can be generated (among all the potential generable melody patterns): *F Ab B C F C Ab F (*common notes between *C Harmonic minor* and *F 6 notes blues* scales*) Ab C Eb F Ab Eb Bb Ab (*common notes between *F 6 notes blues*, *Ab Major (Ionian)* scales*) Bb Db F G Bb G F Db (*common notes between *Ab Major (Ionian)* and *Bb Melodic minor* scales*) Bb Db F A C A F A (*common notes between *Bb Melodic minor* and *Bb Harmonic minor* scales*)*. Then it is possible to iteratively follow the same pattern among nodes of connected scale families starting from the *F Harmonic minor* scale (notice the switch between the *Bb Harmonic minor* and *F Harmonic minor* scales instead of exploiting the common notes between these nodes) and consequently graphically (thus also musically) transpose the same geometric pattern until the same original feature is obtained again on the graph (closing the circle of multiple "tones" each representing a specific group of notes, in this case a specific scale family) as expressed by Figs. 20 and 21. It has to be noted that the reported elemental melody pattern can be considered generated at the same time also by other scale families (other potential elemental geometric patterns in the music network are generators of the same melody pattern).

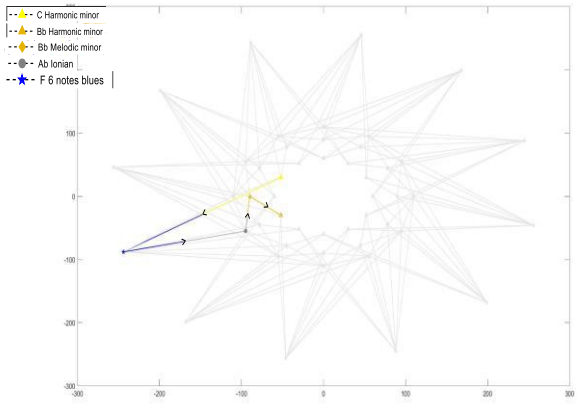

Fig. 19: Generation of a melody pattern following a specific elemental geometric feature/pattern of connected generic groups of notes.

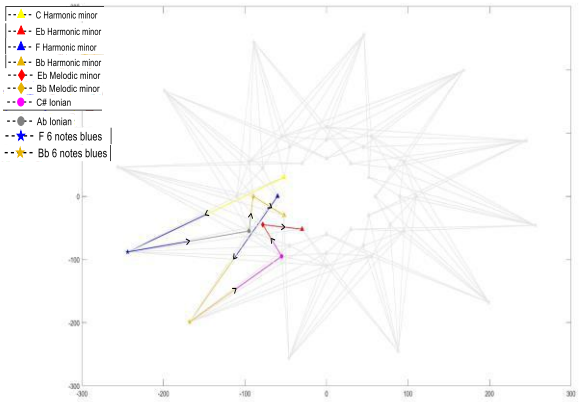

Fig. 20: Iterative transposition of the same elemental geometric melody pattern switching between the Bb Harmonic minor and F Harmonic minor scales.

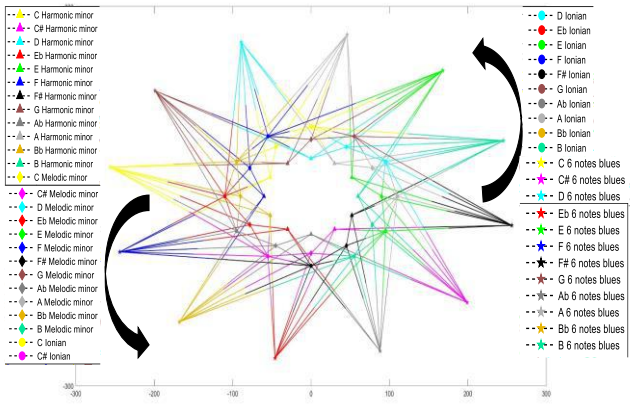

Fig. 21: Iterative transposition of the same elemental geometric melody pattern closing the circle of multiple "tones" each representing a specific group of notes.

A final consideration on chords (and also scales) that have different names even if constituted by the same notes (chord *inversions* and *modes* of a scale). These chord *inversions* or *modes* of a scale can be suitably employed for defining new perspectives in the music network graphs application, specifically for what concerns the inversion substitutions in case of chords and rotations of the main scale in case of scales. In particular, it has to be noted how each node in the music network graphs represents a specific group of notes (scale or chord). Focusing on the tonal scales of seven notes (*major/ionian, melodic minor, harmonic minor, harmonic major, double harmonic* and other scales) and the related *modes*, it is possible to physically place a sort of "handpan resonator" [19, 20] on the conceptual center corresponding to the coordinate of each node of a music network graph. In this manner, a sort of big "handpan" instrument (constituted by the whole group of "handpan resonators" put on each node of the music network) incorporating all the 12 scales of each tonal scale family of seven notes and the related seven *modes* can turn out. Thus, more specifically, seven "handpan resonators" (one for each *mode*) are arranged in a circle around each node of a music network graph conceptually representing in a synthesized/condensed manner each tonal scale of seven notes and related surrounding *modes*. An example of the criteria with which these "handpan resonators" can be mutually arranged is expressed in Fig. 22 considering the nodes' centers coordinates related to the conventional tonal scale families of seven notes (*major/ionian*, *melodic minor*, *harmonic minor*).

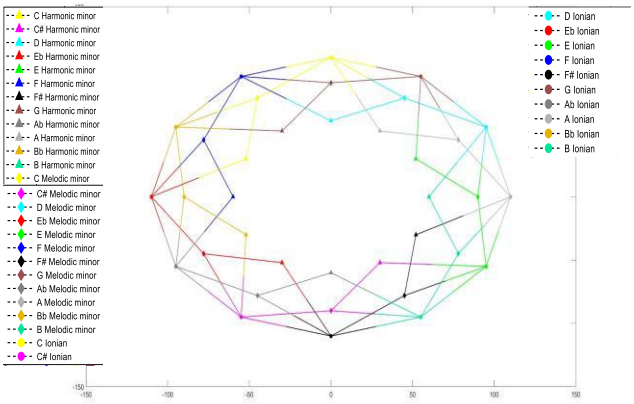

Fig. 22: Harmonic minor, Melodic minor and Major/Ionian scale families as the nodes' centers coordinates corresponding to each "tonal" handpan resonator and the related surrounding "modal" handpan resonators.

From Fig. 22 inspection it is clear how each conceptual music network graph can easily become a physical "handpan resonators" network. The network of "handpan resonators" can be played, for instance, by several people located at the same time on different points/coordinates of the network. Sometimes, also for demonstration purposes, they can resonate the same melody patterns/generic groups of notes arbitrarily emitted from different places in the network, each representing a different scale family whose embedded modes generate one or more specific chords that at the same time are shared by resonating handpans located in the network at different locations. It is evident that the present method introduced in [18] and further developed in the present study considers the circle of fifths only as one of the possible musical dimensions. These dimensions intersect each other and overlap while maintaining alive the mutual nodal interconnections by establishing a network in which mechanisms of self-similarity and symmetries are self-generated recurring periodically. This would demonstrate how in the future, it may no longer be necessary to learn any specific fingering on musical

instruments. The dexterity and the technical research will remain necessary, but the filtering activity will be managed automatically and externally through appropriate switches that electronically and automatically will prevent some notes to be played according to the current tastes preprocessed by the system and possibly modified by the musician user in real-time directly on the instrument equipped with appropriate selectors. Therefore the musician user will not have to worry about what to play and what not to play, thus he will be allowed to focus entirely on the development of technique and rhythmic figurations within the range of notes that he is allowed to play in a dynamic time-varying regime. Therefore he will begin to get used to a new perspective that conceptually facilitates the musician playing of each instrument. The musician of each kind of instrument will be allowed to feel like playing the white keys of the piano all the time thanks to the electronic help that tracks the music in real-time. This logic can also be applied, for example, to an electronic mixer in which the signals of what is played by the band are crossed in real-time. The musician users play harmonic and percussive instruments to which each user can autonomously apply filters. In addition, each instrument emits sounds according to what is played by the others (each electronic filter acting on each instrument "knows" in real time the basic parameters that characterize the filters acting on other instruments that are playing, as they are all connected to a common mixer that may apply, as a Master, its own specific filter or the superposition of the actions operated by electronic filters acting separately on each instrument and constituting the music electronic network on which the band is based).

VII. CONCLUSIONS

The music universe can be considered as constituted by groups and subgroups of vibrations arbitrarily emitted and generated by strings. These vibrations are related to groups and subgroups of frequencies that allow determining the chromo-harmonic structures whose shape is arbitrarily chosen. According to the present model, these groups and subgroups of frequencies can be represented in a condensed form through the use of arbitrary symbols that are mutually interconnected (and also mutually included very often) according to the *melodic connection degree number*.

- Each group and subgroup of frequencies can be considered as a single part of the entire system to be separately (individually) analyzed as if the other groups did not exist. On the other hand, these connected groups and subgroups of frequencies can be considered together as a part of the same investigated bigger system (or of the entire system which is constituted by all the parts parallelly coexisting) even if contained in each other (hence the term subgroups).

- Through an appropriate filtering operation (that is arbitrarily operated by the musician-user subjectivity) and a suitable zooming operation, it is possible to intercept and isolate the specific group of frequencies of interest and observe every single part of the music universe from different perspectives (those arbitrarily observed at that time).

Therefore these perspectives can be different each time (depending on the choice operated by the musician-user at that time). Several possible configurations of groups and subgroups of frequencies to be individually considered and investigated at the same time would turn out if a different filtering operation and the same or a different zoom level were chosen by the musician-user. These configurations can be constituted exclusively by the new groups and subgroups of frequencies that appear as the new filtering and zooming operation is carried out as well as by the same configurations previously investigated as interfaced together with the new ones. These new configurations of groups could even have never been explored by musician-observers and could remain unexplored but their constituting mutual interconnections and logical patterns have already been traced according to the *melodic connection degree number* and therefore have always been parallelly existed in any case. They only had been waiting to be intercepted/tracked by the musician-observer subjectivity. Thus, each group and subgroup of frequencies can be seen and analyzed separately (when considered as a single part of the entire system which is constituted by all the parts parallelly coexisting) or together with the other groups and subgroups as a part of the same investigated bigger system (or of the entire system). It is important to notice that these logical patterns exist regardless of the instrument employed to produce/emit these groups and subgroups of vibrations (and the associated frequencies) since are represented in a symbolical synthetic way through interconnected nodes in a geometrical network. The musician user will be able to start experimenting with improvisational techniques that try to arbitrarily follow the trajectories constituting the structure of the electronic filter on which each instrument will be possibly based. This structure has been determined a priori based on subjective aesthetic choices that lead to a sort of music state diagrams in a dynamic time-varying regime. The present study and the related methodological approach fit into the line of research aimed at making each musician increasingly aware of the many melodic and harmonic possibilities. This can also be useful in order to train/guide an automatic

machine to the composition and study and analysis of improvisation.

NOMENCLATURE

Notes names:

C = (B#)

C# = Db

D

D# = Eb

E = (Fb)

F = (E#)

F# = Gb

G

G# = Ab

A

A# = Bb

B = (Cb)

Interval names:

1/8= perfect unison/perfect octave

(1#)= (augmented unison)

2b/9b= minor second/minor ninth

2/9= major second/major ninth

2#/9# =augmented second/augmented ninth

3b =minor third

3= major third

(3#) =(augmented third)

(4b/11b) =(diminished fourth/diminished eleventh)

4/11= perfect fourth/perfect eleventh

4#/11# = augmented fourth/augmented eleventh

5b=diminished fifth

5=perfect fifth

5#= augmented fifth

6b/13b =minor sixth/minor thirteenth

6/13= major sixth/major thirteenth

(6#/13#)=(augmented sixth/augmented thirteenth)

(7bb)=(diminished seventh)

7b= minor seventh

7 = major seventh

(7#)= (augmented seventh)

(8b) =(diminished octave)

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