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Breaker Depth Analysis Using Critical Wave Steepness

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Abstract— *This research developed a breaker depth equation based on the characteristic of the potential velocity solution of Laplace equation. The breaker length equation was obtained using critical wave steepness as boundary condition. Whereas breaker height was obtained from breaker height index equation. The equation is in the form of linear explicit equation with simple calculation.*
Keywords*— *breaker depth, critical wave steepness.

I. INTRODUCTION

In general, breaker index equation has three shapes, i.e. breaker height index $\frac{H_b}{H_0}$, where H_b is breaker height and H_0 is deep water wave height. The second form is breaker depth index in the form of $\frac{H_b}{h_b}$ where h_b is breaker depth and the third is breaker steepness index in the form of $\frac{H_b}{L_b}$ where L_b is breaker length.

The breaker depth equation developed in this research was obtained by performing one of the conservation characteristics of potential solution wave of Laplace equation, i.e. multiplication between wave number and water depth is constant which means that the multiplication of wave number at breaker depth is similar with the multiplication between wave number and deep water depth. Therefore, there is a relation between breaker depth and the condition of wave at the deep water. There is a breaker length variable at the equation. To eliminate breaker length, the critical wave steepness criteria was performed. Using this method, an equation is obtained based on analysis and the law of conservation.

II. COMPUTATION OF BREAKER HEIGHT H_b

To perform a computation using breaker index in the form of $\frac{H_b}{L_b}$ and $\frac{H_b}{h_b}$, breaker height H_b should be known. The breaker height is obtained using breaker height index equation $\frac{H_b}{H_0}$. There are a lot of breaker height index equations. This research uses a breaker height H_b that is the average of several breaker height indexes that produces adjoining breaker height. The breaker height indexes that were used are as follows:

Komar and Gaughan (1972)

$$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \quad ....(1)$$

Singamsetti and Wind (1980)

$$\frac{H_b}{H_0} = 0.575m^{0.031}\left(\frac{H_0}{L_0}\right)^{-0.254} \quad ....(2)$$

Larson and Kraus (1989),

$$\frac{H_b}{H_0} = 0.53\left(\frac{H_0}{L_0}\right)^{-0.24} \quad ....(3)$$

Smith and Kraus (1990),

$$\frac{H_b}{H_0} = (0.34 + 2.74m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88m} \quad ....(4)$$

Gourlay (1992),

$$\frac{H_b}{H_0} = 0.478\left(\frac{H_0}{L_0}\right)^{-0.28} \quad ....(5)$$

Rattana Pitikon and Shibayama (2000) :

$$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55)\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \quad ....(6)$$

At those equations H_b is breaker height, H_0 is deep water wave height, L_0 is deep water wavelength and m is bottom slope.Table (1) presents the result of breaker height computation using those 6 (six) equations above and their average values. The wave used is the wave with wave period $T = 8\,sec.$, bottom slope $m = 0.005$ and water depth $h_0 = 60\,m$, whereas deep water wave height H_0 varies between 0.60 – 1.8 m.

Table.1: Breaker height from various breaker height index equations

H_0 (m)	Breaker Height H_b (m)						
	(1)	(2)	(3)	(4)	(5)	(6)	Aver.

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0,6	0,93	1,07	0,91	1,09	0,96	0,93	0,98
0,9	1,29	1,45	1,23	1,48	1,28	1,28	1,34
1,2	1,63	1,8	1,53	1,84	1,57	1,62	1,66
1,5	1,94	2,13	1,81	2,18	1,84	1,93	1,97
1,8	2,25	2,44	2,07	2,5	2,09	2,24	2,26

Note : (i), number of breaker height index equation

Table (1) shows that even though each equation provides different breaker height, all of them are close enough to the average value.

III. THE COMPUTATION OF BREAKER DEPTH h_b AND BREAKER LENGTH L_b USING THE EXISTING EQUATIONS.

Relation between breaker depth h_b and breaker length L_b is in the shape of implicit equation where there is a dependency between those two variables. There is an explicit equation to compute breaker depth, i.e. SPM equation (1984) and Van Rijn equation (2011). However, at Van Rijn there is a parameter that has to be tested, therefore this research used SPM equation (1984). The wavelength computation cannot be done using dispersion equation of linear wave theory, considering at breaker depth the value $\frac{H_b}{h_b}$ is quite large, i.e. close to 1, whereas dispersion equation of linear wave theory is formulated at very small $\left(\frac{H}{h}\right)$ condition. As a consequence, the computation is by doing other breaker index equation.

A. The computation of breaker depth h_b and breaker length L_b with SPM equation (1984) and Miche equation (1944)

Breaker depth index from SPM (1984) is in the form of explicit equation for breaker depth as an input.

$$\frac{h_b}{H_b} = \frac{1}{b-\left(\frac{aH_b}{gT^2}\right)} \quad \text{or} \quad h_b = \frac{H_b}{b-\left(\frac{aH_b}{gT^2}\right)} \quad \text{................(7)}$$

$$a = 43.75\left(1 - e^{-19.0m}\right)$$

$$b = \frac{1.56}{1 + e^{-19.5m}}$$

H_b= breaker height, h_b = breaker depth, g = gravity acceleration and m = bottom slope. Using (7) h_b can be calculated using H_b as an input, then L_b is calculated with Miche equation (1944). Whereas breaker index from Miche equation belongs to breaker steepness index with the following form.

$$\frac{H_b}{L_b} = 0.142 tanh\left(\frac{2\pi h_b}{L_b}\right) \quad \text{......................(8)}$$

With the input wave period h_b and H_b then breaker length L_b can be calculated using Newton-Rhapson iteration method.

B. The computation of breaker depth h_b and breaker length L_b with Rattana Pitikon *et al* equation (2003) and Miche equation (1944).

Breaker steepness equation from Rattana Pitikon *et al* (2003) is

$$\frac{H_b}{L_b} = \left(-1.40m^2 + 0.57m + 0.23\right)\left(\frac{H_0}{L_0}\right)^{0.35} \quad \text{..(9)}$$

Using input H_b then L_b can be calculated explicitly with (9),

$$L_b = \frac{H_b}{\left(-1.40m^2 + 0.57m + 0.23\right)\left(\frac{H_0}{L_0}\right)^{0.35}}$$

Then h_b can be calculated using (8), with i Newton-Rhapson iteration method.

IV. THE PROPOSED EQUATION

The potential velocity of linear wave theory (Dean, 1991) is, $\Phi = Gcoskxcoshk(h+z)sin\sigma t$. This equation was obtained by completing Laplace equation with variable separation method, where potential velocity is considered as consists of $X(x)$ which is only a function of x, $Z(z)$ is only a function of z and $T(t)$ is only a function of time, x is horizontal axis and z is vertical axis. In (1), $Z(z) = \cosh k(h+z)$. Applying the velocity potential on small sloping bottom, and derive $\frac{\partial Z(z)}{\partial x} = \frac{\partial \cosh k(h+z)}{\partial x} = \sinh k(h+z)\frac{\partial k(h+z)}{\partial x} = 0$, in this equation $\frac{\partial k(h+z)}{\partial x} = 0$ in every z . With $k = \frac{2\pi}{L}$ and for $z = \frac{H}{2}$, the equation can be written as $\frac{\partial}{\partial x}\left(\frac{h+\frac{H}{2}}{L}\right) = 0$, which means that $\frac{\left(h+\frac{H}{2}\right)}{L} = c$, where c is a constant. From this equation, then if the wave moves from water depth h_1 to shallower water depth h_2 where shoaling took place, then there is a relation, $\frac{h_2+\frac{H_2}{2}}{L_2} = \frac{h_1+\frac{H_1}{2}}{L_1}$ or, $h_2 = \frac{L_2}{L_1}\left(h_1 + \frac{H_1}{2}\right) - \frac{H_2}{2}$. For waves moving from deep water with (h_0, L_0, H_0) to breaker depth with (h_b, L_b, H_b), there is a relation

$$h_b = \frac{L_b}{L_0}\left(h_0 + \frac{H_0}{2}\right) - \frac{H_b}{2} \quad \text{................(10)}$$

In (10) there are 3 (three) unknowns, i.e. h_b,L_b and H_b. Breaker height can be calculated with breaker height index as was done in section 2. Breaker length L_b can be obtained from the criteria of critical steepness curve. Critical steepness from Michel is $\frac{H}{L} = 0.142$, which states

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that if $\frac{H}{L} \geq 0.142$ the wave will be breaking, and this criteria also applies to breaker point, i.e. breaking occurs when $\frac{H_b}{L_b} = 0.142$. However, in this research, $\frac{H_b}{L_b} = \frac{1}{\pi}$ was used, this criteria was obtained by studying breaking condition with non-linear wave theory, which due to spatial limitation could not be shown in this research, and would be shown in the next research. Therefore, the breaker length becomes,

$$L_b = \pi H_b \qquad \text{.................(11)}$$

Substitute to (10),

$$h_b = \frac{\pi H_b}{L_0}\left(h_0 + \frac{H_0}{2}\right) - \frac{H_b}{2} \qquad \text{.................(12)}$$

At the deep water depth, $k_0\left(h_0 + \frac{H_0}{2}\right) = \alpha$, where $tanh(\alpha) = 1$, SPM (1984) uses $\alpha = \pi$ where $\frac{H_0}{L_0} = 0.5$, in this research $\alpha = 1.1\pi$ is used. Substitute $L_0 = \frac{2\pi}{k_0}$ to (12) and $\alpha = 1.1\pi$, produces

$$h_b = (0.55\pi - 0.5)H_b \qquad \text{.................(13)}$$

With input H_b from breaker height index then breaker depth h_b can be calculated with (13)

V. COMPARISON OF THE RESULTS OF THE THREE METHODS

The next section will show the result of breaker depth h_b and breaker length L_b computation using the three methods mentioned above. The bathymetry data is similar to the computation of H_b in section 2. The result of the computation is as follows.

Table.2: h_b and L_b the result of the three methods.

H_0 (m)	H_b (m)	h_b (m)			L_b(m)		
		(3)	(1)	(2)	(3)	(1)	(2)
Wave Period (T) : 7 sec.							
0,6	0,92	1,13	1,14	1,07	2,92	12,85	21,59
0,9	1,25	1,54	1,55	1,47	3,98	17,22	25,5
1,2	1,56	1,92	1,94	1,84	4,95	21,15	28,69
1,5	1,85	2,27	2,3	2,2	5,87	24,75	31,45
1,8	2,12	2,61	2,65	2,56	6,74	28,11	33,89
Wave Period (T) : 8 sec.							
0,6	0,98	1,21	1,21	1,13	3,12	13,81	25,27
0,9	1,34	1,64	1,65	1,55	4,24	18,55	29,83
1,2	1,66	2,04	2,06	1,95	5,28	22,83	33,57
1,5	1,97	2,42	2,45	2,32	6,25	26,77	36,79
1,8	2,26	2,78	2,82	2,69	7,18	30,45	39,65
Wave Period (T) : 9 sec.							
0,6	1,04	1,28	1,28	1,19	3,3	14,68	28,97
0,9	1,41	1,73	1,74	1,63	4,48	19,77	34,2
1,2	1,76	2,16	2,17	2,04	5,58	24,36	38,48
1,5	2,08	2,56	2,58	2,44	6,61	28,61	42,17
1,8	2,39	2,94	2,97	2,82	7,59	32,59	45,45

Note :

(1) Couple computation between (7) and (8)

(2) Couple computation between (8) and (9)

(3) Computation with (11) and (13)

The result of h_b and L_b computation using the three methods shows that h_b produced by (3) , i.e. with the proposed method is quite close with methods (1) and (2). Among the three methods, the smallest breaker wave length is produced by (3) with quite realistic wave length.

The computation result of $\frac{H_b}{h_b}$ and $\frac{H_b}{L_b}$ with the three methods is presented on table (3). It shows that the value of $\frac{H_b}{h_b}$ from method (3) looks constant against the changes of deep water wave height and wave period. At method (1) the value of $\frac{H_b}{h_b}$ is a somewhat affected by the changes in deep water wave height but is constant against wave period. The value of $\frac{H_b}{h_b}$ at the third method changes against deep water wave height as well as against wave period. The value of $\frac{H_b}{L_b}$ at method (1) is constant at $\frac{1}{\pi}$, since it was designated. At method (1) the constant is at 0.07, no change against the changes in deep water wave height as well as wave period. At method (3), the value of $\frac{H_b}{L_b}$ changes against the changes in deep water wave height as well as wave period.

Table.3: $\frac{H_b}{h_b}$ and $\frac{H_b}{L_b}$ the result of the three methods

H_0 (m)	H_b (m)	$\frac{H_b}{h_b}$			$\frac{H_b}{L_b}$		
		(3)	(1)	(2)	(3)	(2)	(1)
Wave Period (T) : 8 sec.							
0,6	0,92	0,81	0,81	0,86	0,32	0,07	0,04
0,9	1,25	0,81	0,81	0,86	0,32	0,07	0,05
1,2	1,56	0,81	0,81	0,85	0,32	0,07	0,05
1,5	1,85	0,81	0,8	0,84	0,32	0,07	0,06
1,8	2,12	0,81	0,8	0,83	0,32	0,08	0,06
Wave Period (T) : 8 sec.							
0,6	0,98	0,81	0,81	0,87	0,32	0,07	0,04
0,9	1,34	0,81	0,81	0,86	0,32	0,07	0,04
1,2	1,66	0,81	0,81	0,85	0,32	0,07	0,05
1,5	1,97	0,81	0,81	0,85	0,32	0,07	0,05

1,8	2,26	0,81	0,8	0,84	0,32	0,07	0,06
Wave Period (T) : 9 sec.							
0,6	1,04	0,81	0,81	0,87	0,32	0,07	0,04
0,9	1,41	0,81	0,81	0,87	0,32	0,07	0,04
1,2	1,76	0,81	0,81	0,86	0,32	0,07	0,05
1,5	2,08	0,81	0,81	0,85	0,32	0,07	0,05
1,8	2,39	0,81	0,81	0,85	0,32	0,07	0,05

Breaker wave length (3) looks very short. This is in accordance with the wavelength produced by the conservation equation $\frac{dk(h+z)}{dx} = 0$. For example, for the ignored wave height, relation $k_2h_2 = k_1h_1$ or $k_2 = \frac{k_1h_1}{h_2}$ applies. For a wave with wave period $T = 8$ second, moved from water depth $h_1 = 60$ m to water depth, $h_2 = 2$ m, then $k_2 = \frac{k_1 60}{2} = 30k_1$, if $k_1$is calculated using dispersion from linear wave theory, it produces k_1 =0.049932 and wave length $L_1 =$ 125.835575, produces $k_2 = 1.497951$ and $L_2 = 4.195$ m. It shows that equation $\frac{dk(h+z)}{dx} = 0$ does produce a very short wave length at shallow water.

From the result above, a simple calculation method of breaker parameter can be produced, i.e.:

a. Breaker height H_b is calculated with one of the breaker height index equation or by taking average values of various breaker height indexes.
b. Breaker depth h_b , is calculated with $\frac{H_b}{h_b} = 0.81$
c. Breaker length L_b, is calculated with $\frac{H_b}{L_b} = \frac{1}{\pi}$

VI. CONCLUSION

The result of breaker wave steepness computation with breaker wave steepness index equation produces breaker wave steepness value that is more or less constant toward wave period as well as deep water height. This shows the presence of critical steepness wave on a wave curve.

The proposed equation uses critical wave steepness criteria. The equation uses wave condition in deep water and in the form of explicit equation that is easy to use. In addition, the equation is an analytical product based on the law of conservation. The critical wave steepness criterion is quite important in the development of a simple breaker index; therefore, a research is needed on critical steepness wave, in the laboratory as well analytical.

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