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Analytical Formulation of Breaker Equations

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***Keywords*— breaker height, breaker length, and breaker depth equations**

***Abstract*— In this study, equations to calculate breaker height, breaker length, and breaker depth were formulated. The analytical formulations were based on the velocity potential of the solution of the Laplace equation. The substitution of velocity potential to weighted kinematic free surface boundary condition yielded equation for wave amplitude function. The dispersion equation was obtained by substituting the velocity potential and wave amplitude function into the weighted momentum Euler equation. The relation between breaker height and breaker wavenumber is extracted from the wave amplitude function. The breaker wavenumber equation was obtained by substituting this relation into the dispersion equation. Furthermore, the breaker depth equation obtained was put to the wavenumber conservation equation with the variables of breaker height and breaker wavenumber. The breaker wave amplitude equation, the breaker wavenumber equation, and the breaker depth equation are simple equations that are very easy to use.*

I. INTRODUCTION

There are quite a lot of empirically formulated equations to calculate the water wave characteristics at the point breaking wave. These equations are called the breaker index. There arebreaker height index to calculate breaker height,breaker depth index to calculate breaker depth with an input of breaker height, andbreaker length index to calculate breaker length with the inputs of breaker height and breaker depth.

Some researchers who put forward equation for the breaker height index include Le Mehaut and Koh (1967), Komar and Gaughan (1972), Sunamura and Horikawa (1974), Singamsetti and Wind (1980), Ogawa and Sutto (1984), Larson and Kraus (1989a), Smith and Kraus (1990), Gourlay (1992), and Pittikon and Shibayama (2000).

Meanwhile, studies dealing with breaker depth index are conducted by Mc Cowan (1984), Galvin (1969), Collins and Weir (1969), Goda (1970), Weggel (1972), Sunamura (1980), Singamsetti and Wind (1980), Seyama and Kimura (1988), Larson and Kraus (1989b), Smith and Kraus (1990).

In terms of breaker length index, researchers proposing the studies include Miche (1944), Battjes and Jansen (1978), Ostendorf and Madsen (1979), Battjes and Stive (1985), Kamphuis (1991), Rattanapitikon and Shibayama (2000) and Rattanapitikon et al. (2003).

This study formulated equations for water wave characteristics at the point breaking wave based on hydrodynamic equations including the equations for breaker height, breaker length, and breaker depth. Using this method, a relationship among the three is elaborated.

In this research, the wave amplitude function was formulated using the weighted kinematic free surface boundary condition and the weighted Euler's momentum equation. Hence,there is a weighting coefficient in the equations of breaker wavenumber, breaker depth, and shoaling in the dispersion equation. The weighting coefficient is obtained by calibrating the critical wave steepness with the critical wave steepness criteria from Michell (1893), where the critical wave height used is the maximum wave height from Wiegell (1949; 1964). In addition to calibrating the critical wave steepness, the weighting coefficient is also obtained by calibrating the breaker height with the breaker height from the breaker height index equation. The breaker depth obtained is also calibrated to the breaker depth from the breaker depth index.

The breaker length generated from the breaker length index equations has a very wide spread and is much different from the critical wave steepness either from Michell (1893) or Toffoli et al (2010). Therefore, breaker length calibration was not carried out in this study. Only comparisons between the breaker length model and the breaker length from the Miche's (1944) breaker formula were made.

II. VELOCITY POTENTIAL

The complete velocity potential obtained from the solution of the Laplace equation with the separation variable method (Dean, 1991) is:

$$\phi(x,z,t) = G(\cos kx + sin\, kx) coshk(h+z)\, sin\, \sigma\, t$$

.....(1)

k is the wavenumber, in general, k and G are wave constants,σ is the angular frequency, $x$is the horizontal axis at the still water levels, while z is the vertical axis, where $z = 0$at the still water levels.

In this equation, there are two components of velocity potential. They are thecos kx and sin kxcomponents. The constants G and $k$must have the same value on the two components of velocity potential. Therefore, the two constants are determined at a value of $kx$where the two sinusoidal functions are the same. The point is called the characteristic point. The characteristic point on the horizontal $x$axis is the point where the value of cos kx is equal to the value of sin kx. Likewise, the wave amplitude which is also a wave constant is determined at the characteristic point.

At this characteristic point, analysis can be done using only one component of the velocity potential, which in this study is the cos kx .

$\Phi(x,z,t) = G\, cos\, kx\, coshk(h+z)\, sin\, \sigma\, t$....(2)

The use of one component implies that the value of G in (2) is the sum of the components cos kx and sin kx, or that G in (2) has double values.

The water particle velocity in the horizontal x direction is,

$$u(x,z,t) = -\frac{\partial\phi}{\partial x} = Gk\, sin\, kx\, coshk(h+z)\, sin\, \sigma\, t$$

...(3)

The water particle velocity in the vertical z direction is,

$$w(x,z,t) = -\frac{\partial\phi}{\partial z} = -Gk\, cos\, kx\, sinhk(h+z)\, sin\, \sigma\, t$$

...(4)

In the velocity potential equation, there are two conservation equations (Hutahaean, 2021), which are the energy-conservation equation and the wavenumber conservation equation.

a. Energy Conservation Equation

$G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x} = 0$ ....(5)

b. Wavenumber Conservation Equation

$\frac{\partial k\left(h+\frac{A}{2}\right)}{\partial x} = 0$.....(6)

In which$A$is the wave amplitude.

III. WAVE AMPLITUDE FUNCTION

The wave amplitude function was formulated using the Weighted Kinematic Free Surface Boundary Condition (KFSBC) from Hutahaean (2021):

$\gamma\frac{\partial\eta}{\partial t} = w_\eta - u_\eta\frac{\partial\eta}{\partial x}$...(7)

γ is the weighted coefficient that is positive and greater than 1, $\eta = \eta(x,t)$ is the water wave surface elevation equation, w_η is the particle velocity in the vertical direction on the water surface, while u_η is the water particle velocity in the horizontal-x direction. . Substitutions (3) and (4) on (7),

$$\gamma\frac{\partial\eta}{\partial t} = -Gk\, cos\, kx\, sinhk(h+\eta)\, sin\, \sigma\, t$$
$$-Gk\, sin\, kx\, coshk(h+\eta)\, sin\, \sigma\, t\frac{\partial\eta}{\partial x}$$

Forcos $kx\ \neq 0$,

$$\gamma\frac{\partial\eta}{\partial t} = -Gkcoshk(h+\eta)\left(tanhk(h+\eta) + \frac{sin\, kx}{cos\, kx}\frac{\partial\eta}{\partial x}\right)$$

cos $kx\, sin\, \sigma\, t$

For a periodic function,

$$Gkcoshk(h+\eta)\left(tanhk(h+\eta) + \frac{sin\, kx}{cos\, kx}\frac{\partial\eta}{\partial x}\right) = constant$$

This equation is a wave constant, whose value is calculated at the characteristic point, the point where cos $kx =$ sin kx,

$$\gamma\frac{\partial\eta}{\partial t} = -Gkcoshk(h+\eta)\left(tanhk(h+\eta) + \frac{\partial\eta}{\partial x}\right)\cos kx\, sin\, \sigma\, t$$

This equation is integrated to time-t,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma} coshk(h+\eta)\left(tanhk(h+\eta) + \frac{\partial\eta}{\partial x}\right)\cos kx \cos\sigma t$$

The wave amplitude function defined is,

$$A = \frac{Gk}{\gamma\sigma} coshk(h+\eta)\left(tanhk(h+\eta) + \frac{\partial\eta}{\partial x}\right)$$

Given that G is doubled, the right-hand side must be divided by 2,

$$A = \frac{Gk}{2\gamma\sigma} coshk(h+\eta)\left(tanhk(h+\eta) + \frac{\partial\eta}{\partial x}\right)$$

The water level equation becomes,

$\eta(x,t) = A\cos kx \cos\sigma t$ ......(8)

The wave amplitude, which is also a wave constant, must be constant and have the same value as the $\cos\sigma t$ and $\sin\sigma t$functions. Thus, the value of the wave amplitude is determined at the characteristic points x and t where, $\cos kx = \sin kx$ and $\cos\sigma t = \sin\sigma t$.

At this characteristic point,

$\eta = \frac{A}{2}$and$\frac{\partial\eta}{\partial x} = -\frac{kA}{2}$

The equation for the wave amplitude function becomes,

$$A = \frac{Gk}{2\gamma\sigma} coshk\left(h+\frac{A}{2}\right)\left(tanhk\left(h+\frac{A}{2}\right) - \frac{kA}{2}\right)$$

Given that the wave amplitude A is constant, thus:

$$tanhk\left(h+\frac{A}{2}\right) = constant$$

and

$$coshk\left(h+\frac{A}{2}\right) = constant$$

Thus,

$$k\left(h+\frac{A}{2}\right) = constant$$

Defined as,

$k\left(h+\frac{A}{2}\right) = \theta\pi$...(9)

$\theta$is a coefficient whose value will be determined by calibrating the breaker depth to the breaker depth from the breaker depth index. It is defined as:

$c_h = \tanh\theta\pi$...(10)

$\beta = \cosh\theta\pi$ dan $\beta_1 = \sinh\theta\pi$....(11)

Wave amplitude function becomes,

$A == \frac{\beta Gk}{2\gamma\sigma}\left(c_h - \frac{kA}{2}\right)$.....(12)

This equation can be written as an equation for G:

$G = \frac{2\gamma\sigma A}{\beta k\left(c_h - \frac{kA}{2}\right)}$ ......(13)

IV. DISPERSION EQUATION

The dispersion equation is formulated using the weighted Euler surface momentum equation (Hutahaean, 2021):

$\gamma^2\frac{\partial u_\eta}{\partial t} + + \frac{1}{2}\frac{\partial}{\partial x}\left(\gamma u_\eta u_\eta + w_\eta w_\eta\right) = -g\frac{\partial\eta}{\partial x}$....(14)

Substituting u from (3) and w from (4), η of (8) with A from (12), and the obtained equation is worked out at the characteristic point.

$$\gamma^2\sigma + (\gamma\beta - \beta_1 c_h)\frac{1}{2}Gk^2 = \frac{gk}{2\gamma\sigma}\left(c_h - \frac{kA}{2}\right)$$

Then, G is substituted by (13) and the equation is multiplied by$\left(c_h - \frac{kA}{2}\right)$,

$\gamma^2\sigma\left(c_h - \frac{kA}{2}\right) + (\gamma - c_h^2)\gamma\sigma kA = \frac{gk}{2\gamma\sigma}\left(c_h - \frac{kA}{2}\right)^2$....(15)

This equation is a dispersion equation, with the wave amplitude A as the variable whose value is known.

To get the appropriate γ weighted coefficient, Michell's (1893) critical wave steepness criteria are used:

$\frac{H}{L} = 0.142$ ......(16)

The wave amplitude at (15), the maximum wave height at a wave period of Wiegel (1949; 1964) is used:

$H = \frac{gT^2}{15.6^2}$ ....(17)

Assuming a sinusoidal wave, then $A = \frac{H}{2}$. In (17), g is the gravitation (m/sec^2), T wave period (sec). The results of calculations with the value of the weighting coefficient $\gamma = 1.211$ and $\theta = 0.625$ are presented in Table (1), with a critical wave steepness referring to Michell's (1893) criteria.

Table.1: Wavelength and Critical Wave Steepness in Deep Water.

T (sec)	H_0 (m)	L_0 (m)	$\frac{H_0}{L_0}$
6	1.451	10.222	0.142
7	1.975	13.913	0.142
8	2.58	18.172	0.142
9	3.265	22.999	0.142
10	4.031	28.393	0.142
11	4.878	34.356	0.142
12	5.805	40.887	0.142
13	6.812	47.985	0.142
14	7.901	55.651	0.142
15	9.07	63.885	0.142

The wavelength in (15) is the wavelength at deep water h_0 which can be calculated by (9) as follows,

$h_0 = \frac{\theta\pi}{k_0} - \frac{A_0}{2}$.....(18)

Table.2: Deep water depth h_0 and $\frac{h_0}{L_0}$

T (sec)	H_0 (m)	h_0 (m)	$\frac{h_0}{L_0}$
6	1.451	2.831	0.277
7	1.975	3.854	0.277
8	2.58	5.034	0.277
9	3.265	6.371	0.277
10	4.031	7.865	0.277
11	4.878	9.517	0.277
12	5.805	11.326	0.277
13	6.812	13.292	0.277
14	7.901	15.416	0.277
15	9.07	17.697	0.277

A_0 was obtained from (17) while k_0 was obtained from (15). The value of deep water depth for each wave period is calculated by (18), where k_0 is calculated by (15) using the weighting coefficient $\gamma = 1.211$ and $\theta = 0.625$ as depicted in Table (2).

Deep water depth $h_0$in (18) and in Table 2, only shows the water depth limits where the wavenumber or wavelength at water depth greater than $h_0$is a constant value. The wavenumber is affected by water depth at a water depth of less than h_0.

The value of $\frac{h_0}{L_0} = 0.277$, half of the criteria of the Shore Protection Manual (1984) which uses $\frac{h_0}{L_0} = 0.5$.

V. BREAKER HEIGHT, $\frac{H_b}{L_b}$ EQUATION

$\frac{H_b}{L_b}$Equation was formulated using the wave amplitude function (12). The equation is differentiated on the x-horizontal axis for sloping bottom,

$$\frac{\partial A}{\partial x} = \frac{\beta}{2\gamma\sigma}\frac{\partial Gk}{\partial x}\left(c_h - \frac{kA}{2}\right) - \frac{\beta Gk}{4\gamma\sigma}\frac{\partial kA}{\partial x}$$

$\frac{\partial Gk}{\partial x}$was substitude by (5) and$\frac{\partial kA}{\partial x}$elaborated,

$$\frac{\partial A}{\partial x} = \frac{\beta}{2\gamma\sigma}\frac{G}{2}\frac{\partial k}{\partial x}\left(c_h - \frac{kA}{2}\right) - \frac{\beta Gk}{4\gamma\sigma}\left(k\frac{\partial A}{\partial x} + A\frac{\partial k}{\partial x}\right)$$

Breaking occurs in condition$\frac{\partial A}{\partial x} = 0$

$$0 = \frac{\beta}{4\gamma\sigma}G\frac{\partial k}{\partial x}\left(c_h - \frac{kA}{2}\right) - \frac{\beta Gk}{4\gamma\sigma}\left(A\frac{\partial k}{\partial x}\right)$$

The relation between wavenumber and wave amplitude at the breaker point is obtained,

$k_b A_b = \frac{2}{3}c_h$ ......(19)

Or,

$$\frac{H_b}{L_b} = \frac{2}{3\pi}c_h$$

If maximum$c_h$is used,where$c_h = 1$, therefore:

$$\frac{H_b}{L_b} = 0.212$$

It was discovered that$\frac{H_b}{L_b}$is a constant value that applies to all wave periods. (19) is substituted into (15) to get the breaker wave number equation,

$k_b = \frac{3}{gc_h}\gamma^2\sigma^2(2\gamma - c_h^2)$...(20)

In this equation, there is no variable of breaker height H_b or breaker amplitude A_b but the formulation used the relation $\frac{H_b}{L_b}$. Thus, it can be said that in (20), there is an effect of breaker wave amplitude, or there is an interaction between breaker wavenumber and breaker amplitude.

The breaker depth equation can be obtained by using the wavenumber conservation equation, which is,

$$k_b\left(h_b + \frac{A_b}{2}\right) = k_0\left(h_0 + \frac{A_0}{2}\right) = \theta\pi$$

So,

$$k_b\left(h_b + \frac{A_b}{2}\right) = \theta\pi$$

Or,

$h_b = \frac{\theta\pi}{k_b} - \frac{A_b}{2}$ ......(21)

From (19), (20), and (21), it can be said that in the breaking equation obtained there is an interaction among breaker characteristics.

VI. DETERMINATION OF THE VALUES OF γ AND θ

To get the weighting coefficientγ and deep water depth coefficientθ, the breaker height model is calibrated to the breaker height from the breaker height index (BHI) and the breaker depth model is calibrated to the breaker depth from the breaker depth index.

Breaker height comparison used was the mean value of the breaker height of five BHI (Gourlay, 1992; Komar&Gaughan,1972; Kraus, 1989; Smith & Kraus, 1990; Pitikon&Shibayama, 2000). Meanwhile, the breaker depth is compared with the breaker depth from SPM (1984).

Komar and Gaughan (1972)

$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$ ....(22)

Larson and Kraus (1989),

$\frac{H_b}{H_0} = 0.53\left(\frac{H_0}{L_0}\right)^{-0.24}$ ....(23)

Smith and Kraus (1990),

$\frac{H_b}{H_0} = (0.34 + 2.74m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88m}$ ....(24)

Gourlay (1992),

$\frac{H_b}{H_0} = 0.478\left(\frac{H_0}{L_0}\right)^{-0.28}$.........................(25)

Pitikon and Shibayama (2000) :

$$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55)\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$$
.......(26)

$H_0$is deep-water wave height, L_0 is deep water wavelength (calculated using linear wave theory, $k_0 = \frac{\sigma^2}{g}, L_0 = \frac{2\pi}{k_0}$, m is bottom slope and H_b is breaker height.

Equation for breaker depth index from SPM (1984) is:
$\frac{h_b}{H_b} = \frac{1}{b-\left(\frac{aH_b}{gT^2}\right)}orh_b = \frac{H_b}{b-\left(\frac{aH_b}{gT^2}\right)}$ ....(27)

$$a = 43.75(1 - e^{-19.0m})b = \frac{1.56}{1 + e^{-19.5m}}$$

h_b is breaker depth.

For the deep-water wave height H_0 for the calculation input with BHI, the maximum breaker height (17) is used. In Tables (3) and (4), the results of calculations and comparisons between the results of the model and the results of BHI are presented, where the model used were$\gamma = 1.201$and $\theta = 0.623$.

Table.3: The Comparison ofBreaker Height H_bModel andH$_b$ BHI.

T (sec)	H_0 (m)	H_b (m)	
		Model	BHI
6	1.451	1.721	1.721
7	1.975	2.343	2.343
8	2.58	3.06	3.06
9	3.265	3.872	3.873
10	4.031	4.781	4.781
11	4.878	5.785	5.785
12	5.805	6.884	6.885
13	6.812	8.079	8.08
14	7.901	9.37	9.371
15	9.07	10.757	10.757

In Table (4), the breaker depth index of $\frac{H_b}{h_b}$ model is compatible with $\frac{H_b}{h_b}$ of SPM, and also in accordance with Mc Cowan's equation (1894):

$\frac{H_b}{h_b} = 0.78$ ......(28)

Table.4: The Comparison of breaker depth h_bModel andh_b SPM.

T (sec)	h_b (m)		$\frac{H_b}{h_b}$	
	Model	SPM	model	SPM
6	2.199	2.207	0.783	0.78
7	2.993	3.003	0.783	0.78
8	3.909	3.923	0.783	0.78
9	4.948	4.965	0.783	0.78
10	6.108	6.129	0.783	0.78
11	7.391	7.417	0.783	0.78
12	8.796	8.826	0.783	0.78
13	10.323	10.359	0.783	0.78
14	11.972	12.014	0.783	0.78
15	13.743	13.791	0.783	0.78

There is a relatively small difference in the value of γ. In deep water, the value of $\gamma = 1.211$ is obtained by calibrating to Michell's (1893) criteria. While the value of $\gamma = 1.201$is obtained the calibration of breaker height and breaker depth. However, this difference is also caused by the conditions of the Michell's criteria (1893) and the conditions of the breaker height index. If Michell's criteria are considered more accurate, $\gamma = 1.211$can be used, while if the breaker height index is more accurate, $\gamma = 1.201$ can be used. However, with this relatively small difference, the mean value of the two can be used, which slightly increasing the Michell's criteria and lowering the breaker height.

VII. THE COMPARISON OF BREAKER LENGTH

There are a number of breaker length indexes which have almost the same basic form. They are:

a. Miche (1944)

$$\frac{H_b}{L_b} = 0.142 \tanh\left(\frac{2\pi h_b}{L_b}\right) \qquad ......(29)$$

b. Battjes and Jansen (19878)

$$\frac{H_b}{L_b} = 0.142 \tanh\left(\frac{0.8}{0.88}\frac{2\pi h_b}{L_b}\right).....(30)$$

c. Battjes and Stive (1985)

$$\frac{H_b}{L_b} = 0.14 \tanh\left(\left(0.5 + 0.4 \tanh 33\frac{H_0}{L_0}\right)\frac{2\pi h_b}{0.88 L_b}\right)$$

.......(31)

d. Kamphuis (1991)

$$\frac{H_b}{L_b} = 0.127\, exp(4m) \tanh\left(\frac{2\pi h_b}{L_b}\right)......(32)$$

e. Rattanapitikon and Shibayama (2000)

$$\frac{H_b}{L_b} = 0.14 \tanh\left((-11.21\, m^2 + 5.01\, m + 0.91\,)\frac{2\pi h_b}{L_b}\right)$$

.....(33)

In the Breaker Length Index equations, m is the bottom slope. In (31), H_0 is the deep-water wave height. While L_0 is the deep-water wave length of the linear wave theory dispersion equation of $k = \frac{\sigma^2}{g}$.

The breaker height H_b was calculated using the Breaker Height Index. The breaker depth is obtained using (28). With the input breaker height H_b and breaker depth h_b, the breaker length can be calculated by (29) – (33).

Table.5: Breaker Length Index value $\frac{H_b}{L_b}$ from (29)-(30)

T (sec)	$\frac{H_b}{L_b}$ (29)	(30)	(31)	(32)	(33)
6	0.083	0.038	0.087	0.03	0.036
7	0.083	0.038	0.087	0.03	0.036
8	0.083	0.038	0.087	0.03	0.036
9	0.083	0.038	0.087	0.03	0.036
10	0.083	0.038	0.087	0.03	0.036
11	0.083	0.038	0.087	0.03	0.036
12	0.083	0.038	0.087	0.03	0.036
13	0.083	0.038	0.087	0.03	0.036
14	0.083	0.038	0.087	0.03	0.036
15	0.083	0.038	0.087	0.03	0.036

The results of the calculation of $\frac{H_b}{L_b}$ with a number of breaker length index equations vary in value, with a very widespread. In addition, it is also much different from the critical wave steepness criteria both from Michell (1893) and Toffoli et al. (2010) where the criteria of Toffoli et al. (2010):

$$\frac{H_b}{L_b} = 0.170.....(34)$$

Therefore, the value of the weighting coefficient γ and deep water coefficient θ in this study were adjusted to the breaker height index and breaker depth index only.

Comparison of breaker length model with $\gamma = 1.201$ and $\theta = 0.623$, with breaker length (29), is presented in Table (6). Where the breaker length of the model is much shorter with the breaker length index index $\frac{H_b}{L_b}$ also much bigger.

Table.6: The Comparison ofBreaker Length Model andMiche'sBreaker Length

T (sec)	L_b (m) model	L_b (m) Miche	$\frac{H_b}{L_b}$ model	$\frac{H_b}{L_b}$ Miche
6	8.44	20.802	0.204	0.083
7	11.488	28.314	0.204	0.083
8	15.005	36.982	0.204	0.083
9	18.991	46.805	0.204	0.083
10	23.446	57.785	0.204	0.083
11	28.369	69.919	0.204	0.083
12	33.762	83.21	0.204	0.083
13	39.623	97.656	0.204	0.083
14	45.954	113.258	0.204	0.083
15	52.753	130.015	0.204	0.083

The breaker length index of the model is also much greater than the critical wave steepness criteria of Michel (1893) and Toffoli et al. (2010). This is due to the wave energy compression when the waves move from deeper waters to shallower waters making the waves are stronger.

VIII. CONCLUSION

The equations for breaking characteristics in this study were formulated based on hydrodynamic equations, continuity equations, momentum equations, and conservation equations attached to the velocity potential, which were the energy conservation equation and the wavenumber conservation equation. Hence,first, it can be concluded that the obtained equations satisfy the conservation laws of fluid dynamics.

The equations formulated are in line with the previous researchers in terms of breaker height and breaker depth. Considering that the breaker height is determined by the breaker depth and the breaker depth is determined by the breaker height and breaker length, it can be concluded that the breaker length formulated is appropriate or accurate. Thus, in general, it can be concluded that the breaking equations formulated can provide good information regarding the condition of the breaking wave characteristic.

There are still physical parameters that have not been included as a variable in the breaking equations obtained, which is the bottom slope. The next research is expected to develop a breaking equation involving bottom slope as the variable.

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