Acoustic analysis and Modeling of the Group and phase Velocities of an Acoustic circumferential waves by an Adaptative Neuro-Fuzzy Inference System (ANFIS)

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Abstract— In this work, an Adaptative Neuro-Fuzzy Inference System (ANFIS) is applied to predict the velocity dispersion curves of the antisymmetric (A₁) circumferential waves propagating around an elastic copper cylindrical shell of various radius ratio b/a (a: outer radius and b: inner radius) for an infinite length cylindrical shell excited perpendicularly to its axis. The group and phase velocities, are determined from the values calculated using the eigenmode theory of resonances. These data are used to train and to test the performances of these models. This technique is able to model and to predict the group and phase velocities, of the anti-symmetric circumferential waves, with a high precision, based on different estimation errors such as mean relative error (MRE), mean absolute error (MAE) and standard error (SE). A good agreement is obtained between the output values predicted using ANFIS model and those computed by the eigenmode theory. It is found that the ANFIS networks are good tools for simulation and prediction of some parameters that carry most of the information available from the response of the shell. Such parameters may be found from the velocity dispersion of the anti-symmetric circumferential waves A₁ propagating around the cooper cylindrical shell of different radius ratio b/a, since many studies, experimental and theoretical showed that acoustic resonances of a cylindrical shell are related to its physical and geometrical properties[1-18]. In the phase to train and test, the group velocity obtained from the values calculated using the eigenmode theory of resonances is used as parameters in ANFIS networks. It is possible to recognize an unknown cylindrical target detected experimentally with an Adaptative neuro-fuzzy Inference System network technique, This model use neuro-adaptive learning techniques, which are similar to those of neural networks [6]. An understanding of the acoustic scattering is used to study wave dispersion. These studies allow us to redraw the phase velocity and group velocity of circumferential waves propagating around a tube which is still a complicated task experimentally. During this work that focus on the prediction of group velocity that characterize submerged tubes, the ANFIS approach has shown good range characterization and computational efficiency. Its robustness, speed and accuracy of its

I. INTRODUCTION

Several theoretical and experimental studies show that there is a generation of circumferential waves in the shell and in the water/shell interface when an air-filled tube immersed in water is excited by a plane acoustic wave perpendicularly to its axis. These circumferential waves are two types that are equivalent to the Lamb waves on a plate: the antisymmetric (A₁) and symmetric (S₁) circumferential waves (i = 0, 1, 2, . . . : index of wave) [1]. For some frequencies, these circumferential waves form standing waves on the circumference of the tube constituting resonances. These resonances are observed on the spectrum of the acoustic pressure backscattered by the tube [2-5]. For a tube made in a given material, the resonance dimensionless frequencies of these waves essentially depend on the radius ratio b/a (a, outer radius; b, inner radius of tube). One of the most precious points for the characterization of an elastic, air-filled tube immersed in water can be made from some parameters that carry most of the information available from the response of the shell. Such parameters may be found from the velocity dispersion of the anti-symmetric circumferential waves A₁ propagating around the cooper cylindrical shell of different radius ratio b/a, since many studies, experimental and theoretical showed that acoustic resonances of a cylindrical shell are related to its physical and geometrical properties[1-18]. In the phase to train and test, the group velocity obtained from the values calculated using the eigenmode theory of resonances is used as parameters in ANFIS networks. It is possible to recognize an unknown cylindrical target detected experimentally with an Adaptative neuro-fuzzy Inference System network technique, This model use neuro-adaptive learning techniques, which are similar to those of neural networks [6]. An understanding of the acoustic scattering is used to study wave dispersion. These studies allow us to redraw the phase velocity and group velocity of circumferential waves propagating around a tube which is still a complicated task experimentally. During this work that focus on the prediction of group velocity that characterize submerged tubes, the ANFIS approach has shown good range characterization and computational efficiency. Its robustness, speed and accuracy of its
outputs enable it to give correct decisions and avoid cases of indecision. ANFIS with their ability to adapt to unknown situations through learning to model imprecise knowledge and uncertainty management. But it depends strongly on the data used to train the ANFIS network [7, 19]. In this study, and without seeking to establish a mathematical equation which sometimes remains a difficult task for this purpose, ANFIS model is developed to predict the phase and group velocity of antisymmetric wave $A_1$. Several models of ANFIS have been tested, and to evaluate the performance of these models, a comparative study between the proposed model and the theoretical method was performed. It shows a good agreement between the values predicted by ANFIS model and those calculated by the theoretical method.

II. THEORETICAL STUDY

A. Acoustic backscattering from a cylindrical shell

The scattering of an infinite plane wave by an air-filled cylindrical shell of radius ratio $b/a$ is investigated through the solution of the wave equation and the associated boundary conditions. Fig.1 shows the cylindrical coordinate orientation and the direction of a plane wave incident on an infinitely long cylindrical shell in a fluid medium. The fluid (1) outside the shell has a density of $\rho_1$ and the acoustic propagation velocity $C_1$. In general, the inner fluid (2) will be different and is described by the parameters $\rho_3$ and $C_3$. The parameters for the two fluids outside and inside the shell are given in Table 1.

The axis of the cylindrical shell is taken to be the z-axis of the cylindrical coordinate system $(r, \theta, z)$. Let a plane wave incident on an infinite cylindrical shell with air-filled cavity (fluid 2), be submerged in water (fluid 1), see figure 1.

The backscattered complex pressure $P_{\text{diff}}$ from a cylindrical shell in a far field ($r >> a$) is the summation of the incident wave, the reflective wave $\circ$, surface waves shell waves $\mathbb{Q}$, the symmetric waves $S_0, S_1, S_2, \ldots$, and the antisymmetric waves $A_0, A_1, A_2, \ldots$) and interface Scholte waves (A) $\mathfrak{O}$. The waves $\mathbb{Q}$ and $\mathfrak{O}$ are the circumferential waves connected to the geometry of the object (Fig. 2).

The module of the normalized backscattered complex pressure in a far field is called form function. This function is obtained by the relation [2-18]:

$$|P_{\text{diff}}(\omega)| = \frac{2}{\sqrt{\pi kr}} \sum_{n=0}^{\infty} \varepsilon_n (-1)^n \frac{D_n^{(1)}(\omega)}{D_n(\omega)}$$

where $\varepsilon_n$ is the Neumann factor ($\varepsilon_n = 1$, if $n = 0$; $\varepsilon_n = 2$, if $n > 0$).

Table 1: Physical parameters

<table>
<thead>
<tr>
<th></th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Longitudinal velocity $c_L$ (m/s)</th>
<th>Transverse Velocity $c_T$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooper</td>
<td>8930</td>
<td>4760</td>
<td>2325</td>
</tr>
<tr>
<td>Water  (fluid 1)</td>
<td>1000</td>
<td>1470</td>
<td>---------</td>
</tr>
<tr>
<td>Air    (fluid 2)</td>
<td>1.29</td>
<td>334</td>
<td>---------</td>
</tr>
</tbody>
</table>

Fig. 1: Geometry used for formulating the sound backscattering from a cylindrical shell

The physical parameters used in the calculation of the backscattered complex pressure are illustrated in table 1.

Fig. 2 Mechanisms of the formation of echoes showing the specular reflection $\circ$ and shell waves $\mathbb{Q}$ and Scholte wave (A) $\mathfrak{O}$.
Figure 3 shows the module of the normalized backscattered complex pressure as function of the reduced frequency $ka$ (without unit) given by:

$$ka = \frac{2\pi f d}{c(1 - \frac{b}{a})}$$  \hspace{0.5cm} (3)

where $d=a-b$ is the thickness of a cylindrical shell and $f$ is the frequency of the incidence wave in Hz.

The time signal $P(t)$ of a cylindrical shell is computed from the Inverse of Fourier Transform of the backscattered complex pressure:

$$P(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} h(\omega) P_{\text{diff}}(\omega) e^{-i\omega t} d\omega$$  \hspace{0.5cm} (4)

where $h(\omega)$ is a smoothing window.

The sharp transitions of shell (corresponding to frequencies of resonances) in the spectrum of Fig 3 are connected with the propagation of acoustic circumferential waves: Scholte wave (A) and shell waves ($S_0$, $A_1$, $S_1$, $S_2$, $A_2$ ...). The time signal backscattered by a cooper cylindrical shell is from the module of the Inverse Fourier Transform of the backscattered complex pressure using the equation (4). Figure 4 presents this time signal. It is constituted by the specular reflection echo (large amplitude and short duration) and several wave packets associated with different circumferential waves ($A$, $S_0$, $A_1$, $S_1$, $S_2$, $A_2$ ...).

The observation of this signal shows a succession of components more or less distinct that one seeks then to identify. The different echoes finish by overlapping and in these conditions, the identifications and the measures of arrival times of echoes (this time depends on the radii of the tube a and b) become difficult, perhaps impossible. This constitutes a major disadvantage of the temporal approach. An important feature of the acoustic circumferential waves is the velocity dispersions that lead to determine the scattering time of wave packets.

### III. MATERIALS AND METHOD

#### A. Fuzzy Inference System

Fuzzy logic is an extension of Boolean logic that allows intermediate values between “True” and “False”. In this approach the classical theory of binary membership in a set, is modified to incorporate the memberships between “0” and “1”. The fuzzy models are means of capturing human’s expert knowledge about the process, in terms of fuzzy (if–then) rules. The fuzzy inference system (FIS) can initialize and learn linguistic and semi-linguistic rules; hence it can be considered as direct transfer knowledge, which is the main advantage of fuzzy inference systems over classical learning systems and Neural Networks [19-21]. Often the rules of the fuzzy system are designated a priori and the parameters of the membership functions are adapted in the learning process from input–output data sets.
Basically, a fuzzy inference system is composed of five functional blocks, shown in Fig.10, as follows [19-21]:

(i) A rule base containing a number of fuzzy if–then rules. All the uncertainties, non linear relationships, or model complications are included in the descriptive fuzzy inference procedure in the form of if–then statements. In general, a fuzzy if–then rule has two constitutes; first the if part and the second the then part; which are called premise and consequent, respectively. The general form of a fuzzy if–then rule is as follows; Rule: if Z is A then f is B.

(ii) A database, which defines the membership functions of the fuzzy sets used in the fuzzy rules.

(iii) A decision-making unit, which performs the inference operations on the rules.

(iv) A fuzzification inference, which transforms the crisp inputs into degree of match with linguistic values.

(v) A defuzzification inference, which transforms the fuzzy results of the inference into a crisp output.

Several types of FIS have been proposed in the literature [22], which, vary due to differences between the specification of the consequent part and the defuzzification schemes. This paper incorporates one of these types, the so-called Takagi and Sugeno FIS [23], to propose a systematic scheme for the development of fuzzy rules using the input/output data sets.

A typical fuzzy rule in a sugeno fuzzy model has the format:

If x is A and y is B then z = f(x, y)

where A and B are fuzzy sets in the antecedent; z =f(x, y) is a crisp function in the consequent. Usually f(x, y) is a polynomial in the input variable x and y, but it can be any other functions that can appropriately describe the output of the system within the fuzzy region specified by the antecedent of the rule. When f(x, y) is a first order polynomial, we have the first-order sugeno fuzzy model. When f is a constant, we then have the zero-order Sugeno fuzzy model. Consider first-order Sugeno fuzzy inference systems which contain two rules:

Rule 1: if \( x \) is \( A_1 \) and \( y \) is \( B_1 \) then \( f_1 = p_1 x + q_1 y + r_1 \)

Rule 2: if \( x \) is \( A_2 \) and \( y \) is \( B_2 \) then \( f_2 = p_2 x + q_2 y + r_2 \)

Weighted averages are used in order to avoid complexity in deffuzification processes. Fig.8 illustrates graphically the fuzzy reasoning mechanism to derive an output \( f \) from a given input vector \((x, y)\). The firing strengths \( \alpha_1 \) and \( \alpha_2 \) are usually obtained as the product of the membership grades in the premise part, and the output \( f \) is the weighted average of each rule’s output. To facility the learning of the sugeno fuzzy model, into the framework of adaptative networks we can compute gradient vectors systematically. The resultant network architecture is called Adaptive Neuro Fuzzy Inference system (ANFIS).

B. Adaptive neuro-fuzzy inference system architecture

The Adaptive Network-based Fuzzy Inference System (ANFIS) is developed by Jang in 1993 [19]. This model use neuro-adaptive learning techniques, which are similar to those of neural networks. Given an input/output data set, the ANFIS can construct a Fuzzy Inference System whose membership function parameters were adjusted.
Layer 0: It consists of plain input variable set.

Layer 1: It is the fuzzy layer. Each node in this layer generates a membership grade of a linguistic label. For instance, the node function of the ith node may be generalized bell membership function:

\[ \mu_{A_i} = \frac{1}{1 + \left( \frac{x - c_i}{a_i} \right)^b} \]  

(10)

where \( x \) is the input to node \( i \); \( A_i \) is the linguistic label (small, large, etc.) associated with this node; and \( \{a_i, b, c_i\} \) is the parameter set that changes the shapes of the membership function. Parameters in this layer are referred to as the premise parameters.

Layer 2: The function is T-norm operator that performs the firing strength of the rule, e.g., fuzzy conjunctive AND and OR. The simplest implementation just calculates the product of all incoming signals.

\[ \omega_i = \mu_{A_i}(x) \mu_{B_i}(y) \quad , i=1, 2 \]  

(11)

Layer 3: Every node in this layer is fixed and determines a normalized firing strength. It calculates the ratio of the firing strength of the jth rule’s firing strength to the sum of all rules firing strength.

\[ \sigma_i = \frac{\omega_i}{\sum_{j=1}^{2} \omega_j} \quad , i=1,2 \]  

(12)

Layer 4: The nodes in this layer are adaptive and are connected with the input nodes and the preceding node of layer 3. The result is the weighted output of the rule j.

\[ \omega_j f_j = \omega_j (r_p x + q_j y + r_j) \]  

(13)

where \( \omega_j f_j \) is the output of layer 3 and \( \{p_j, q_j, r_j\} \) is the parameter set. Parameters in this layer are referred to as the consequent parameters.

Layer 5: This layer consists of one single node which computes the overall output as the summation of all incoming signals.

\[ \text{Overall Output} \sum_j \sigma_j f_j = \frac{\sum_i \omega_i f_i}{\sum_i \omega_i} \]  

(14)

The constructed adaptive network in figure 9 is functionally equivalent to a fuzzy inference system in figure 7. The basic learning rule of ANFIS is a combination of last square error and the back-propagation gradient descent, which calculates error signals (the derivative of the squared error with respect to each node’s output) recursively from the output layer backward to the input nodes. This learning rule is exactly the same as the back-propagation learning rule used in the common feed-forward neural networks.

IV. COLLECTION OF DATA

The conception of the ANFIS model requires the determination of the relevant entries that have a significant influence on the required model. In this work, a data base is collected to involve and test the performance of these models starting from the results obtained by the proper modes theory of the circumferential waves. The density of material, the radius ratio, the index of the anti-symmetric circumferential waves, and longitudinal and transverse velocities, of the material constituting the cylindrical shell, are retained like relevant entries of the model because these parameters characterize the cylindrical shell and the types of circumferential waves propagating around this one. The phase and the group velocity, of the anti-symmetric circumferential waves (\( A_i, i=1, 2 \)) for a cooper cylindrical shell with different radius ratios b/a, constitutes the output of ANFIS. The collected data for the training and
Validation phases of these models are represented in tables II and III. For example, for cooper cylindrical shell, the density is 8930 kg/m³, the transverse velocity is 2325 m/s and the longitudinal velocity is 4760 m/s. For the anti-symmetric circumferential wave A₁, the group velocity predicted by ANFIS is 2645.8 for a frequency equal 24.87 and a radius ratio b/a equal to 0.9.

ANFIS networks method requires for its training a dataset of phase and group velocities calculated by the analytical method or obtained by experiments. This dataset is divided into two sets. The first 2/3 training data set was used for training the ANFIS while the remaining 1/3 checking data set were used for validating the identified model, the number of membership function is fixed to 16 MF, so the rule number is 16. The ANFIS used here contains a total number of parameters: 64, of which number of linear parameters: 32 and number of nonlinear parameters: 32. The desired and predicted values for both training data and checking data are essentially the same in fig 13-16.

V. RESULTS AND DISCUSSION

The performance of ANFIS model for a set of data in both training and testing phases were evaluated according to statistical criteria such as the correlation coefficient R, MAE, MRE, SE, and the root error root mean square (RMSE). The selection of different models is performed based comparison of different error MAE, MRE, SE and RMSE between the values of velocities predicted by ANFIS and those desired. The correlation coefficient R and R² of the linear regression of the determination are used as measures of the performance of the model between the predicted values and the desired values. The various measures of error and the correlation coefficient are given by the following relationships:

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |D_i - P_i| \tag{16}
\]

\[
MRE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{D_i - P_i}{D_i} \right| \tag{17}
\]

\[
R = 1 - \frac{\sum_{i=1}^{n} (D_i - P_i)^2}{\sum_{i=1}^{n} (D_i - P_m)^2} \tag{18}
\]

\[
SE = \sqrt{\frac{\sum_{i=1}^{n} (D_i - P_i)^2}{n-1}} \tag{19}
\]

where n is the number of data, Pᵢ and Dᵢ is the predicted and desired of phase and group velocities respectively and Pₘ is the mean of predicted values.

The correlation coefficient is a statistical criterion commonly used and which provides information about the strength of the linear relationship between the observed and the calculated values. The performance of ANFIS models of the training and test data are represented in Fig. 13-16.

The analysis is repeated several times. Indeed, the error values are measured for each ANFIS architecture based on the number of rules and the type of the membership function used.

![Fig.10: Errors for the prediction of the group velocity with different ANFIS configuration.](image)

In this work, we tried to change the number of rules and the number of epoch; we found that the error of our models values decrease more than the number of rules, and the number of times is increased. The results of the measured errors shown in Figures 10, 11 and 12 for the circumferential wave A₁. Tables II and III show that the results obtained by ANFIS method is good agreement with those determined from the results calculated using the theory of natural modes of resonance.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>calculated by the TM</th>
<th>Predicted by ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>931.29</td>
<td>927.12</td>
</tr>
<tr>
<td>14.427</td>
<td>2470.6</td>
<td>2470.8</td>
</tr>
<tr>
<td>18.47</td>
<td>2798.7</td>
<td>2798.7</td>
</tr>
<tr>
<td>20.08</td>
<td>2794.1</td>
<td>2794</td>
</tr>
<tr>
<td>24.13</td>
<td>2664</td>
<td>2664</td>
</tr>
<tr>
<td>28.17</td>
<td>2234.1</td>
<td>2234.1</td>
</tr>
<tr>
<td>32.22</td>
<td>1794.3</td>
<td>1794.3</td>
</tr>
<tr>
<td>37.88</td>
<td>1745.3</td>
<td>1745.3</td>
</tr>
</tbody>
</table>

Table II: phase velocity of Antisymmetric Waves for the cooper tube (b/a=0.9)
Table III: group velocity of Antisymmetric Waves for the cooper tube (b/a=0.9)

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>values of the group velocity Vg(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>calculated by the TM</td>
</tr>
<tr>
<td></td>
<td>Predicted by ANFIS</td>
</tr>
<tr>
<td>11.73</td>
<td>411.55</td>
</tr>
<tr>
<td>13.57</td>
<td>2175.9</td>
</tr>
<tr>
<td>20.57</td>
<td>2763.4</td>
</tr>
<tr>
<td>24.87</td>
<td>2645.9</td>
</tr>
<tr>
<td>28.31</td>
<td>2234.3</td>
</tr>
<tr>
<td>30.89</td>
<td>1822.8</td>
</tr>
<tr>
<td>35.37</td>
<td>1705.3</td>
</tr>
<tr>
<td>38.42</td>
<td>1763.7</td>
</tr>
</tbody>
</table>

The results of individual measurements of the error and the correlation coefficient (MRE, MAE, SE and R) are given in Table IV. And also are illustrated in Fig. 13-16. Thus, it is advantageous to use the ANFIS approach. The best configuration is found for a network with 16 rules.

Fig.11: Evolution of errors of training and testing as a function of the number of epoch for an ANFIS with 16 rules.

Fig.12: correlation between the group velocity calculated by the theoretical method and that predicted by ANFIS

Table IV: results of the different error measures and the coefficient of correlation (MRE, MAE, SE and R) with 16 rules

<table>
<thead>
<tr>
<th>Error measures</th>
<th>Phase velocity Vph</th>
<th>Group velocity Vg</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>ANFIS</td>
<td>ANFIS</td>
</tr>
<tr>
<td>MRE</td>
<td>5.9.10^{-4}</td>
<td>3.4.10^{-6}</td>
</tr>
<tr>
<td>SE</td>
<td>0.3</td>
<td>0.01</td>
</tr>
<tr>
<td>R</td>
<td>0.99</td>
<td>1</td>
</tr>
</tbody>
</table>

The predicted values are plotted against the desired values in Figures 12. The results show good agreement between the predicted values and the desired values of the group velocity and group. The coefficient of determination $R^2$ of this optimal configuration is 1 Fig. 13-16 shows that the phase velocity decreases as the frequencies increase, against by the group velocity increases with the frequency up to a maximum point then decrease from the value of the frequency approximately 20kHz. The evolution of the mean square error (RMSE) of the training during the training and testing phase as a function of the number of epoch is shown in figure 11.
VI. CONCLUSION

In this work, we presented a strategy of hybridization between two techniques: neural networks and fuzzy logic to develop a system for modeling some parameters characterizing the signals backscattered by a cylindrical shell as phase and group velocities of antisymmetric circumferential waves. This technique can be considered as simple and flexible tools that adapt to data modeling rather than seeking to establish mathematical equations that require more time or may be sometimes difficult to establish. During this work that focus on the modeling and prediction of group and phase velocity that characterize submerged tubes, the ANFIS approach has shown its effectiveness. The use of neuro-fuzzy approach (ANFIS) allowed automatic generation of fuzzy rules. According to the results, we can conclude that the neuro-fuzzy system shows a good range characterization and computational efficiency. Its robustness, speed and accuracy of its outputs enable it to give correct decisions and avoid cases of indecision. The use of the ANFIS does not present any approximation as in the case of the natural modes method which assimilates the tubes to the plates with the same thickness and that is not sullied with errors as in the case of the time-frequency representations of Wigner-Ville that determines the group velocity manually starting from the time-frequency image [9]. This article can be used as a new tool for characterization of an elastic tube. This method allows one to determine automatically and with good precision the group velocity of an antisymmetric wave propagating around the tube. The $R^2$ value in figure is about 1, which can be considered as very satisfactory.

The results obtained in our work encourage further research in this direction, we can also consider improving. This work does not seek to condemn conventional methods! The approach presented is primarily enriched the family of methods for modeling and prediction of physical processes.

REFERENCES


