

Operational Solution to Economic Load Dispatch (ELD) of power plants by different deterministic methods and Particle Swarm Optimization.

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Abstract—Decision-making for operational optimization Economic Load Dispatch (ELD) is one of the most important tasks in thermal power plants, which provides an economic condition for power generation systems. The aim of this paper is to analyze the application of evolutionary computational methods to determine the best situation of generation of the different units in a plant so that the total cost of fuel to be minimal and at the same time, ensuring that demand and total losses any time be equal to the total power generated. Various traditional methods have been developed for solving the Economic Load Dispatch, among them: lambda iteration, the gradient method, the Newton's method, and so others. They allow determining the ideal combination of output power of all generating units in order to meet the required demand without violation of the generators restrictions. This article presents an analysis of different mathematical methods to solve the problem of optimization in ELD. The results show a case study applied in a thermal power plant with 10 generating units considering the loss of power and its restrictions, using MATLAB tools by developed techniques with particle swarm algorithm.

Keywords —Particle Swarm Algorithm, Load Dispatch, Mathematical Methods, Thermal Power Plants.

I. INTRODUCTION

The Economic Load Dispatch Problem is to minimize the total cost and at the same time to guarantee the power plant demand. Thus, the classic problem of economic dispatch is to provide the required amount of power at the lowest possible cost[1], taking into account the load and operation restrictions.

Because of its massive size, this problem becomes very complex for solving, because it contains a non-linear objective function, and a large number of constraints.

Several techniques such as the integer programming[2, 3], the dynamic programming [4, 5], and the Lagrange functions[6]have been applied to solve the economic dispatch problem.

Other optimization methods such as "Differential evolution based on truncated Levy-type flights and population diversity measure[7], Artificial Immune algorithm [8], Harmonic differential search algorithms[9], Oppositional invasive weed optimization [10], Neural Networks[11-13], Genetic Algorithms[14, 15], and Real Coded Chemical Reaction algorithm[16], are also used to solve the economic dispatch optimization problem. Methods based on mathematical approaches have also been developed to provide a faster solution [17, 18].The multi-objective evolutionary algorithms (MOEA) [19, 20]were applied to solve the ELD problem.

Research papers included emission constrains on economic dispatch and selection of machines, but only focused in cost minimization [21, 22].Recently, in order to use the most appropriate numerical methods for solving the ELD problems, modern optimization techniques[23-26] have been successfully employed to resolve the ELD as a non-smooth optimization problem.

In [27]is presented a particle swarm optimization model (PSO) with an aging leader and challengers (ALC-PSO) to solve the optimization problem of the reactive power.

According to [28] the convexity of the optimal load dispatch problem makes it difficult to guarantee the global optimum. In [29] an evolutionary algorithm named "Cuckoo Search algorithm" was applied to not convex economic load dispatch problems.

In [30]it is presented a new hybrid algorithm that combines the Firefly Algorithm (FA) and Nelder Mead (NM) simplex method to solve Optimal Reactive Power

Dispatch problems (ORPD).The program is developed in Matlab and the proposed hybrid algorithm is examined in two IEEE standard test systems to solve ORPD problems.

A methodology to solve the economic load dispatch problem (ELD), considering the generation of reliability uncertainty of wind power generators is presented in [31].

The corresponding probability distribution function (PDF) of available wind power generation is discretized and introduced into the optimization problem in order to describe probabilistically the power generation of each thermal unit, the limitations of wind energy, ENS (energy not supplied), the excess of power generation, and the total cost of generation. The proposed method is compared to the Monte Carlo Simulation (MCS) approach, being able to reproduce the PDF in a reasonable way, especially when system reliability is not taken into account.

Comparing the different demand response strategies, using heuristics and linear programming, it was found that the one that minimizes the daily operation costs is the linear programming model, although it presents the highest increase in energy demand. [32].

In [33] it is presented a new variant of the optimization algorithm called "teaching-learning-based optimization (TLBO)", the authors called this new algorithm "Gaussian bare-bones TLBO (GBTLBO)" and in addition they make a modified version of the same (MGBTLBO) for optimal reactive power dispatch (ORPD) with discrete and continuous variables.

According [34]the dynamic economic dispatch (DED) is one of the most complicated nonlinear problems showing the non-convex characteristic in energy systems.

This is due to the "valve point" effect in cost functions for the generating units, to the velocity gradient limits and transmission losses.

The proposal of an effective method of solution for this optimization problem is of great interest, and the solution of Economic Dispatch (ED) problems mainly depends on the modeling of the thermal generators [35].

Physical changes such as generators aging and environmental temperature affect the modeling parameters and are unavoidable. Because these parameters are the backbone of the ED solution, periodic estimation of these characteristic coefficients is required for a precise economical load dispatch.

II. MATERIALS AND METHODS

1.1. EconomicLoadDispatch.

The economic load dispatch problem is to minimize the overall cost rate and meet the demand load of a power system. The classic economic load dispatch problem is intended to provide the necessary amount of energy at a lower cost possible[1]. The dispatch problem can be

stated mathematically as follows: To minimize the total cost of fuel for thermal plants[29, 36-39]:

$$Min = F_{P_i} \sum_{i=1}^n (a_i + b_i P_i + c_i P_i^2) \quad (1)$$

The previous expression depends on the equality of constraints balance of real power.

1.2. Economic Load Dispatch taking into account the "valve point" effect.

The cost function of a fossil fuel generating unit is obtained from data points taken during the unit "run" tests when the input and output data are measured as the unit is varying slowly through its operating area. In the case of steam turbines these effects occur each time that the intake valve in a steam turbine begins to open, and produce a ripple effect on the unit power versus consumption curve.

The generating units based on multivalve steam turbines are characterized by a complex nonlinear function of the fuel cost. This is mainly due to the induced load ripples produced by the valve throttling or valve point. To simulate this complex phenomenon, a sinusoidal component is imposed on supplies quadratic curve of the engines.

In fact, a sharp increase in the loss of fuel is added to fuel cost curve due to the throttling effect when the steam inlet valve begins to open or close. This procedure is named as valve point. To model the effects of valve-point, a rectified sine function is added to the quadratic one [10, 40], as is showed in Figure 1.

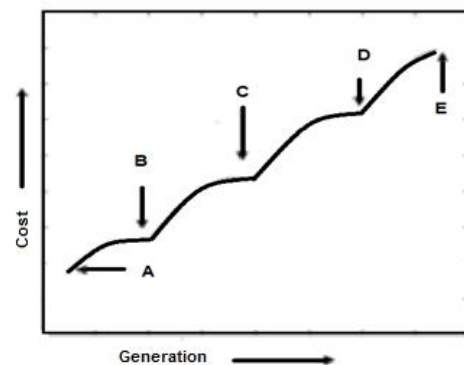


Fig. 1. Cost function taking into account the valve point effect with 5 valves. A: Primary valve, B: Secondary valve, C: Tertiary valve, D: Quaternary valve, E: Quinary valve.

Source: [40].

The cost expression taking into account the valve point effect can be expressed as[36, 41, 42]:

$$F_i(P_{i,t}) = a_i P_{i,t}^2 + b_i P_{i,t} + c_i + \left| e_i \sin \left(f_i (P_{i,t}^{min} - P_{i,t}) \right) \right| \left(\frac{\$}{h} \right) \quad (2)$$

Where a_i , b_i and c_i are the fuel cost coefficients of the (ith) generating unit, and e_i and f_i are the fuel cost

coefficients of the (ith) generating unit, but taking into account the valve point effect.

In [40, 43] it is established that the fuel cost function of each heat generating unit taking into account the valve point effects is expressed as the sum of a quadratic function and a sine function. The total fuel cost then can be expressed as

$$F_c = \sum_{m=1}^M \sum_{i=1}^N t_m [a_i + b_i P_{im} + c_i P_{im}^2 + |d_i \sin\{e_i(P_i^{min} - P_{i,t})\}|] \quad (\$/h) \quad (3)$$

Where a_i, b_i and c_i are the fuel cost coefficients of the (ith) generating unit, and d_i and e_i are the fuel cost coefficients of the (ith) generating unit, but taking into account the valve point effect.

1.3. Economic Load Dispatch Constrains.

Some constrains are considered in this paper:

- An equality constrains of power balance.

For stable operation, the real power of each generator is limited by lower and upper limits. The following equation is the equality restriction [37, 39, 44]:

$$\sum_{i=1}^n P_i - P^D - P^L = 0 \quad (4)$$

Where P_i is the output power of each i generator, P^D is the load demand and P^L are the transmission losses.

In other words the total generation of power must cover the total demand P^D and the real power losses in transmission lines P^L . Thus:

$$\sum_{i=1}^n P_i = P^D + P^L \quad (5)$$

The calculation of the power loss P^L implies the resolution of the load flow problem, which has equality constrains on active and reactive power on each bar as follows [44, 45]:

$$P^L = \sum_{i=1}^n B_i P_i^2 \quad (6)$$

A reduction is applied to shape the transmission loss as a function of the output of the generators through the Kron loss coefficients derived from the Kron losses formula.

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^M B_{0i} P_{Gi} + B_{00} \quad (7)$$

Where B_{ij}, B_{0i} e B_{00} are the power loss coefficients of the transmission network. Reasonable accuracy can be obtained when the actual operating conditions are close to the base case, from where the coefficients - B were derived [44, 45].

- An inequality constraint in terms of generation capacity. For stable operation, the real power of each generator is limited by upper and lower limits. Inequality constraint limits of the output of the generator is:

$$P_{min.i} \leq P_i \leq P_{max.i} \quad (8)$$

Where:

P_i – Output Power of i generator

$P_{min.i}$ – Minimal Power of the i generator

$P_{max.i}$ – Maximal Power of the i generator

- An inequality constraint in terms of delivery of fuel.
- At each interval, the amount of fuel supplied to all units must be less than or equal to the fuel supplied by the supplier, ie the fuel delivered to each unit in each interval should be within its lower limit F_{min} and its upper limit F_{max} . Thus:

$$\sum_{i=1}^N F_{im} - F_{Dm} = 0 \quad m \in M \quad (9)$$

Where:

F_{im} – Fuel supplied to the engine i in the range m

F_i^{min} – Minimum limit of fuel delivery to the engine i

F_i^{max} – Maximal limit of fuel delivery to the engine i

F_{Dm} – Fuel supplied in the range m

- An inequality constraint in terms of fuel storage limits.

The fuel storage limit of each unit in each range should be within its lower limit V_{min} and the upper limit V_{max} , so that:

$$V_{im} = V_{(m-1)} + F_{im} - t_m [\eta_i + \delta_i P_i + \mu_i P_i^2 + |\lambda_i \sin\{\rho_i (P_i^{min} - P_i)\}|] \quad i \in N, m \in M \quad (10)$$

Where:

η_i, δ_i and μ_i are the fuel consumption coefficients for each generating unit and λ_i and ρ_i are the fuel consumption coefficients for each generating units, taking into account the valve point effect.

1.4. Economic Load Dispatch Problem Formulation

1.4.1. Incremental fuel cost method.

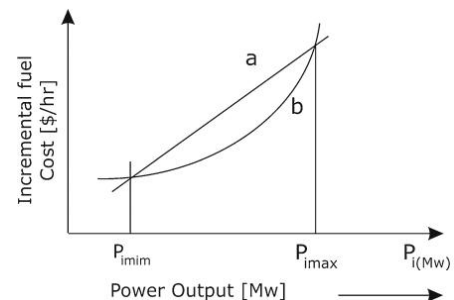


Fig. 2. Incremental Fuel Cost curve of generator i .

Source: [48]

The incremental fuel cost can be computed by the following expression [46]:

$$IC_i = (2 \cdot a_i \cdot P_{gi} + b_i) \quad \$/hr \quad (11)$$

Where IC is the incremental fuel cost a_i is the actual incremental cost curve, and b_i is the approximate

incremental fuel cost curve (linear). P_{gi} is the total power generated[47].

The incremental fuel cost curve is showed in Figure 2.

To load dispatch purposes, the cost is usually approximated by a quadratic or more segments, then the fuel cost curve in the generation of active power, takes a quadratic form

1.4.2. Lambda iteration Method

One of the most popular traditional techniques for solving the economic load dispatch problem (ELD) for minimizing the cost of the generating unit is the lambda iteration method. Although the computational procedure for lambda iteration technique is complex it converges very fast for this type of optimization problem [1], [49]. The Lambda iteration method is more conventional to deal with the minimization of cost at any power generation demand. For a large number of units Lambda iteration method is more precise and more incremental cost functions of all units are stored in memory.

The detailed algorithm for the lambda iteration method for the ELD problem is given below:

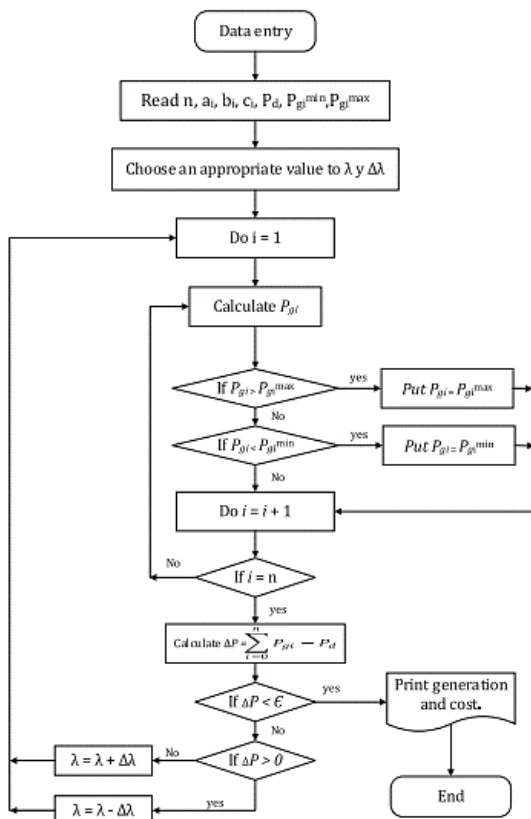


Fig. 3. Iteration Lambda (λ) Algorithm for solving the Economical Load Dispatch.

Source: Authors.

The steps for solving the Lambda (λ) iteration method algorithm are the following:

1. To Read the problem data:
 - The cost coefficients (a_i, b_i, c_i)

- The coefficients of losses B
 - The power limits
 - The power demand.
2. To Assume an initial value of λ and $\Delta\lambda$ using the equations of cost curves.
 3. To Calculate the power generated by each unit P_{gi}
 4. To Check the limits of generation of each unit:

$$\text{if } P_{gi} > P_{gi}^{max}, \quad \text{set } P_{gi} = P_{gi}^{max}$$

$$\text{if } P_{gi} < P_{gi}^{min}, \quad \text{set } P_{gi} = P_{gi}^{min}$$

5. To Calculate the generated power.
6. To Calculate the difference in power, which is given by the following equation:

$$\Delta P = \sum_{i=1}^{Ng} P_{gi} - P_d \quad (12)$$

7. if $\Delta P < \epsilon$ (Tolerance value), then stop calculations and to calculate the generation costs. Otherwise, go to the next step.
8. if $\Delta P > 0$, then $\lambda = \lambda - \Delta\lambda$
9. if $\Delta P < 0$, then $\lambda = \lambda + \Delta\lambda$
10. Repeat the procedure from step 3

1.4.3. Sequential Quadratic Programming

An efficient and accurate solution to the economic load dispatch problem does not only depend on the size of the problem in terms of the number of constraints and design variables but also depends on the characteristics of the objective function and constraints. When both objective functions and constraints are linear functions of the design variables, the economic dispatch problem is known as a linear programming problem. The quadratic programming problem (QP) refers to minimizing or maximizing an objective quadratic function that is linearly restricted.

The most difficult problem to solve is the non-linear programming problem where the objective function and constraints can be nonlinear functions of the design variables.

The solution of the latter problem requires an iterative procedure to obtain a search direction at each iteration. This direction can be found by solving a QP sub problem.

Methods for solving these problems are commonly referred to as SQP since a QP sub problem is solved at each greater iteration, they are also known as iterative quadratic programming, recursive quadratic programming, or constrained variable metric

The problems solved in this work, are from quadratic and nonlinear programming because the objective

function is quadratic and nonlinear, respectively, according to their equations. The SQP is in many cases superior to other methods of nonlinear programming optimization with constraints, having advantages in terms of efficiency, accuracy and success of obtaining solutions for a large number of test problems available in the literature[50-52].

1.4.4. Quadratic Programming Algorithm.

The quadratic programming is an effective optimization method to find the global solution if the objective function is quadratic and the constraints are linear. It can be applied to the optimization problems with non-quadratic objective functions and non-linear constraints[53]. For all the problems with quadratic objective and constraints, imposed restrictions should be linear

The non-linear equations and inequalities are addressed through the following steps:

Step 1:To start the procedure it is necessary to allocate the lower limit of generation of each plant and evaluating the transmission losses and loss coefficients and update the incremental demand.

$$P_i = P_i^{min}, x_i = 1 - \sum_{j=1}^n B_{ij} P_j \quad (13)$$

and

$$PD^{new} = PD + P_L^{old} \quad (14)$$

Step 2:Replace the incremental costs coefficients and solve the set of linear equations to determine the incremental cost of fuel λ as:

$$\lambda = \frac{\sum_i^n 0.5 \times \frac{b_i}{a_i}}{P_D^{new} + \sum_i^n 0.5 \times \frac{b_i}{a_i}} \quad (15)$$

Step 3:Determining the power allocation of each generator

$$P_1^{new} = \frac{\lambda - \frac{b_1}{a_1}}{2 \times \left(\frac{a_1}{x_1}\right)} \quad (16)$$

If the generator violates its limits should be set this limit and only the remaining generators should only be considered for the next iteration.

Step 4:To Check the convergence

$$|\sum_i^n P_i - PD^{new} - P_L| \leq \epsilon \quad (17)$$

ϵ - is the value of tolerance for the violation of power balance.

Step 5:Perform steps 2-4 until convergence is achieved. For all four steps above the objective is quadratic, and the restrictions are quadratic too, so the restrictions should be made linear:

$$\text{Minimize: } XH X^T + f^T X \quad (18)$$

$$\begin{aligned} \text{Subject to: } & KX \leq R, \quad X^{min} \leq X \leq X^{max} \\ & X = [x_1, x_2, x_3, \dots, \dots, x_n]^n \\ & f = [f_1, f_2, f_3, \dots, \dots, f_n]^n \\ & R = [R_1, R_2, R_3, \dots, \dots, R_n]^T \end{aligned}$$

H is the Hessian matrix of size $n \times n$ and A is the matrix $m \times n$ representing inequalities. For economic load dispatch with losses the quadratic programming algorithm can be effectively implemented, defining matrices H, f, K and R .

$$H = \text{diag} \left(\left[\frac{a_1}{x_1}, \frac{a_2}{x_2}, \dots, \dots, \frac{a_n}{x_n} \right] \right) \quad (19)$$

$$f = \left[\frac{b_1}{x_1}, \frac{b_2}{x_2}, \dots, \dots, \frac{b_n}{x_n} \right]$$

K is a Matrix: $1 \times nK = [1, 1, \dots, \dots, 1]$, $eR = PD + P_L^{old}$

1.4.5. Newton Method

The economic load dispatch may also be solved by observing that the objective is to ensure that $\nabla L_x = 0$.

Since this is a vector function, the problem may be formulated as seek to take exactly the gradient to zero (i.e., a vector whose elements are equal to zero). The Newton method can be used to find this.

Newton's method to a more than one variable is developed as follows [54-59]. Assume that the function $g(x)$ will be conducted to zero. The function g is a vector and the unknowns, x , are also vectors. So to use Newton's method, must be done the following:

$$g(x + \Delta x) = g(x) + [g'(x)] \Delta x = 0 \quad (20)$$

If the function is defined as:

$$g(x) = \begin{bmatrix} g_1(x_1, x_2, x_3) \\ g_2(x_1, x_2, x_3) \\ g_3(x_1, x_2, x_3) \end{bmatrix} \quad (21)$$

Then:

$$g'(x) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{bmatrix} \quad (22)$$

That is the well-known Jacobean matrix. The adjustment to each step then is:

$$\Delta x = -[g'(x)]^{-1} g(x) \quad (23)$$

But, if the function g and the gradient vector ∇L_x , so:

$$\Delta x = -inv \left[\frac{\partial}{\partial x} \nabla L_x \right] \cdot \nabla L \quad (24)$$

For the problem of economic load dispatch, the expression to use is:

$$L = \sum_{i=1}^N F_i(P_i) + \lambda(P_{load} - \sum_{i=1}^N P_i) \quad (25)$$

and ∇L is as was defined above. The Jacobian matrix now becomes a compound of second derivatives and is called the Hessian matrix:

$$\left[\frac{\partial}{\partial x} \nabla L_x \right] = \begin{bmatrix} \frac{d^2 L}{dx_1^2} & \frac{d^2 L}{dx_1 dx_2} & \dots \\ \frac{d^2 L}{dx_2 dx_1} & & \dots \\ \vdots & & \vdots \\ & \frac{d^2 L}{d\lambda dx_1} & \end{bmatrix} \quad (26)$$

Generally, the Newton method will solve a problem with a correction that is much closer to the minimum value at a generation step than would be with the gradient method.

1.4.6. Dynamic Programming Method

The application of computational methods to solve a wide range of control problems and dynamic optimization in the late 1950 led to Dr. Richard Bellman and their associated to the development of dynamic programming. These techniques are useful in solving a variety of problems and can greatly reduce the computational effort to find the best paths or control policies. The theoretical mathematical background based on the calculus of variations, is a bit difficult. The applications are not, however, since they will depend on the particular expression of the optimization problem in appropriate terms for formulating a dynamic programming (DP)[1].

When programming power generation systems, DP techniques have been developed for the economic load dispatch of thermal systems, the solution of economic problems scheduling of hydrothermal plants and a practical solution to the problem of the commitment of units. If the valve points are considered at the input-output curve, should be considered the possibility of not convex curves if it is desired extreme precision

If the non-convex input-output curves are going to be used, the same incremental cost methodology cannot be used since there are several MW of output values for any given value of incremental cost. Under such circumstances, there is a way to find an optimal dispatch

using dynamic programming (DP). The dynamic programming solution to the economic load dispatch is made as an allocation problem.

Using this approach, not only one set of optimal power (Mw) output of the generator is calculated for a specific total load, but a set of outputs are generated at discrete points to a whole range of load values [60]. A problem that is common to the economic load dispatch with dynamic programming is the poor performance of control of generators.

The only way to produce an order of load that is acceptable to the control system as well as being the best economically, is to add the ramp rate limits or velocity gradient for the formulation of economic load dispatch. This requires a short load forecasting interval to determine the most likely best load requirements and the ramp loading units. This problem can be approached as follows [61, 62]:

Given a load to be provided to increments of time $t = 1 \dots t_{max}$ With load levels of P_{load}^t and N generators on-line for supplying the load

$$\sum_{i=1}^N P_i^t = P_{load}^t \quad (27)$$

Each unit must comply with a limit relation, such that:

$$P_i^{t+1} = P_i^t + \Delta P_i \quad (28)$$

and:

$$-\Delta P_i^{max} \leq \Delta P_i \leq \Delta P_i^{max} \quad (29)$$

Then, the units must be programmed to minimize the cost of power supply during the time period in which:

$$F^{total} = \sum_{t=1}^{T_{max}} \sum_{i=1}^N F_i(P_i^t) \quad (30)$$

Subjected to:

$$\sum_{i=1}^N P_i^t = P_{load}^t \quad (31)$$

for $t = 1 \dots \dots t_{max}$ and:

$$P_i^{t+1} = P_i^t + \Delta P_i \quad (32)$$

with:

$$-\Delta P_i^{max} \leq \Delta P_i \leq \Delta P_i^{max} \quad (33)$$

1.4.7. Representation of Particle Swarm Optimization (PSO).

Let p be the coordinates (position) of a particle and v its corresponding flight speed (speed) in a search space,

respectively. Therefore, the i th particle is represented as $P_i = [P_{i1}, P_{i2}, P_{i3}, \dots, P_{iNG}]$, in the NP-dimensional space. The best previous position of each particle is recorded and represented as $Pb_i = [Pb_{i1}, Pb_{i2}, Pb_{i3}, \dots, Pb_{iNG}]$. The index of the best particle among the particles of the group is represented by $[G_1, G_2, G_3, \dots, G_{NG}]$, the particle speed ratio is represented as: $v_i = [v_{i1}, v_{i2}, v_{i3}, \dots, v_{iNP}]$ The new velocity and position of each particle can be calculated using the current speed and distance from Pb_{ij} to G_j , as shown in the following expression[47][63][64]:

$$v_{ij}^{r+1} = w \times v_{ij}^r + C_1 \times R_1 \times (Pb_{ij}^r - P_{ij}^r) + C_2 \times R_2 \times (G_{jr} - P_{ij}^r) \quad (i = 1, 2, \dots, NP; j = 1, 2, \dots, NG) \quad (34)$$

$$P_{ij}^{r+1} = P_{ij}^r + v_{ij}^{r+1} \quad (i = 1, 2, \dots, NP; j = 1, 2, \dots, NG) \quad (35)$$

Where:

- NP - The number of particles in the group.
- NG- The number of members on the particle
- R- The interaction pointer (generation)
- w- The inertia weight factor
- C_1 and C_2 - The acceleration constants
- R_1 and R_2 - The random values in the range [0,1]
- v_{ij}^r -The speed of the j th member of the i th particle in r th iteration.

$$v_j^{min} \leq v_{ij}^r \leq v_j^{max} \quad (36)$$

P_{ij}^r - The current position of the j th particle in the r th iteration.

The parameters v_j^{min} and v_j^{max} determine the resolution, or ability, to look for regions between the current position and the target position.. If v_j^{max} is too high, the particles may fly through good solutions. If v_j^{max} is very low, the particles cannot explore sufficiently and can lead to local solutions.

The C_1 and C_2 constants represent the weighting of stochastic acceleration pulling each particle toward the Pb_{ij}^r, G_{jr} . Low values allow the particles to move away from the target area before being lured back. Moreover, high values result an abrupt movement toward or passing the target regions [47].

A. Solution to the Economic Load Dispatch by the criterion of incremental cost (λ) and particleswarmalgorithms, case study.

The problem to be solved by particle swarm algorithms can be formulated as follows:

$$\text{Minimize } \sum_{i=1}^n F_i(P_i) \quad (37)$$

$F_i(P_i)$ is the fuel cost equation of the i th engine. It is the change in the cost of fuel (\$) versus the generated power (Mw). Normally, it is expressed by the continuous quadratic equation:

$$F_i(P_i) = \sum_{i=1}^n (a_i + b_i P_i + c_i P_i^2) \quad (38)$$

Power losses are calculated by the expression:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^M B_{0i} P_{Gi} + B_{00} \quad (39)$$

The restrictions used in this case are the following:

$$P_{min.i} \leq P_i \leq P_{max.i} \quad (40)$$

$$\sum_{i=1}^n P_i - P^D - P^L = 0 \quad (41)$$

The selected reference plant for the case study is composed of 10 motors with their characteristics described in Table 1, The first three columns are the coefficients a_i, b_i , and c_i , and the last two the minimum and maximum power of each engine.

TABLE 1, contains 10 lines (each line is a machine of the plant), and 5 columns (representing the coefficients of the cost of fuel and the limits of the plant).

1. Coefficient “a” (\$/Mw²)
2. Coefficient “b” \$/Mw
3. Coefficient “c” (\$)
4. Lower power limit in (Mw)
5. Upper power limit in (Mw)

TABLE 1: Technical Specifications Of Plant Engines.

Motor	a_i (\$/Mw ^{^2})	b_i (\$/Mw)	c_i (\$)	P_{min} (Mw)	P_{max} (Mw)
1.	0.007	7	240	0.66	3.35
2.	0.0095	10	200	0.9	3.7
3.	0.009	8.5	220	0.8	3.6
4.	0.009	11	200	0.66	3.35
5.	0.008	10.5	220	0.72	3.45
6.	0.0075	12	120	0.66	2.97
7.	0.0075	14	130	0.88	3.5
8.	0.0075	14	130	0.754	3.33
9.	0.0075	14	130	0.9	3.9
10.	0.0075	14	130	0.56	2.35

Source: Authors.

Loss coefficients (Bm) are determined by a square matrix of size $n \times n$, where n is the number of engines, demonstrated in TABLE 2.

TABLE 2: Matrix of Losses of the 10 Plant Engines (All Values Must be Multiplied By $10 E^{-4}$).

M	1	2	3	4	5	6	7	8	9	10
1	0.14	0.17	0.15	0.19	0.26	0.22	0.34	0.38	0.43	0.45
2	0.17	0.6	0.13	0.16	0.15	0.2	0.23	0.56	0.23	0.51
3	0.15	0.13	0.65	0.17	0.24	0.19	0.25	0.38	0.43	0.45
4	0.19	0.16	0.17	0.71	0.3	0.25	0.43	0.56	0.23	0.51
5	0.26	0.15	0.24	0.3	0.69	0.32	0.18	0.37	0.42	0.48
6	0.22	0.2	0.19	0.25	0.32	0.85	0.97	0.55	0.27	0.58
7	0.22	0.2	0.19	0.25	0.32	0.85	0.67	0.38	0.43	0.45
8	0.19	0.7	0.13	0.18	0.16	0.21	0.28	0.56	0.23	0.51

9	0.26	0.15	0.24	0.3	0.69	0.32	0.18	0.37	0.42	0.48
10	0.15	0.13	0.65	0.17	0.24	0.19	0.25	0.38	0.43	0.45

Source: Authors.

The demand for power to be provided by the plant is 20Mw

Demand (Mw), $P_d = 20$;

III. RESULT ANALYSIS AND DISCUSSIONS.

According to the non-compliance with any of the restrictions, the program offers the following messages:

=====

ERROR! The demanded power is less than the minimum power.

Minimal Power: 0.56 Mw > 0.50 Mw of Demanded Power.

=====

This restriction is related to the minimum capacity of the engine of smaller capacity among all of them, ensuring that the demanded capacity is greater than minimum generation capacity of the plant.

=====

ERROR! It is not possible to reach the demand with the current capacity of the plant.

Current capacity: 33.50 Mw < 200.00 Mw of power Demand.

=====

This restriction is related to the maximum capacity of the set of all machines, ensuring that the required capacity is less than the maximum capacity of power plant generation

There was used a population of 300 individuals and 1500 generations.

The results after running the program were the following:

=====

Economic Load Dispatch using Particle Swarm Optimization -Results:

Demanded Power: 20 Mw

Minimal Power: 0.56 Mw

Maximal Power: 3.9 Mw

Fuel Cost: 1922.72

Power losses: 0.01

Power of each motor:

Pm1	Pm2	Pm3	Pm4	Pm5	Pm6	Pm7
3.35	3.70	3.60	2.16	3.45	0.66	0.88
Pm8	Pm9	Pm10	Total P.			
0.75	0.90	0.56	20.01			

=====

The solutions report shows the input parameters to run the program, as power demand, minimum and maximum capacity of power of the engines and the results of the

total cost of fuel, total power loss as well as the optimum power for each one of the plant engines.

Fig. 4 shows the convergence of the particles (values), in green, particles with better trajectories, in blue, current positions of the particles and in red the axes with the Global best position to be found. Each particle establishes its path combining their past experiences with the experiences of their neighbors (other particles with which they communicate), obtained by PSO, generated by MATLAB.

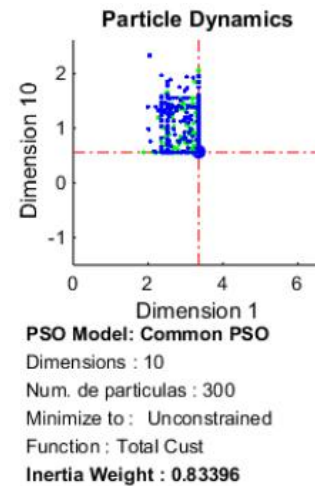


Fig. 4. Particles convergence.

Source: Authors.

Fig. 5, shows the convergence of the cost function for the lowest total cost considering the ten generating machines, obtained by PSO, generated by MATLAB.

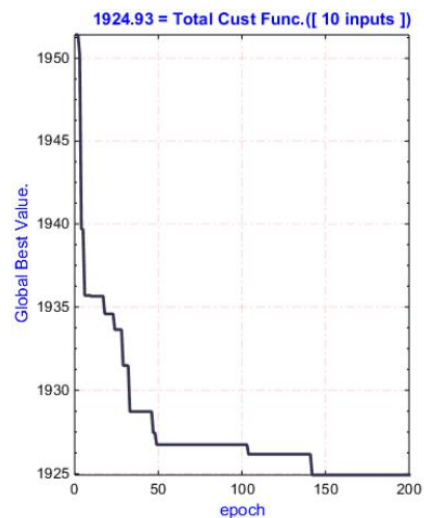


Fig. 5. Total cost convergence.

Source: Authors.

In Fig. 6, there are offered the graphics of the lowest Total Cost of Fuel and Best Overall Value, obtained by PSO, generated by MATLAB.

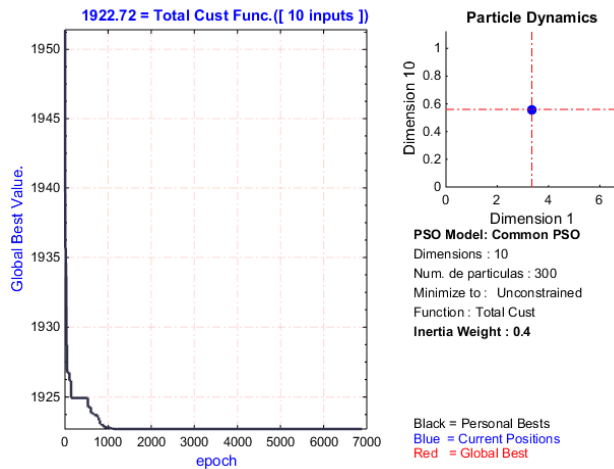


Fig. 6. Consumo total de Combustível e Melhor Valor Global.

Source: Authors.

IV. CONCLUSIONS

In this paper it was developed an analysis of ELD problems and different approaches to solving the problem. Conventional methods such as lambda iteration method converge rapidly, but the complexity increases as the system size increases. Furthermore, the lambda method always requires provide or meet the power output of a generator and then to assign an incremental cost for this generator. In cases where the cost function is much more complex, it can be used Newton's method. If the input-output curves are not convex, then can be used the dynamic programming to solve economic dispatch problems. Hence, different methods have different applications. In this paper the operational optimization problem of Economic Load Dispatch (ELD) was solved using the lambda iteration technique and the Particle Swarm algorithm. It was analyzed as a case study a generating plant with 10 units or motors. The results agree with the actual load dispatch. The lambda iteration method applying the particle swarm algorithm is a simple way to solve the ELD problem with good results.

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