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New Weighted Total Acceleration on Momentum Euler Equation for Formulating Water Wave Dispersion Equation

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***Abstract*— In this present study, weighted total acceleration for Kinematic Free Surface Boundary Condition (KFSBC) and in momentum Euler equation was formulated. Furthermore, by using both aforementioned equations, the nonlinear water wave dispersion equation was then formulated.**

The wavelength obtained from dispersion equation is determined by weighting coefficient. The weighting coefficient value was determined by using the maximum wave height and critical wave steepness criteria which have been obtained from the previous studies.

I. INTRODUCTION

Wavelength L and wave number k in the $k=\frac{2\pi}{L}$ is a very important water wave parameter, where the accuracy of water wave analysis is largely determined by these parameters. The equation for calculating these parameters is called the dispersion equation and the most used is the dispersion equation from the linear wave theory (Dean (1991)).

Hutahaean (2019) formulated the dispersion equation for water waves using the weighted total acceleration in the KFSBC equation and the Euler momentum equation, which the weighted total acceleration in both equations has the same form. In this study, the weighted total acceleration was developed in different forms.

With the KFSBC equation and Euler's momentum equation using weighted total acceleration, the dispersion equation for water waves was formulated. To get the weighted coefficient value, the critical wave steepness of the equation is calibrated against that of the previous research results. There are two critical criteria for wave steepness, and the first criteria of Michell, J.H. (1893). The results of Michell's research were widely used by other researchers in developing breaking criteria in shallow waters, including Miche (1944), Battjes and Jansen (1978), Ostendorf and Madsen (1979), Battjes and Stive (1985) and Rattanapittikon and Shibayama (2000).).

The newest criteria of critical wave steepness is the criteria proposed by Toffoli, A., Babanin, A., Onorato and Waseda,T. (2010), with the same form as Michell's (1893) criteria, but having different coefficients. In this study, the criteria from Toffoli et all will be used.

Wiegel (1949,1964) proposed an equation for the maximum wave height for a wave period. By using this wave height maximum, wavelengths that meet the criteria of Toffoli, et al (2010) can be calculated. So that we get a wavelength value that meets the maximum wave height criteria and the critical wave steepness criteria.

II. MAXIMUM WAVE HEIGHT AND CRITICAL WAVE STEEPNESS

a. Wiegel's criteria (1949,1964)

According to Weigel (1949,1964) , the maximum wave height in a wave period is,

$H_{max} = \frac{gT^2}{15.6^2}$ ....(1)

T = wave period (sec)

g = gravitational force (m/sec^2)

b. Michell's criteria (1893)

According to Michell (1893) critical wave steepness is

$\frac{H}{L} = 0.142$ ....(2)

c. Toffoli, A., Babanin, A., Onorato and Waseda,T. (2010)

According to Toffoli, A., Babanin, A., Onorato and Waseda,T. (2010), critical wave steepness is

$\frac{H}{L} = 0.170$ ....(3)

L= wavelength

With the input wave height from (1), it can be calculated the wavelength with (2) or (3), it is then compared with the deep water wavelength of the dispersion equation from the linear wave theory (Dean (1991)),

$L_0 = \frac{gT^2}{2\pi}$....(4)

Table.1: The wavelength from (2), (3) and (4)

T (sec)	H_{max} (m)	L_{Tof} (m)	L_{Mich} (m)	L_0 (m)
6	1.45	8.54	10.22	56.21
7	1.98	11.62	13.91	76.5
8	2.58	15.18	18.17	99.92
9	3.27	19.21	22.99	126.47
10	4.03	23.71	28.39	156.13
11	4.88	28.69	34.35	188.92
12	5.8	34.15	40.88	224.83
13	6.81	40.07	47.98	263.86
14	7.9	46.48	55.64	306.02
15	9.07	53.35	63.87	351.29

On table (1), L_{Tof} is the wavelength that is calculated by (3), L_{Mich} wave length that is calculated by using (2), and L_0 is the wavelength that is calculated by using (4). The results of the wavelength calculation are presented in table (1). It can be seen that the wavelength produced by the linear wave theory is too long, so that the critical wave steepness criteria of both Michel and Toffoli will never be achieved.

Furthermore, if the wavelength of (4) is used to calculate the critical wave height with (2) and (3), a very large critical wave height is obtained as presented in table (2) below.

Table.2: Wave height maximum according to (2) and (3), wave length is calculated by (4)

T (sec)	H_{Mich} (m)	H_{Tof} (m)
6	7.98	9.56
7	10.86	13.01
8	14.19	16.99
9	17.96	21.5
10	22.17	26.54
11	26.83	32.12
12	31.93	38.22
13	37.47	44.86
14	43.45	52.02
15	49.88	59.72

In table (2), it can be seen that the maximum wave height that is calculated using the wavelength of the linear wave theory produces a very large wave height. In table (2), H_{mich} the wave height is calculated using the Michell's criteria, while the wave height H_{Tof} is calculated using the Toffoli's criteria. Considering that Toffoli et al's criteria produce a greater maximum wave height than Michell's criteria, this research uses the Toffoli et al's critical wave steepness criteria to calibrate the wavelengths produced by the dispersion equations of this study.

III. WEIGHTED TOTAL ACCELERATION.

Courant (1928) stated that the relation between δx and δt in the water wave equation is $\delta x = 3.0\ C\ \delta t$ where C is wave celerity, x is the horizontal axis and t is the time.

Hutahaean (2021) who used the Taylor series on a sinusoidal function

$f(t,x) = A\cos\sigma t \cos kx$ ....(5)

A= amplitude

σ = angular frequency= $\frac{2\pi}{T}$, T = period

k = wave number

gained a relationship that is more or less the same as the Courrant relationship, that is $\delta x = 3.01519\ C\ \delta t$

Hutahaean (2021) used the Taylor series for functions that take the form of:
$f(t,x,z) = A\cos\sigma t\cos kx\cosh k(h+z)$ ......(6)
which is the solution to Laplace's equation (Dean 1991), where z is the vertical axis, get the relation or $\delta z = 3.0012\ C\ \delta x$ or $\delta z = 3.012^2\ C\ \delta t$.
Based on Courrant (1928) and Hutahaean (2021), the relationship obtained
$\delta x = \gamma\ C\ \delta t$ ......(7)
$\delta z = \gamma^2\ C\ \delta t$....(8)

where γ is called as weighting coefficient.

Relationship between δx and δt and between δz and δt were done on Taylor series order 1, in a function of time and space $f(t,x)$ by working on the weighting coefficient on time differential terms.

$$f(t+\delta t, x+\delta x) = f(t,x) + \gamma\delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x}$$

While on a function $f(t,x,z)$

$$f(t+\delta t, x+\delta x, z+\delta z) = f(t,x,z) + \gamma^2\delta t\frac{\partial f}{\partial t} + \gamma\delta x\frac{\partial f}{\partial x} + \delta z\frac{\partial f}{\partial z}$$

a. Function $F = f(t,x)$

$$f(t+\delta t, x+\delta x) = f(t,x) + \gamma\delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x}$$

$$\frac{f(t+\delta t, x+\delta x) - f(t,x)}{\delta t} = \gamma\frac{\partial f}{\partial t} + \frac{\delta x}{\delta t}\frac{\partial f}{\partial x}$$

$$\frac{Df}{dt} = \gamma\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} \quad ......(9)$$

For the water level equation $\eta = \eta(x,t)$

$$\frac{D\eta}{dt} = \gamma\frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{\partial x}$$

KFSBC, Dean (1991) is :

$$w_\eta = \frac{\partial\eta}{\partial t} + u_\eta\frac{\partial\eta}{\partial x} \quad ....(10)$$

Where w_η is the the velocity of water is in the direction of the vertical axis-z - on the surface of the water, whereas u_η is the velocity of water particles in the direction of the horizontal axis-x on the surface of the water. It can be seen that KFSBC is the total derivative in the Taylor series for a function$F = f(t,x)$ as in (9). Therefore, by using the weighting coefficient as in (9), KFSBC becomes

$$w_\eta = \gamma\frac{\partial\eta}{\partial t} + u_\eta\frac{\partial\eta}{\partial x} \quad ....(11)$$

b. Function $F = f(t,x,z)$

Taylor series firstorder for function $f(t,x,z)$

$$f(t+\delta t, x+\delta x, z+\delta z) = f(t,x,z) + \gamma^2\delta t\frac{\partial f}{\partial t} + \gamma\delta x\frac{\partial f}{\partial x} + \delta z\frac{\partial f}{\partial z} \quad ....(12)$$

The total acceleration is,

$$\frac{Df}{dt} = \gamma^2\frac{\partial f}{\partial t} + \gamma u\frac{\partial f}{\partial x} + w\frac{\partial f}{\partial z}$$

The total acceleration of the water particles in the horizontal direction-x is,

$$\frac{Du}{dt} = \gamma^2\frac{\partial u}{\partial t} + \gamma u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} \quad .....(13)$$

In the same way, the total weighted total acceleration can be obtained in the vertical direction.-z

$$\frac{Dw}{dt} = \gamma^2\frac{\partial w}{\partial t} + \gamma u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} \quad ....(14)$$

With (13) and (14), then the equation for Euler's momentum in the horizontal direction-x and the vertical direction-z becomes,

$$\gamma^2\frac{\partial u}{\partial t} + \gamma u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad ....(15)$$

$$\gamma^2\frac{\partial w}{\partial t} + \gamma u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad ....(16)$$

Equations (15) and (16) are a modification of Euler's momentum equation in the total acceleration term, by adding the weighted coefficient γ. The difference between this study and Hutahaean's (2019a) is that in this study the weighting coefficient is not only in time differential terms.

IV. FORMULATION OF SURFACE MOMENTUM EQUATIONS

The irrotational flow condition is substituted in (15) and (16) where, $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$,

$$\gamma^2\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(\gamma uu + ww) = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad ......(17)$$

$$\gamma^2\frac{\partial w}{\partial t} + \frac{1}{2}\frac{\partial}{\partial z}(\gamma uu + ww) = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad ......(18)$$

Eq. (18) is written into an equation for p and integrated with respect to the vertical axis z from the elevation z to the water surface η, and the dynamic boundary condition of the surface where the surface pressure $p_\eta = 0$ is applied, thenthe equation of pressure p is obtained

$$\frac{p}{\rho} = \gamma^2\int_z^\eta \frac{\partial w}{\partial t}dz + \frac{1}{2}(\gamma u_\eta u_\eta + w_\eta w_\eta) - \frac{1}{2}(\gamma uu + ww) + g(\eta - z)$$

This equation is differentiated with respect to the horizontal axis-x,

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \gamma^2 \frac{\partial}{\partial x}\int_z^\eta \frac{\partial w}{\partial t}dz + \frac{1}{2}\frac{\partial}{\partial x}\left(\gamma u_\eta u_\eta + w_\eta w_\eta\right) - \frac{1}{2}\frac{\partial}{\partial x}(\gamma uu + ww) + g\frac{\partial \eta}{\partial x} \quad .....(19)$$

Substituting (19) by (17),

$$\gamma^2 \frac{\partial u}{\partial t} = -\gamma^2 \frac{\partial}{\partial x}\int_z^\eta \frac{\partial w}{\partial t}dz - \frac{1}{2}\frac{\partial}{\partial x}\left(\gamma u_\eta u_\eta + w_\eta w_\eta\right) - g\frac{\partial \eta}{\partial x} \quad ...(20)$$

The solution to $\frac{\partial}{\partial x}\int_z^\eta \frac{\partial w}{\partial t}dz$ was done using the equation of the potential velocity of water waves (Dean (1991)), that is

$$\Phi(x,z,t) = Gcoshk(h+z)coskxsin\sigma t \quad ....(21)$$

The velocity and acceleration in the vertical direction-z are,

$$w = -\frac{\partial \Phi}{\partial z} = -Gksinhk(h+z)coskxsin\sigma t \quad ..(22)$$

$$\frac{\partial w}{\partial t} = -Gksinhk(h+z)\sigma coskxcos\sigma t \quad ....(23)$$

(23) is integrated with respect vertical axis$-z$,

$$\int_z^\eta \frac{\partial w}{\partial t}dz = -G\left(coshk(h+\eta) - coshk(h+z)\right)coskxcos\sigma t$$

then was differentiated with respect to the horizontal axis-x,

$$\frac{\partial}{\partial x}\int_z^\eta \frac{\partial w}{\partial t}dz = Gk\left(coshk(h+\eta) - coshk(h+z)\right)\sin kx \; cos\sigma t \quad ......(24)$$

Horizontal direction velocity and acceleration-x is,

$$u = -\frac{\partial \Phi}{\partial x} = Gkcoshk(h+z)sinkxsin\sigma t \quad .....(25)$$

$$\frac{\partial u}{\partial t} = Gk\sigma coshk(h+z)sinkxcos\sigma t \quad .....(26)$$

From(24) and (26) the following relationship is obtained

$$\frac{\partial}{\partial x}\int_z^\eta \frac{\partial w}{\partial t}dz = \frac{\partial u_\eta}{\partial t} - \frac{\partial u}{\partial t} \quad ......(27)$$

Substituting (27) to (20) the equation of surface momentum is obtained,

$$\gamma^2 \frac{\partial u_\eta}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\left(\gamma u_\eta u_\eta + w_\eta w_\eta\right) = -g\frac{\partial \eta}{\partial x} \quad ....(28)$$

V. FORMULATION OF THE DISPERSION EQUATION

The dispersion equation is formulated at the characteristic point, where $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t = \frac{\sqrt{2}}{2}$. At this characteristic point, water surface elevation $\eta = \frac{A}{2}$.

The dispersion equation will be formulated using two equations, namely the KFSBC and the surface momentum equation, (28).

a. KFSBC operation.

Hutahaean (2021), by integrating KFSBC (11) against time t, obtained

$$\eta(x,t) = Acoskxcos\sigma t \quad ...(29)$$

At the characteristic point,

$$\frac{\partial \eta}{\partial x} = -\frac{kA}{2} \quad ...(30)$$

Where,

$$A = \frac{Gk}{2\sigma\gamma}coshk\left(h+\frac{A}{2}\right)\left(1-\frac{kA}{2}\right) \quad ....(31)$$

Substituting (31) to (30)

$$\frac{\partial \eta}{\partial x} = -\frac{k}{2}\frac{Gk}{\sigma\gamma}coshk\left(h+\frac{A}{2}\right)\left(1-\frac{kA}{2}\right) \quad .....(32)$$

Equation (31) can be written as an equation for G,

$$G = \frac{2\sigma\gamma A}{kcoshk\left(h+\frac{A}{2}\right)\left(1-\frac{kA}{2}\right)} \quad ....(33)$$

b. Operation of Surface Momentum Equations

Substituting (25) for horizontal velocity u, (22) for vertical velocity$w$ and (32) for $\frac{\partial \eta}{\partial x}$to (28), where at the characteristic point $\eta = \frac{A}{2}$,

$$\gamma^2\sigma + \frac{1}{2}(\gamma-1)Gk^2coshk\left(h+\frac{A}{2}\right) + \frac{1}{2}\frac{Gk^2}{coshk\left(h+\frac{A}{2}\right)} = \frac{gk}{2\sigma\gamma}\left(1-\frac{kA}{2}\right)$$

Substituting G by (33),

$$\gamma^2\sigma\left(1-\frac{kA}{2}\right) + \sigma\gamma Ak\left((\gamma-1) + \frac{1}{cosh^2k\left(h+\frac{A}{2}\right)}\right) = \frac{gk}{2\sigma\gamma}\left(1-\frac{kA}{2}\right)^2$$

Hutahaean (2021) found that $\frac{\partial k\left(h+\frac{A}{2}\right)}{\partial x} = 0$, so the $\cosh k\left(h+\frac{A}{2}\right)$ values is constant. In deep water, element $\frac{1}{cosh^2k\left(h+\frac{A}{2}\right)} = 0$, is also constant. The final equation becomes,

$$\gamma^2\sigma\left(1-\frac{kA}{2}\right) + \sigma\gamma Ak(\gamma-1) = \frac{gk}{2\sigma\gamma}\left(1-\frac{kA}{2}\right)^2 \quad ....(34)$$

This equation is the dispersion equation for deep water, which can be solved by the Newton-Rhapson method. The solution using the Newton-Rhapson method requires an initial estimate value of k. The initial estimation value can be obtained by working with the assumption that in deep water the convective acceleration term, the second term on the left side (34) can be ignored, so that a simpler equation is obtained,

$$2\gamma^3\sigma^2 = gk\left(1 - \frac{kA}{2}\right)$$

Or,

$\frac{gA}{2}k^2 - gk + 2\gamma^3\sigma^2 = 0$ ......(35)

Calculating k using (35), when the formulas a, b, c is used are:

$$k = \frac{-b - \sqrt{d}}{2a}$$

The determinant d in (35) is:

$d = g^2 - 4gA\gamma^3\sigma^2$

At the determinant value $d = 0$, the maximum amplitude for a wave period will be obtained, that is

$A_{max} = \frac{g}{4\gamma^3\sigma^2}$ ......(36)

VI. ADJUSTMENT TO CRITICAL WAVE STEEPNESS CRITERIA AND MAXIMUM WAVE HEIGHT IN DEEP WATER.

In (34) there is a weighting coefficient γ whose value will determine the resulting wavelength. In this section the weighting coefficient value γ will be determined using the maximum wave height criteria and critical wave steepness criteria.

The calculation was done by trial and error that is by determining γ, the wave amplitude A is calculated using (36), then the wave number $k$is calculated using (34), where the wavelength $L = \frac{2\pi}{k}$, then the wave height H is calculated using (37)-(40). Trial and error was carried out until wave height $H = H_{max}$ was obtained, whereH_{max} was calculated by using (1).

Calculation of wave height H was done using the water level equation

$\eta_0(x,t) = A_0 coskxcos\sigma t$....(37)

A_0 is the wave amplitude calculated by (36). Then the wave amplitude A was calculated,

$A = \frac{Gk}{2\sigma\gamma} coshk(h + k\eta_0)(1 - k\eta_0)$.....(38)

Water wave surface elevation is:

$\eta(x,t) = Acoskxcos\sigma t$....(39)

The calculation was done fort $= 0$. By using the water wave surface equation the η-maximum and η-minimum were calculated. Wave height H is,

$H = \eta_{max} - \eta_{min}$....(40)

The trial error on value γ was done until $H = H_{max}$ where the condition was achieved at $\gamma = 1.4614$, but the critical wave steepness $\frac{H}{L} = 0.210$ is greater than the Toffoli et all's (2010) criteria that is $\frac{H}{L} = 0.170$.

Table.3: Wave height and critical wave steepness on$\gamma = 1.4614$

T (sec)	L (m)	A (m)	H (m)	H_{max} (m)	$\frac{H}{L}$
6	6.894	0.717	1.451	1.451	0.21
7	9.383	0.975	1.975	1.975	0.21
8	12.256	1.274	2.58	2.58	0.21
9	15.511	1.612	3.265	3.265	0.21
10	19.149	1.99	4.031	4.031	0.21
11	23.171	2.408	4.877	4.878	0.21
12	27.575	2.866	5.804	5.805	0.21
13	32.362	3.364	6.812	6.812	0.21
14	37.533	3.901	7.9	7.901	0.21
15	43.086	4.478	9.069	9.07	0.21

The water wave surface profiles was calculated by (37), (38) and (39) are sinusoidal in shape where$\frac{H}{A} = 2$, but with truncated peaks. This condition is in accordance with the characteristics of the sinusoidal solution to Laplace's equation (Dean (1991)).

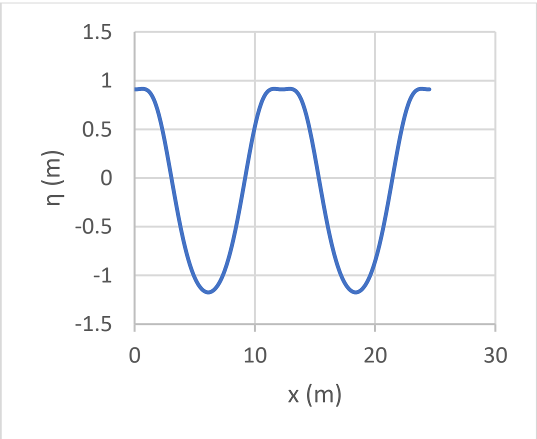

Fig.1: Wave profile for $T = 8$ sec., $\gamma = 1.4614$

In this following section, trial-error of the weighting coefficient value was done until wave steepness value $\frac{H_{max}}{L} = 0.170$, which is the Toffoli criteria. This condition was achieved atthe weighting coefficient value $\gamma = 1.45$, but wave height H is less than H_{max}, as can be seen in Table (4).

Table.4: Wave height and critical wave steepness on $\gamma = 1.45$

T (sec)	L (m)	A (m)	H (m)	H_{max} (m)	$\frac{H_{max}}{L}$
6	7.101	0.734	1.206	1.451	0.17
7	9.665	0.998	1.642	1.975	0.17
8	12.623	1.304	2.144	2.58	0.17
9	15.976	1.651	2.714	3.265	0.17
10	19.724	2.038	3.35	4.031	0.17
11	23.866	2.466	4.054	4.878	0.17
12	28.403	2.934	4.825	5.805	0.17
13	33.334	3.444	5.662	6.812	0.17
14	38.659	3.994	6.567	7.901	0.17
15	44.379	4.585	7.538	9.07	0.17

It can be concluded thatstudy on deep water results weighting coefficient valueγ in the range 1.45-1.464, with a critical wave steepness in the range 0.17-0.21. Where with this weighting coefficient value the wave height maximum is in accordance with the wave height maximum criteria of Wiegel (1949,1964) and the wave steepness is in accordance with the Toffoli's (2010) criteria .

VII. THE EFFECT OF WAVE AMPLITUDE ON WAVELENGTH

In the previous section, it has been found that the wavelength and wave amplitude are determined by the weighting coefficient value γ. In (34), there is an interaction between wave ampltude$A$and wavelength$L$. In the following section, the effect of wave amplitude on wavelength is investigated. This is important, considering that (34) there is no water depth parameter h. So it is necessary to explain how the effect of water depth h on wavelength L.

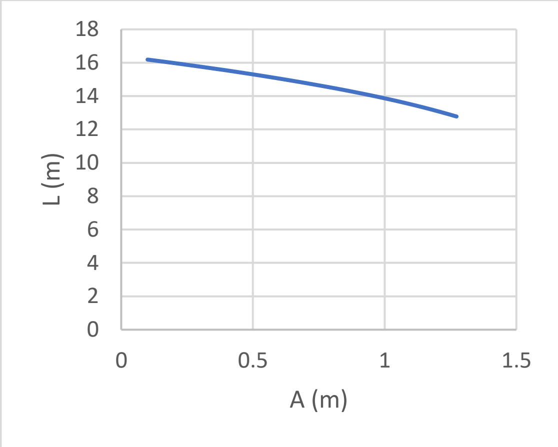

Fig. 2: Wavelength graphic L towards the wave amplitude A, wave perod 8 sec., $\gamma = 1.45$.

The effect of wave amplitude on wavelength is shown in Fig. (2) where the larger the wave amplitude, the shorter the wavelength. When waves move from deeper waters to shallower waters, an enlargement of the wave height and shortening of the wavelength occurs, known as the shoaling phenomenon. Thus the effect of water depth on wavelength is through the wave amplitude, where the wave amplitude is influenced by the water depth. In other words, to obtain wavelengths in shallow water, shoaling analysis must be carried out. However, the wavelength calculation can be done in a simple way by ignoring the wave amplitude enlargement, which will be discussed in section 8.

Table.5: Wave height and critical wavelength on $\gamma = 1.00$

T (sec)	L (m)	A (m)	H (m)	H_{max} (m)	$\frac{H_{max}}{L}$
6	28.104	2.236	4.027	1.451	0.143
7	38.252	3.044	5.481	1.975	0.143
8	49.962	3.976	7.159	2.58	0.143
9	63.233	5.032	9.061	3.265	0.143
10	78.066	6.212	11.187	4.031	0.143
11	94.459	7.517	13.536	4.878	0.143
12	112.414	8.946	16.109	5.805	0.143
13	131.931	10.499	18.905	6.812	0.143
14	153.008	12.176	21.926	7.901	0.143
15	175.647	13.977	25.17	9.07	0.143

To further clarify the effect of wave amplitude on wave length, the calculation is done with the weighting coefficient value $\gamma = 1.00$. With this weighting coefficient value, the effect of the convective acceleration in (34) is neglected with the calculation results presented in Table (5). It can be seen that even though it is used $\gamma = 1.00$, the wavelength of (34) is still much shorter than the wavelength of the linear wave theory (4), where the wavelength of the equation can be seen in Table (1). This shows the magnitude of the wave amplitude influence on wavelength. With this weighting coefficient value, a very large wave height is produced, much bigger than H_{max} Wiegel's criteria (1949,1964), but with a critical wave steepness that is very close to Michell's (1893) criteria, namely $\frac{H}{L} = 0.142$.

VIII. SIMPLE METHODS FOR CALCULATING WAVELENGTHS ON SHALLOW WATER

The dispersion equation obtained was the equation for deep water. In the previous section, it has been found that the effect of waterdepth on wavelength is through wave amplitude, where the calculation of wavelength in shallow waters must go through shoaling analysis. In this section analysisof wavelength was done by ignoring the phenomenon of wave amplitude enlargement, or by working on constant wave amplitudes. The calculation was done using the wave number conservation equation (Hutahaean (2021)), namely,

$$\frac{\partial k\left(h+\frac{A}{2}\right)}{\partial x} = 0 \text{.....(41)}$$

For waves moving from water depth h_1 to shallower water depth h_2 this applies

$$k_2\left(h_2 + \frac{A_2}{2}\right) = k_1\left(h_1 + \frac{A_1}{2}\right)$$

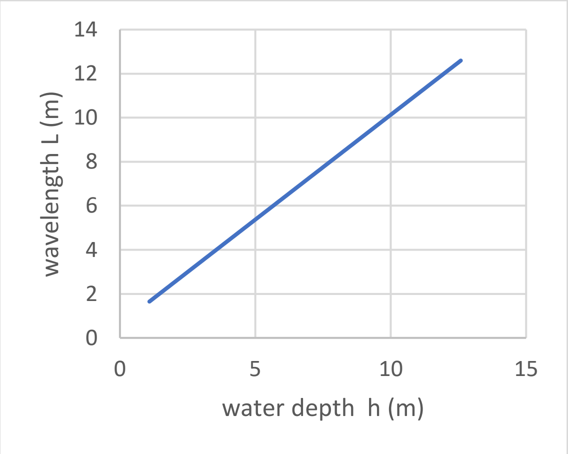

Fig.3: Wavelength graphic on water depth, T = 8 sec., $\gamma = 1.45$

By ignoring the change in wave amplitude

$$k_2 = \frac{k_1\left(h_1+\frac{A_1}{2}\right)}{\left(h_2+\frac{A_1}{2}\right)} \text{.....(42)}$$

In (43) it can be seen that forh_2 less than h_1 there will be an enlargement of the wave number k or shortening on wavelength L.

On Fig. (3), it presents the calculation results with (42) for waves with wave period $T = 8$ sec., weighting coefficient $\gamma = 1.45$, with a linear change in water depth, it is found that the wavelength also changes linearly.

IX. CONCLUSION

The main conclusion is that the total acceleration equation in both the KFSBC equation and the momentum equation, which uses the weighting coefficient produces a dispersion equation that can produce a wavelength that is close enough to the criterion of critical wave steepness.

The weighting coefficient value obtained was 1.45-1.464, where with this weighting coefficient value the critical wave steepness and maximum wave height were obtained according to the existing criteria of previous researchers.

In the dispersion equation obtained, there is no effect of water depth, but there is an influence of wave amplitude. The effect of wave amplitude on wavelength is quite significant, that even though a weighting coefficient of 1.0 is used, the wavelength is still much shorter than the wavelength of the linear wave theory.

The effect of wave amplitude on wavelength is that the bigger the wave amplitude is, the shorter the wave length. This is consistent with the shoaling-breaking phenomenon in waves moving from a deeper water depth to a shallower water depth, where an enlargement of the wave amplitude occurs which results in a shortening of wavelengths which eventually breaking occurs when the critical wave steepness is exceeded.

To get a more definite weighting coefficient value, it is necessary to examine the shoaling-breaking phenomenon in a model developed using the KFSBC equations and Euler's momentum using weighting total acceleration and comparing the model results with the breaker index equation.

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