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Modeling of a new Probabilistic Education Quality Indicator - CPP_QEdu

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***Abstract*—** *This research aims to propose a new probabilistic indicator of quality of education, CPP_QEdu. The proposed indicator intends to be more accurate than the existing ones, as it covers four dimensions of the quality of education: pedagogical, cultural, social, and financial, based on the definition of quality of education proposed by the United Nations Educational, Scientific, and Cultural Organization (UNESCO). The problem that arises in the construction of an indicator of educational quality is that its components seek to measure subjective values. This makes it necessary to introduce imprecision in the analysis to reduce evaluation errors. Thus, in addition to the comprehensiveness of dimensions, an important aspect of this study is the use of the Composition of Probabilistic Preferences (CPP), a multicriteria method to support the manager in the decision-making process that considers imprecision from the beginning. Finally, it is worth highlighting the possibility of application of the proposed indicator, using the variables of the quality panel proposed by the United Nations Development Program (UNDP) and the incorporation of this index in the Human Development Index (HDI).*

I. INTRODUCTION

According to the United Nations Development Programme (UNDP), the Human Development Index (HDI), created by the Pakistani economist Mahbub ul Haq with the collaboration of the Indian Nobel laureate in Economics Amartya Sen, is intended to contrast with a widely used indicator, the Gross Domestic Product (GDP), which considers only the economic dimension of development. The HDI has been measured annually since 1990, incorporating some changes over the years. The HDI is composed of three dimensions: income, health, and education.

The HDI uses school attendance in the education dimension, but this measure may not be enough to evaluate the quality of education provided in each country. This justifies the need to insert a quality indicator for education, motivating this research, which will also have the objective of incorporating a probabilistic component in the evaluation of the quality of education.

The most widely used definition of quality in education is that of citizens who can read and interpret in their mother tongue and have the skills to apply mathematical logic reasoning in their daily lives [28]. In this sense, there are tests such as the Programme for International Student Assessment (PISA) and, at the national level, the BrazilianSystem for Basic Education Assessment (SAEB), the BrazilianHigh School Exam (ENEM) and the BrazilianStudent Performance Exam (ENADE), focused on higher education.

But can these evaluations alone provide a good picture of quality education? This work proposes to look beyond these evaluations, considering other factors that can have an

impact on this quality. Such as the teacher's qualification and how the school environment with internet access broadens the classroom education. The ratio of the number of students per teacher is also an important factor, because its reduction enables teaching with a more individualized look.

The present research aims to build a probabilistic indicator of quality of education more comprehensive than the existing ones from an interdisciplinary approach, using the structural bases of probabilistic modeling, which in this work will be the Composition of Probabilistic Preferences (CPP), developed by Annibal P. Sant'Anna since 2001. The incorporation of this indicator of the quality of education, contemplating the dimensions of pedagogical, cultural, socialand financial quality, allows to amplify the relevance of the HDI.

The concept of quality of education used in this work may change over time. Thus, we can reformulate the criteria according to the new needs of society. However, he methodology used to construct the index can continue to be used.

In a search in the literature, no probabilistic index of the quality of education was found with the breadth of the developed index, contemplating the pedagogical, cultural, social and financial dimensions. Most of the indexes are not probabilistic and contemplate only the pedagogical dimension.

The structure of this research is divided into five sections detailed below.

Section 1 is composed of the introduction to the theme, through the contextualization of the problem, justifying the need to create an index of quality of education that comprises other criteria and that can be aggregated to the HDI.

Section 2 describes the criteria that are being used to construct the CPP_QEdu indicator.

Section 3 presents the modeling and methodology used to construct the indicator.

Section 4 shows the calculation for extending the HDI by incorporating the proposed education quality index.

Section 5 presents the analysis of the results of the application of the developed index to the states and municipalities of Brazil.

II. DESCRIPTION OF THE CPP_QEdu CRITERIA

As seen earlier, the quality of education can be understood from various perspectives. Based on this diversity, UNESCO points out four dimensions that make up quality: pedagogical, cultural, social, and financial.

The pedagogical dimension is the effective fulfillment of the curriculum. For this, we can use evaluation tools like PISA, SAEB, ENEM, and ENADE.

In the cultural dimension, curricula need to be in accordance with the diverse regional cultures, and for this the teacher needs to have adequate training. The well qualified teacher who uses an appropriate curriculum develops strategies to make school more interesting and, consequently, reduces dropouts and increases school attendance. Thus, the teacher's training will be the criterion used in the cultural perspective.

From the social perspective, UNESCO associates the quality of education with its contribution to equity. In this sense, it checks whether the school is, in a democratic way, giving the opportunity for students of the same age group to have access to the same content, as anywhere on the planet. The use of the Internet in the teaching-learning process provides equal access to information. For this reason, the percentage of schools with Internet access will be used as an indicator to represent the social indicator.

From the economic point of view, quality refers to efficiency in the use of resources allocated to education. To represent this criterion, the number of students per teacher will be used. In this item, there is a counterpoint: more students per teacher would be less costly, but would have lesserbenefit. Thus, reducing the number of students per class characterizes the quality in the use of resources [59][60].

The United Nations Development Programme (UNDP) Human Development Report 2019[56], which has a central theme of human development inequalities in the 21st century, elaborates a quality of human development dashboard with three quality indicators. This Quality of Human Development dashboard is a table containing a selection of 14 indicators associated with the quality of health, education, and income. The three health quality indicators are: health expectancy lost, number of doctors, and number of hospital beds. The seven education quality indicators are: pupil-teacher ratios in elementary school, primary teachers trained to teach, proportion of elementary school with Internet access, proportion of secondary schools with Internet access, and the Programme for International Student Assessment (PISA) results in math, reading, and science. The four quality of life indicators are: the proportion of vulnerable jobs, the proportion of the rural population with access to electricity, the proportion of the population using improved drinking water sources, and the proportion of the population using improved sanitation facilities.

Based on the quality of education indicators used by UNDP in the Quality of Human Development panel and the

Quality of Education dimensions pointed out by UNESCO, seven criteria were used to compose the CPP_Qedu, covering these four dimensions. The criteria are the number of students per teacher, percentage of schools with Internet access, percentage of teachers with higher education, SAEB Mathematics score for the Elementary School - Early Years, SAEB Portuguese Language score for the Elementary School - Early Years, SAEB Mathematics score for the Elementary School - Final Years, and SAEB Portuguese Language score for the Elementary School - Final Years.

In the UNDP indicator, the pupil-teacher ratio in elementary school is used, while in the CPP_Qedu the ratio between the number of pupils per teacher in primary and secondary education in Brazil is used as an equivalent criterion. The second criterion used is the percentage of teachers with higher education in primary and secondary schools in Brazil, corresponding to the ratio of primary teachers trained to teach. The third criterion, percentage of primary and secondary schools in Brazil with Internet access, is analogous to two indicators: the proportion of elementary school with Internet access and the proportion of secondary schools with Internet access. The remaining four criteria are the SAEB (Basic Education Evaluation System) scores for Mathematics and Portuguese Language of the two phases of elementary school. This assessment developed by the Brazilian government is similar to the Programme for International Student Assessment (PISA) in mathematics and reading.

III. CPP_Qedu DATA MODELING

The modeling will use Composition of Probabilistic Preferences(CPP) [45], based on the application of probability theory in decision support. CPP is a methodology developed to take into account, in the composition of multiple criteria, the presence of imprecision in preference evaluations. Inaccuracy is inherent in the subjectivity and errors of evaluation in decision making processes. Thus, a random component is recognized for the evaluation of each alternative under each criterion.

CPP deals with rules for combining evaluations by different criteria or different experts in terms of choice probabilities. It starts bythe evaluation of the preference of each alternative by the probability of that alternative being chosen among the others. To arrive at this result, the preference given by the value of a performance attribute is not treated as a precise and definite measure of preference, but as a random variable.

Thus, a probability distribution is associated with each measurement, in a manner similar to that employed in fuzzy sets theory [64] to replace exact numbers with measures of relevance to the points in the intervals around them. The initial exact measurements are considered as parameters for locating probability distributions of the possible values that in other evaluations under similar circumstances would be assigned to the same option. This replacement of exact measurements with probability distributions constitutes the **first step** of CPP, according to Sant'Anna (2015)[46].

The **second step** refers to the choice of a probability distribution that is identified, or even approximately assumed, to be characteristic of the disturbances affecting the measurements, with the exact values as location parameters.

The **third step** involves two computations. **T**he probabilities that the i-th alternative is superior (M_{ij}) and inferior (m_{ij}) to the others are calculated for each j-th criterion. According to equations (1) and (2),

$$M_{ij} = \int_{D_{X_i}} \left[\prod_{j \neq i} F_{X_j}(x_j) \right] f_{X_i}(x_i)\, dx_i \quad (1)$$

$$m_{ij} = \int_{D_{X_i}} \left[\prod_{j \neq i} \left(1 - F_{X_j}(x_j)\right) \right] f_{X_i}(x_i)\, dx_i \quad (2)$$

where F_{X_j} represents the cumulative distribution function (cdf), f_{X_i} the probability density function (pdf) and D_{X_i} the domain of the random variable x_j, as in Sant'Anna et al (2012)[42]. The calculations are performed for each alternative under each criterion.

In the **fourth step**, the joint probabilities M_{ij} and m_{ij} are composed into global preference scores under different decision points of view. Compositions based on M_{ij} are called progressive because they are associated with gain maximization. Compositions based on m_{ij} are called conservative, because they are associated with loss minimization. On the other hand, maximizing M_{ij} and minimizing m_{ij} in all criteria are called pessimistic compositions, while optimistic ones are based on satisfaction in only at least one criterion with the maximum or minimum preference. Four points of view then emerge.

The Progressive-Pessimistic (PP) viewpoint calculates preference by the probability of being the best according to all the criteria considered. The calculation is performed according to the hypotheses of independence and maximum dependence, which portray the extremes of correlations between variables, respectively, by Equations (3) and (4).

$$PP_i = \prod M_{ij} \quad (3)$$

$$PP_i = \min M_{ij} \quad (4)$$

The Progressive-Optimistic (PO) viewpoint uses the probability of being the best according to at least one of the criteria considered, calculated according to Equations (5) and (6).

$$PO_i = 1 - \prod \left(1 - M_{ij}\right) \quad (5)$$

$$PO_i = \max M_{ij} \quad (6)$$

The Conservative-Pessimistic (CP) viewpoint uses the probability of not being the worst in all the criteria consideredand is calculated from Equations (7) and (8).

$$CP_i = \prod \left(1 - m_{ij}\right) \quad (7)$$

$$CP_i = 1 - \max m_{ij} \quad (8)$$

Finally, the Conservative-Optimistic (CO) viewpoint uses the probability of not being the worst in at least one of the criteria considered, and is calculated according to Equations (9) and (10).

$$CO_i = 1 - \prod m_{ij} \quad (9)$$

$$CO_i = 1 - \min m_{ij} \quad (10)$$

The problem that arises in the construction of an indicator of quality of education is that its components seek to measure subjective values, which are affected by political and methodological problems. It is also necessary to consider the aspect of sustainability in education, contemplating the balance between the dimensions of quality of education proposed in this work. All aspects considered are equally important in the composition of this indicator.

The initial transformation into preference probabilities solves the problem of choice of scales, transforming the measures of each dimension into probabilities of reaching a frontier of better or worse performance in that dimension. And it contributes to improving the efficiency of decisions, since it improves the level of precision with respect to quality.

In this paper, the conservative-optimistic probabilistic composition (CO) approach will be used: in which the alternative is evaluated by the probability that it is no worse than the sample by at least one of the criteria. Conservative-Optimistic composition seeks the goal of escaping the large social differences that exist in Brazil and in the world in the particular aspects considered, while hoping that the momentum generated in some dimension will have the effect of causing advances in others.

IV. HDI EXPANSION

4.1 HDI calculation

The Human Development Index (HDI) is a summary of measures in three dimensions, which are the key to human development. A long and healthy life in the health dimension, access to knowledge represented in the education dimension, and a decent economic standard, which is the income dimension. The HDI is the geometric mean of the normalized indices for each of the three dimensions.

As of 2010, a new calculation method was developed to construct the index, according to Technical Note 1 of the HDI report. In this new calculation, UNDP continues to combine the three dimensions, health, education and income, but uses new variables in some dimensions:

In the health dimension, it uses life expectancy at birth in the locality in question. The variable is the same, but there was a change in the calculation, because there was a change in life expectancy in the world.

To construct the life expectancy index (IEV), we use equation (11):

$$(IEV) = \frac{EV-20}{85-20} \quad (11)$$

Where EV is life expectancy at birth, IEV is 1 when life expectancy at birth is equal to or greater than 85 years and 0 when life expectancy at birth is equal to or less than 20 years.

In the Education dimension, today it uses the arithmetic average of the observed number of years of schooling and the expected number of years of schooling for that location, truncated at 15 and 18, respectively. Previously, the literacy rate and the schooling rate were used. As literacy is no longer a problem in most countries, this value has become less significant.

To construct the education index (IE), we do the following calculations:

First, we construct the index of average years of study (IAME), that is, years that a person aged 25 or older spent in formal education, equation (12).

$$(IAME) = \frac{AME}{15} \quad (12)$$

where 15 is the maximum projected in this indicator for 2025.

Next, we construct the expected years of schooling index (IAEE), that is, the expected number of years of schooling a child will achieve at birth, equation (13).

$$(IAEE) = \frac{AEE}{18} \quad (13)$$

where 18 years is the equivalent of obtaining a master's degree in most countries.

The education index (IE) is calculated by taking the simple arithmetic average of the two indexes (IAME) and (IAEE), represented in equation (14).

$$(IE) = \frac{(IAME)+(IAEE)}{2} \quad (14)$$

In the income dimension, Gross National Income (GNI) per capita (GNIpc) is used, with a new calculation.

$$(IR) = \frac{\ln(GNIpc)-\ln(100)}{\ln(75000)-\ln(100)} \quad (15)$$

where IR is 1 when GNI per capita is equal to or greater than $75000 and 0 when GNI per capita is $100. GNI per capita is the dollar value of a country's final income in a year, divided by its population.

Finally, the HDI is calculated by the geometric mean of the indices, from equations (11), (14) and (15).

$$HDI = \sqrt[3]{(IEV).(IE).(IR)} \quad (16)$$

To extend the HDI with the education quality index proposed in this paper, at first, the 7 education quality indicators from the human development quality dashboard proposed by UNDP in 2018 are used.

4.2 Calculation of CPP_QEdu with data from UNDP's Human Development Quality Panel

In a first step, the missing data were replaced by the median of the human development quality scoreboard proposed by UNDP in 2018, Table 1. This includes most countries as far as PISA data is concerned.

Table 1: Human Development Quality Scoreboard - UNDP - 2018

https://docs.google.com/document/d/1giKf9_E2huxDWLX3K4E0UdEfJcMhCFrAAFwKgzGwPck/edit?usp=sharing

We use the seven criteria from the panel, Table 1, to compose the index. These data are normalized, passing the minimum to zero and the maximum to one. After this standardization of the data, we randomize them, using these values as the mode of a triangular distribution with extremes zero and one. These values are then used to calculate the probabilities of maximizing (Mij) and minimizing (mij) the data. Assuming independence, we use the following combinations of maximization and minimization to find the four profiles:

Conservative/Optimistic: COi = 1-∏jmij, (17)

Conservative/Pessimistic: CPi = ∏j(1-mij), (18)

Progressive/Optimistic: POi = 1-∏j(1-Mij), (19)

Progressive/Pessimistic: PPi = ∏jMij. (20)

where ∏ denotes the product operator.

The comparison rules adopted are the conservative ones, which are based on the distances to the worst performing extremes, equations (17) and (18). The justification is that the goal of the analysis is to identify the countries that need more help with respect to education quality. The optimistic conservative composition and the pessimistic conservative composition are analyzed.

In the optimistic approach, the country is evaluated using the probability that it is not the worst in the sample by at least one of the criteria. This assumption captures the hope of escaping underdevelopment in any of the particular characteristics considered by generating favorable momentum to advance in the other dimensions. This optimistic assumption, combined with an assumption of statistical independence, results in a score that is calculated using the complement of the product of the probabilities of showing the worst assessment in each dimension. In the pessimistic approach, the country is evaluated using the probability of not being the worst in all criteria. As there was no significant difference in the ranking, Table 2, we opted to present only the result of applying the Conservative/Optimistic approach to compose the HDI.

In Table 2, the following countries were excluded: Armenia, Gabon, Haiti, Iraq, Libya, Palau, Sudan, Syrian Arab Republic, Timor-Leste, Turkmenistan, Venezuela, which did not have at least one data in the UNDP quality panel. For this reason, the HDI ranking was recalculated without these countries to compare with the IDH_QEdu.

4.3 CalculationofIDH_QEdu

To extend the HDI education index, we use the index CPP_QEdu(CO), which will be introduced in the arithmetic mean IE of the HDI, equation (21), introducing the quality of education in the indicator, equation (22).

$$(IEQ) = \frac{(IAME) + (IAEE) + CPP\_QEdu(CO)}{3} \quad (21)$$

$$IDH\_QEdu = \sqrt[3]{(IEV).(IEQ).(IR)} \quad (22)$$

In Table 2, we present the results comparing the HDI ranks with the extended HDI (IDH_QEdu), removing the 11 countries that did not have quality data.

Table 2: Expanded HDI ranking - 2018, compared to the HDI.

https://docs.google.com/document/d/1kx2JeulYE2m2jFrOtDRLFDhHl1iWKXh8DmxSEIfxtT0/edit?usp=sharing

The quality indicator did not impact, significantly, the rank of the first quartile countries; there were small changes in positions. But it had a positive impact on some countries in the second quartile, such as Panama, which rose 16 positions in the ranking. Costa Rica also had a significant position change. However, the countries that were impacted the most were those in the 4th quartile, many of which had significant changes in their positions for better or worse. Thus, the indicator is able to identify the countries that have been seeking improvements in the quality of education; a change in some criterion can impact the change in position.

V. ANALYSIS OF THE RESULTS

5.1 Analysis of the Brazilian States

For a first analysis of the adequacy of CPP_Qedu with SAEB data, the data of Brazilian states from the year 2017 were used, Table 3:

Table 3: Indicators per BrazilianState

https://docs.google.com/document/d/1iA5y8soOP2PVkQZyINEldTI04agJT9l7Uu6Ow1scs6g/edit?usp=sharing

To analyze the results obtained with thesedata, we began with a comparison of the 2017 Municipal Human Development Index (MHDI) results by states with the results of the Conservative-Optimistic axis of the CPP. We started with the MHDI Education, a synthetic index of the education dimension that is part of the MHDI composition, which is composed of three dimensions: longevity, education, and income. This indicator is obtained through the geometric average of the subindex of children and young people's school attendance, with a weight of 2/3, and the subindex of the adult population's schooling, with a weight of 1/3. The CPP-QEdu has seven criteria, based on the new education quality indicator, these results are described in Table 4.

In this first analysis, three states show significant differences in the comparison. The state of Roraima appears in fourth place with the MHDI-Education methodology, demonstrating an excellent job in getting children into school and improving the schooling level of the adult population. However, this state does not seem to have given much importance to the quality of the education offered, since with the CPP_QEdu it appears in 20th position, highlighting the importance of analyzing the quality criteria in the human development indexes.

Table 4: Comparison of the Indexes per Brazilian State

https://docs.google.com/document/d/1AKr2pA6B614QEqmt2gzKV7qDKFA0m50Sn0TuVTBJiq4/edit?usp=sharing

Looking at Ceará and Mato Grosso do Sul, we observe that these states still need to improve the inclusion of all children in school and increase the schooling of the adult population, as they appear in the 14th and 15th positions in the MHDI Education ranking.However,it can be seen that they are moving in this direction, as they are in a median position in this ranking. When we examine the results of the CPP_QEdu, which focuses on the quality of education, we see that, although this inclusion is happening more slowly, these states appear in the 5th and 6th positions in this ranking, which indicates that these states are investing in the quality of education offered.

In Figure 1, in the radar graph that compares the rankings of the two methodologies, the Education MHDI ranking with the CPP_QEdu ranking from the conservative-optimistic point of view, a very distinct design can be seen in the methodologies, that is, the Education HDI ranking is very different from the CPP_Qedu ranking in most states, confirming the importance of using indicators that evaluate the quality of the education offered and not only how long a student has been in school. There is no point in staying a long time in a school that offers a precarious teaching-learning process.

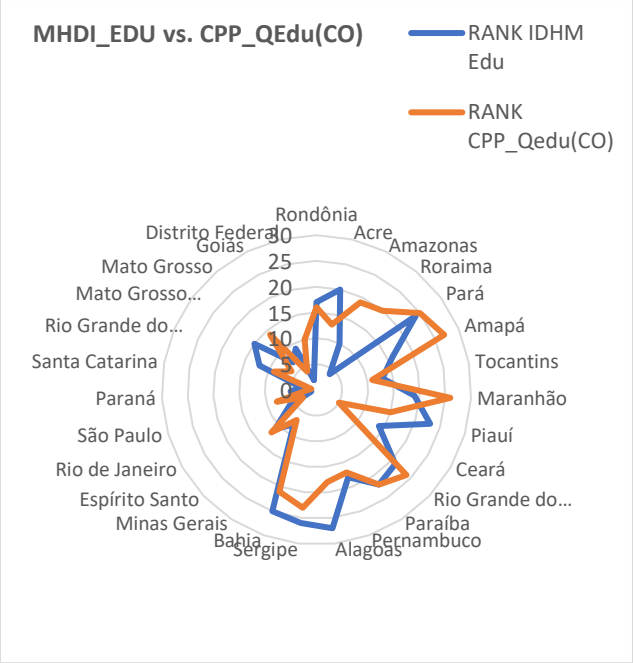

Fig.1: Spider graph comparing the results of the MHDI_EDU with the CPP_QEdu

Source: Authors

To verify the influence of SAEB scores on the CPP_QEdu indicator, we compared the performance of the CPP_QEdu indicator with the CPP_SAEB. The CPP_SAEB was calculated with the four SAEB score criteria per state (SAEB Math score for Primary School Mathematics, SAEB Portuguese Language score for Primary School Mathematics, SAEB Math score for Primary School Mathematics, Final Years, and SAEB Portuguese Language score for Primary School Portuguese Language, Final Years), as shown in Table 5.

It can be seen that the states of São Paulo and Minas Gerais were strongly influenced by the SAEB scores. However, when the results of Rio de Janeiro and Rio Grande do Sul are analyzed, it is observed that the SAEB score had little influence on the CPP_QEdu ranking, so the other three criteria had a strong influence on the result. This analysis is important to show that the CPP_QEdu index is not focused only on content quality and that the other criteria are also influencing the ranking.

Observing, in Figure 2, the radar chart, which compares the CPP_QEdu with the CPP_SAEB with the same conservative-optimistic point of view, there is a very close approximation of the two curves. In this view, there is a strong influence of SAEB scores on the CPP_QEdu indicator.

Table 5: Comparison of CPP_QEdu with the SAEB performance index

https://docs.google.com/document/d/1vyScEYQh7WJBnnTvK8UuRh-zF_itRJ1tBd7oy7K9VLc/edit?usp=sharing

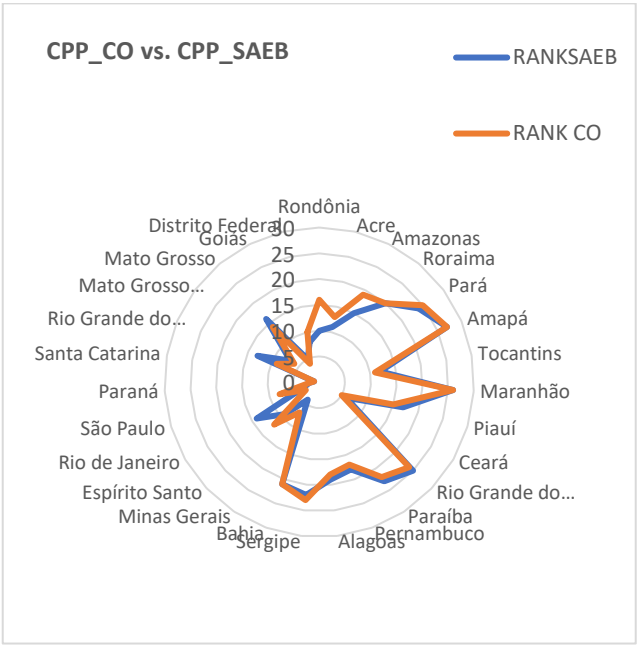

Fig.2: Spider graph comparing CPP_SAEB results with CPP_QEdu (CO)

Source: Authors

Although the context of the problem better fits the Conservative-Optimistic point of view, it is considered relevant to verify the sensitivity of the model, comparing it with the result of the application of the compositions by the other points of view, Table 6.

In this sensitivity analysis, we verify the consistency of the state of Santa Catarina which, under different points of view, remains in first place in the ranking, that is, the state with the best quality of education in the country. In this same analysis, we find the state of Pará among the worst in Brazil from all points of view.

Table 6: Comparison of the Four Viewpoints of CPP_QEdu

https://docs.google.com/document/d/1A-wsP2VPg4zf_22yW-5Rc-T9SKymN8VTwh8xBpsonq8/edit?usp=sharing

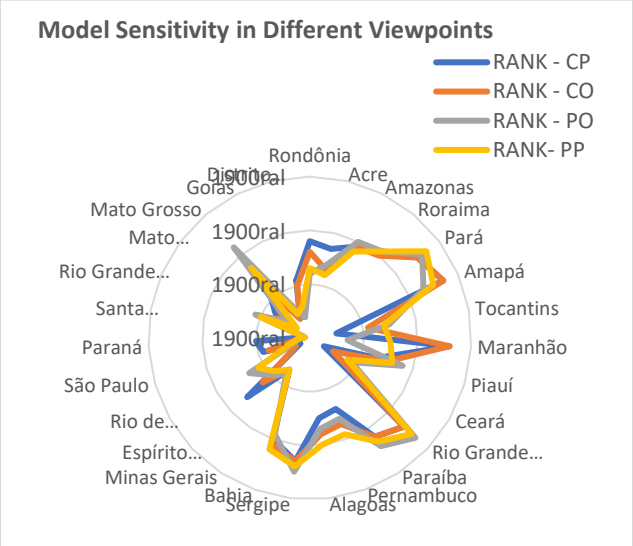

Figure 3: Spider graph comparing the Four Viewpoints of CPP_QEdu

Source: Authors

At the same time, the states of Rio de Janeiro and Ceará behave much better in the conservative profiles, where losses are minimized. As most of the criteria are close to the maximum, the states of São Paulo and the Federal District behave much better in the progressive profile.

Figure 3 shows the behavior of the rankings in the four approaches analyzed. It can be seen that the four approaches have similar results, with the distortions previously analyzed.

5.2 AnalysisofBrazil'sMunicipalities

An analysis of the municipalities was also carried out. The same variables were used: number of students per teacher, percentage of schools with Internet access, percentage of teachers with higher education, SAEB Mathematics score for the Elementary Beginning Years, SAEB Portuguese Language score for the Elementary Beginning Years, SAEB Mathematics score for the Elementary End Years, and SAEB Portuguese Language score for the Elementary End Years.

As for missing data, to be expected in a dataset of all Brazilian municipalities, including representation from municipalities far away from urban centers, it is interesting to note that few municipalities had missing data. To impute values, we used the following procedure. First, the values of each variable were rescaled to the interval [0,1]. Then, the missing values were imputed with the median of the values of the other variables in the municipality.

Two analyses were conducted keeping the data for the explanatory variables of performance quality fixed at their 2019 values. For the variables assessing the results of curriculum delivery effectively, i.e., the SAEB variables, 2017 data were considered in the first analysis and 2019 data in the second.

To avoid the effects of data imprecision from variables representing smaller presences, the comparison was limited to the municipalities in the top quartile in the ordering of municipalities by size, represented by the number of students enrolled in 2019. In this quartile are municipalities with 6180 students or more. Of the total 5570 municipalities in the database, the top quartile consists of 1393 municipalities.

To provide an objective comparison of the differences, a selection of data from extreme municipalities from the extreme states is presented here, that is, the states of Santa Catarina, Paraná and Rio de Janeiro on one side, and the states of Amapá, Maranhão and Pará on the other.

Initially, the differences in the size of these states should be noted. Amapá has only 6 municipalities in the analyzed quartile, Maranhão 96 and Pará 102. On the other side, Santa Catarina has 52 municipalities in the upper quartile, Paraná has 71 and Rio de Janeiro 54.

Table 7: CPP_QEdu Resources and Rank Values of Municipalities in the Extremes

https://docs.google.com/document/d/1AslEHAeyaLnksM9jGSJZMTt0FEmGKEpsOxkSVc9oJcE/edit?usp=sharing

In Table 7, one can observe the expected distance between the values corresponding to municipalities at the two extremes, but one can also highlight the variation between the availability of educational resources within the

states. Thus, in Macapá, capital of Amapá, the percentage of teachers without higher education is much lower than in the other municipalities of the state, while the proportion of schools with Internet resources is much higher. In Maranhão and Pará, the internal variability comes from the fact that, while municipalities like Santa Quitéria do Maranhão and Porto de Moz rank last among the municipalities in all the states, the municipalities in Maranhão and Pará, as a whole, rank better than those in Amapá, Table 5.

Tables 8 and 9 allow us to compare, respectively, the performances of the municipalities in the lower end and the upper end listed in Table 7. The values in these tables are, respectively, the probabilities of presenting the worst and the best score in each of the tests considered in the index. These probabilities allow the comparison to be free from the effect of the possible variation in the difficulty of the tests from one year to another.

Table 8: Odds of Lowest Scoring Municipalities in the Worst-Ranked States

https://docs.google.com/document/d/1sKG7xltLu1GpHjogGViN8cfCRQxuUu0lGtBQnfZcr9g/edit?usp=sharing

Table 9: Odds of Highest Score of Top-Ranked Municipalities in Top-Ranked States

https://docs.google.com/document/d/1klvcvc3mnO0l9e_gXNx9sa-e-OC-NI9n9KGL77TRC-A/edit?usp=sharing

From the point of view of the values recorded in these tables, the highlight is the agreement between the results of the two years.

It should also be noted that these probabilities are calculated in R with remarkable precision despite the large size of the data set, 1393 municipalities. Even with the separation into quartiles, we still have over 1000 (almost 1400 in the top quartile) municipalities in the same group. This comparison of such large samples, however, proved feasible, even though CPP compared each alternative with all the others.

VI. CONCLUSION

The present research achieved the goal of constructing a probabilistic indicator of quality of education that is more comprehensive than existing ones. From an interdisciplinary approach, using the structural basis of the probabilistic modeling offered by CPP. To extend the HDI, the developed index, CPP_QEdu, was incorporated. The incorporation of this education quality indicator contemplates the dimensions of quality, pedagogical, cultural, social, and financial, proposed by UNESCO, extends the relevance of the HDI.

The variables representing these dimensions were based on the human development quality dashboard proposed by UNDP in 2018. That body developed the dashboard, but did not develop a measure that could be incorporated into the HDI, as was done in this research. These variables that were used to construct the indicator per country are adequate, as they contemplate the dimensions of quality proposed by UNESCO.

In this work, a CPP_QEdu indicator was also constructed for the states and municipalities of Brazil, where the Ministry of Education works with a quality indicator, the IDEB, which, however, only considers the pedagogical and social dimensions. To construct the CPP_QEdu at the national level, variables equivalent to those in the UNDP panel were used. All variables used to compose the index cover all basic education in the states or municipalities, without restricting the segment of education. The PISA scores (Mathematics, Portuguese Language and Science) were substituted by SAEB scores (Mathematics, Portuguese Language) in two segments, for greater national coverage. The indicator constructed is more comprehensive than the IDEB and can contribute to the construction of better public policies that provide higher quality education in the country.

The dimensions of quality of education that were incorporated into the CPP_QEdu were based on the dimensions of quality proposed by UNESCO and on the variables used in the panel on quality of human development developed by the UNDP. At the beginning of the research, other variables were evaluated to compose the index, but as the research evolved, we concluded that these dimensions were already broad enough. The standardized tests in the case of PISA and SAEB to compose the pedagogical dimension are appropriate, because they evaluate the knowledge acquired by the students. In the ratio of students per teacher to compose the financial dimension, the balance of this ratio ratifies the efficiency in the use of resources directed to education. The teacher needs to have adequate knowledge to develop a curriculum in accordance with cultural differences, making the school more attractive.

In this way, the teacher's training is propitious to compose the cultural dimension. And the access to the Internet to compose the social dimension is justified by the equality in the reach of information.

The previously existing indexes do not have probabilistic features and, for the most part, evaluate the quality of education only from the pedagogical point of view. Only the IDEB, a national indicator, is calculated from crossing of student performance data with approval rate, evaluating two dimensions.

As future work, it is suggested to consider the importance of introducing a variable that evaluates gender inequalities in access to education, in Brazil and in the world, since these differences are relevant in several places. Another analysis can be done by separating the municipalities that are in rural regions from the essentially urban ones; the evaluation was done by the number of students attended in the region, but there was no separation by type of region. The approval rate and the grade/age distortion are variables that can be introduced in the index, because, when composed with the standardized tests, they help to identify when students are approved without having acquired knowledge. An important study at the moment would be an analysis of the impact of the COVID 19 pandemic on the quality of education, as well as including new educational modalities as criteria.

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