

Discrete PI controller applied on a brushless motor with a coupled load

Ricardo Breganon^{1,2}, Uiliam Nelson Lenzion Tomaz Alves^{1,2}, João Paulo Lima Silva de Almeida¹, Gustavo Vendrame Barbara¹, Fernando Sabino Fontequê Ribeiro¹, Luis Fabiano Barone Martins¹, Luiz Eduardo Pivovar^{1,2}, Rodrigo Henrique Cunha Palácios², Marcio Mendonça²

¹Instituto Federal do Paraná, IFPR, Jacarezinho, Paraná, Brazil

²Universidade Tecnológica Federal do Paraná, UTFPR, Cornélio Procopio, Paraná, Brazil

Abstract—Direct current motors are widely used on the actioning of the electromechanical systems. One of the reasons for that usage is due to the possibility of a precise control of its axis rotation and therefore of the actioning performed by the motor. Due to the important role of that motors, this work presents a study about the control of a direct current motor using the discrete Proportional (P) – Integral (I) control technique, that encompassed: (i) the system dynamics description through differential equation and its equivalent transfer function; (ii) the transfer function discretization; (iii) the project of a discrete PI controller for the system; (iv) the control implementation and experimental tests, considering simulations and the real system; and (v) the comparative analysis of the obtained results. The motor considered in the real experiments is one from Maxon® manufacturer, that is a brushless motor of 12V DC integrated with an encoder of 500 pulses per revolution. An arm was connected to the motor's axis in order to represent the load. For the electronic instrumentation of the system, a data acquisition board from National Instruments, model NI-PCI-6602, was used. The data obtained from that board were processed in the Matlab/Simulink® software, in order to generate the control signal to be sent to the system. The results obtained from the simulations and from the real system show that the used control strategy is appropriate for the presented application. Furthermore, the results also support futures comparative studies, considering other control techniques to be implemented in the system described in this paper.

Keywords—Discrete PI Controller, Instrumentation, Brushless Motor.

I. INTRODUCTION

Direct current (DC) motors are widely used on the actioning of the electromechanical systems. Especially, brushless motors are highlighted due to their better indicators of efficiency, mass/power relation, maintenance cost, lower noises, besides a widely range of operational velocities [1].

A Proportional-Integral (PI) controller consists of a control strategy characterized by the jointly actuation of the Proportional (P) and Integral (I) portions related to the error measured/computed of a process to be controlled.

The tuning step of these controllers is defined by the adjust of the K_p and K_i gains, that correspond to the P and I actions, respectively. In general, while P aims a significant improvement in the control's response time, the

part I aims to meet the prerequisites about the steady-state error that, ideally, should be null [2].

Although the PI controllers have a widely application potential, this paper has its theoretical background inspired on correlated works that consider the brushless motor control. In [3, 4], for example, the mathematical modeling and a PI controller implementation on a brushless motor are presented to show its applicability on electrical cars. A Proportional-Integral-Derivative (PID) controller is designed to control spindle-type electromechanical actuators, that are used on a movement platform of 6 degrees of freedom [5], in which the Derivative (D) term helps it to decrease the overshoot and also the steady-state error. A comparative study between PI and Fuzzy controllers' performance is addressed in [6], where both of them are applied on a DC motor. In [7], PI and PID

controllers are designed. They are discretized by the Zero Order Holder (ZOH), Tustin and zeros and pole mapping methods. The objective in this work was to control the angular velocity of the differential drive of a mobile robot. In all mentioned correlated works, the designed PI and PID controllers met the control requirements of each system, a fact that highlights the promising performance of these control approaches. Other examples considering discrete PID controllers and their variations are shown in [8, 9, 10].

This work presents the project of a discrete PI controller to control a DC motor that has a load connected to its axis. This motor is the main component of a didactical prototype for experiments involving control strategies, available at the Automation Laboratory of the Instituto Federal do Paraná, Jacarezinho, Paraná, Brazil. Moreover, the main steps for the control discretization, that are necessary to its implementation on a digital control hardware, are highlighted. In this way, the main objectives of this work is to validate an appropriate and promising control strategy to problems involving the control of DC motors with coupled load.

The rest of this paper is divided as follows: in Section II the main structural characteristics and functioning of the mentioned didactical prototype is presented, highlighting the DC motor. Moreover, the control objectives are also described in this section; in Section III, the details about the discrete PI controller are described; the results are discussed in Section IV and finally the main conclusions are presented in Section V.

II. PROTOTYPE DESCRIPTION

An overview of the didactical prototype for control experiments is presented in Fig. 1, with a special highlight to the motor, the encoder system and to the load coupled in its axis. Specifically, a brushless motor (12V) manufactured by Maxon® along with an incremental encoder of 500 pulses per revolution are used.

The prototype shown in Fig. 1 can be represented by a Single-Input and Single-Output (SISO) system, in which the controlled variable is the angular position of the load (arm), that it connected to the motor's axis, while the manipulated variable is the voltage applied on the motor. The incremental encoder is used to measure the angular position of the arm. The control objective is to stabilize the arm in a desired angle by means of an adequate voltage level applied on the motor.

A data acquisition board from National Instruments manufacturer, model NI-PCI 6602, is used for the angle acquisition from the encoder and for the control signal

application. The control signal is computed by means of the Matlab/Simulink® software using the desired reference and the current arm angle. After this step, the resulted signal is sent to the motor driver and then the appropriate voltage is applied on the motor.

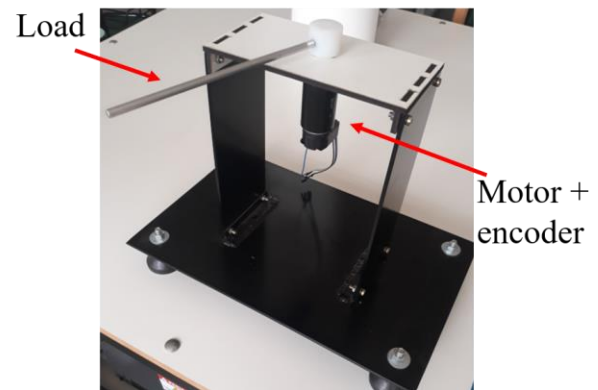


Fig. 1: Didactic prototype for control experiments.

In this work, a PI controller is applied on the described system, according to the block diagram presented in Fig. 2, with the highlights to the prototype (Plant), to the data acquisition system (NI-PCI 6602) and to the software used to the control implementation (Matlab/Simulink®). The details about the project and implementation of the PI controller are presented in the next section.

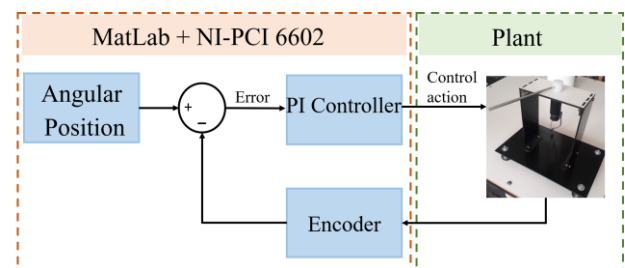


Fig. 2: Control diagram.

III. DISCRETE PI CONTROLLER

Due to its simple implementation, the PI controller is widely used in control systems, mainly fulfilling industrial demands. Different manufacturers of industrial controllers use, basically, variations of the PI control algorithm [1].

Motivated by the constructive simplicity combined with expected satisfactory results, the project of a discrete PI controller is considered in this work, according to the Fig. 2 ("PI Controller" block).

The PI controller in continuous time is defined as [2],

$$u(t) = K_p e(t) + K_i \int_0^t e(\eta) d\eta, \quad (1)$$

in which K_p and K_i are the proportional and integral gains, respectively, and $e(t)$ is the error between the desired reference and the current value of the controlled variable.

In general, a discretization procedure consists of transform a continuous-time signal (analog signal) into a discrete signal, according to the required format (sampling period, quantization, etc) for the computational system. In this sense, the controller can only access the error samples, $e(k.T_s)$, and then it calculates the control signal at the instant $k.T_s$, that is, $u(k.T_s)$, where T_s is the sampling period. The proportional part of the signal $u(t)$ in (1), given by $K_p e(t)$, is computed in discrete time as

$$u_p(k.T_s) = K_p e(k.T_s). \quad (2)$$

However, a special analysis is required to the integral part of the control signal in (1). The Fig. 3 illustrates the integral approximation process (area under the curve) through Euler method. In this case, the area increment is given by the product between the sampling period and the function value at the previous sampling instant. According to this, the following approximation is founded:

$$\int_0^t e(\eta) d\eta \approx \sum_{n=0}^{k-1} T_s e(nT_s) = T_s \sum_{n=0}^{k-1} e(nT_s). \quad (3)$$

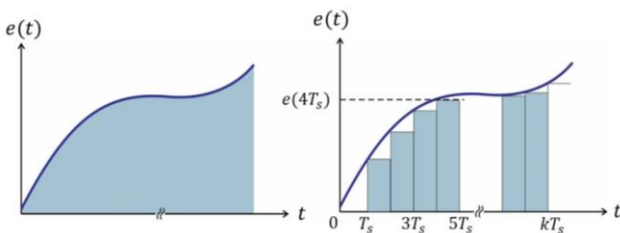


Fig. 3–Integral representation and its approximation through Euler method.

Using (3), the control signal defined in (1), related to the integral part, is computed as

$$u_i(k.T_s) = K_i T_s \sum_{n=0}^{k-1} e(nT_s). \quad (4)$$

Thus, the discrete approximation of the control law (1) can be computed as

$$\begin{aligned} u(t) &\approx u(k.T_s) = u_p(k.T_s) + u_i(k.T_s) \\ &= K_p e(k.T_s) + K_i T_s \sum_{n=0}^{k-1} e(nT_s). \end{aligned} \quad (5)$$

A more efficient way to compute the control signal (5) consists of calculating the increment on the control signal instead of calculating its total value at each instant. For that, consider the increment in the control signal

$$\Delta u(k.T_s) = u(k.T_s) - u((k-1).T_s). \quad (6)$$

From (5) e (6) it follows that

$$\begin{aligned} \Delta u(k.T_s) &= K_p e(k.T_s) + K_i T_s \sum_{n=0}^{k-1} e(nT_s) \\ &\quad - K_p e((k-1).T_s) - K_i T_s \sum_{n=0}^{k-2} e(nT_s) \\ &= K_p [e(k.T_s) - e((k-1).T_s)] \\ &\quad + K_i T_s e((k-1).T_s), \end{aligned} \quad (7)$$

that can be used to compute the control signal. Taking the Z-Transform in (7), with null initial conditions, it is found

$$\begin{aligned} (1 - z^{-1})U(z) &= K_p(1 - z^{-1})E(z) \\ &\quad + K_i T_s z^{-1}E(z) \end{aligned}$$

and then

$$\begin{aligned} \frac{U(z)}{E(z)} &= K_p + K_i T_s \frac{z^{-1}}{(1 - z^{-1})} \\ &= K_p + K_i T_s \frac{1}{z - 1}, \end{aligned} \quad (8)$$

where $U(z)$ is the Z-Transform of the control signal and $E(z)$ is the Z-Transform of the error between the reference and the controlled variable. The Equation (8) describes the transfer function of the discrete PI controller.

Table.1: Parameters of the system.

Parameter	Symbol	Value
Motor's armature resistance	R_a	1.966 Ω
Motor's armature inductance	L_a	0.000424 H
Motor's torque constant	K_m	0.0518 N.m/A
Damping opposing the motor's axis movement	b	2.69e-5 N.m.s/rad
Inertia in the motor's axis	J	1.887e-04 kg.m ²

The controller was designed from the discretized transfer function of the motor system presented in Fig. 1. For that, the parameters presented in Table 1 were considered.

The motor and load dynamic can be described according to the following differential equations [11]:

$$R_a i(t) + L_a \frac{di(t)}{dt} + K_m \frac{d\theta(t)}{dt} = v(t), \quad (9)$$

$$\tau(t) = K_m i(t) = b \frac{d\theta(t)}{dt} + J \frac{d^2\theta(t)}{dt^2}, \quad (10)$$

in which $i(t)$ represents the electric current in the DC motor's armature, $\theta(t)$ is the angle of the motor's axis, $v(t)$ is the voltage applied on the motor, and the other parameter are described in the Table 1.

By some algebraic manipulations in (9) and (10), the dynamic of the motor along with the load is described by the differential equation

$$\frac{d^3\theta(t)}{dt^3} = - \left[\frac{R_a b + K_m^2}{L_a J} \right] \frac{d\theta(t)}{dt} - \left[\frac{R_a J + L_a b}{L_a J} \right] \frac{d^2\theta(t)}{dt^2} + \frac{K_m}{L_a J} v(t). \quad (11)$$

Taking the Laplace Transform in (11), with null initial conditions, it yields

$$\frac{\Theta(s)}{V(s)} = \frac{6.475e5}{s^3 + 4639s^2 + 3.419e4 s}, \quad (12)$$

where $\Theta(s)$ is the Laplace Transform of the angle in the motor's axis, $V(s)$ is the Laplace Transform of the voltage applied on the motor, and the parameter values listed in Table I were already replaced.

Using a Zero Order Holder (ZOH) in the control input, the transfer function (12) was discretized with sampling time of $T_s = 0.02$ s [12], obtaining the transfer function

$$\frac{\Theta(z)}{V(z)} = \frac{0.0261z^2 + 0.0259z}{z^3 - 1.863z^2 + 0.8627z}. \quad (13)$$

Thus, the transfer function (13) was used together with Matlab® in the tuning process of the K_p and K_i gains. The calculated gains for the discrete PI controller are presented in Table 2.

Table 2: Controller's parameters.

K_p	K_i	T_s
0.165	0.115	0.02

For the controller implementation, the software Matlab/Simulink® was used, and the simulation was based on the system transfer function (13) and on the controller transfer function (8).

IV. RESULTS AND DISCUSSION

For the validation of the designed controller and the considered methodology, the control strategy was applied both in simulated experiments and in the real system (prototype shown in Fig. 1), in order to present a comparative study between the results. The experiments consider two distinct cases, named "Case 1" and "Case 2". In the Case 1, a step input (Ref_{step} in Equation (14)) was considered as reference signal. In Case 2, a sinusoidal reference signal (Ref_{sin} in Equation (15)), with a frequency of 0.5 rad/sec, was considered as reference. In (14) and (15) t is the time in seconds.

$$Ref_{step} = \begin{cases} 2\pi, & t \geq 2 \\ 0, & t < 2 \end{cases} \quad (14)$$

$$Ref_{sin} = \begin{cases} \frac{\pi \sin(t/2)}{2}, & t \geq \pi \\ 0, & t < \pi. \end{cases} \quad (15)$$

The results of using the proposed controller in the Case 1 (reference signal (14)) is presented in Fig. 4. Although the real system's response presents an oscillatory characteristic, when it is compared with the simulation's response, in both situations the system's output has stabilized in, approximately, $t = 6.5$ seconds.

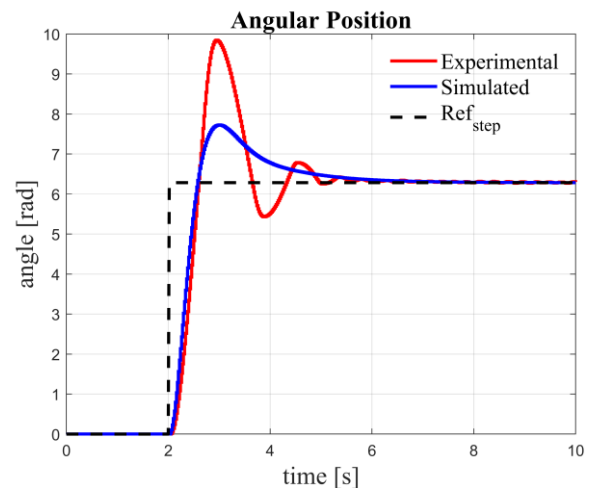


Fig. 4: System's responses with step signal reference of Case 1.

A second analysis of the results of Fig. 4 is about the overshoot part, as summarized in Table 3. It is observed that there is a greater overshoot in the real system.

Probably, these analyzed values can be mitigated with a derivative part inserted in the controller project, or with the application of a more sophisticated tuning method, however more complex. It is noteworthy that this paper reports initial experiments with classical control, which aim the validation of the mathematical model. Obviously, these data suggest other control techniques, such as Fuzzy-PI (intelligent control) for example, can be tested in order to improve the results.

Table.3: Overshoot values for Case 1.

	Maximum value	Overshoot
Experimental	9.795	55.89%
Simulated	7.724	22.93%
Reference	6.283	

Still regarding to the Case 1, the evolution of the angular position's errors along the time is presented in Fig. 5. It is possible to note that the error tends to zero in both situations.

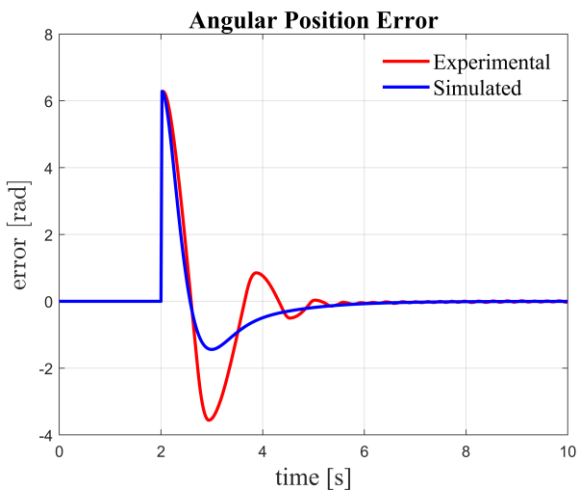


Fig. 5: Angular position's errors for Case 1.

Finally, the control actions (real and simulated) related to the experiments of Case 1 are shown in Fig. 6. It is important to highlight that the motor has a dead zone of 0.27V. This value is compensated in the final control action send to the motor. Moreover, this fact implies an oscillatory response in the real system around the stabilization point.

A similar analysis was addressed considering Case 2 (reference signal in Equation (15)). In this way, the response of the simulation and of the real system experiment is presented in Fig. 7. Although the reference

signal has an oscillatory characteristic, it is possible to observe that the PI controller presented a satisfactory performance in both experiments. However, overshoot still occurs, as shown in Table 4, whose difference between the simulated and the real experiments in this case (30.49%) is practically the same as that found in Table 3 (32.96%).

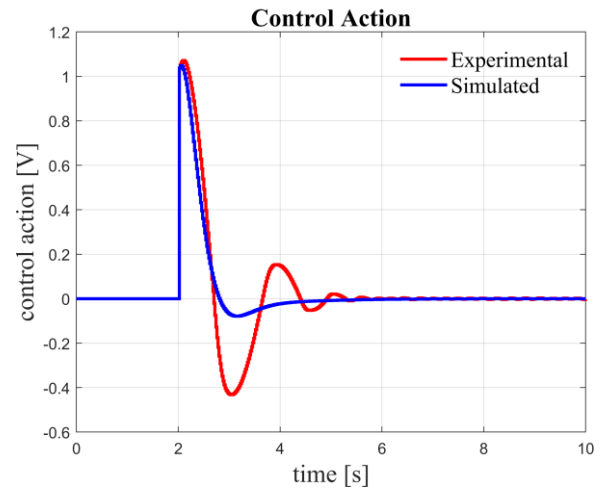


Fig. 6: Control actions for Case 1.

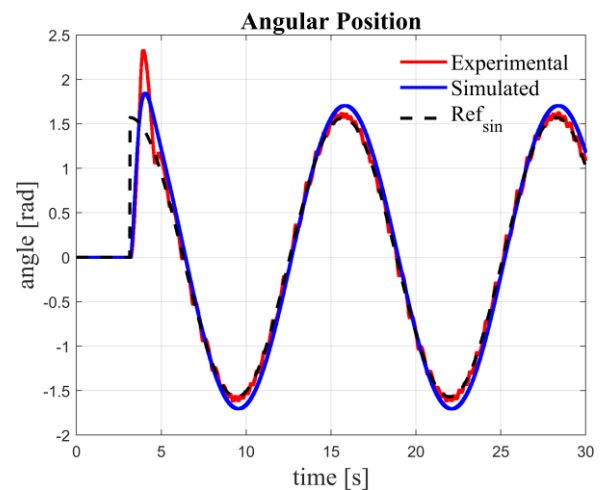


Fig.7: System's responses with sinusoidal reference signal from Case 2.

Table.4: Overshoot values for Case 2.

	Maximum value	Overshoot
Experimental	2.322	47.80%
Simulated	1.843	17.31%
Reference	1.571	

When the system follows the oscillatory characteristic of the reference signal, the experimental result presents a

trajectory error smaller than the simulated results. These values can be verified in Table 5.

Table.5: Trajectory errors for Case 2.

	Maximum amplitude	Trajectory error
Experimental	1.615	2.80%
Simulated	1.705	8.53%
Reference	1.571	

The angular position's errors for Case 2 are presented in Fig. 8, while the respective control actions are shown in Fig. 9.

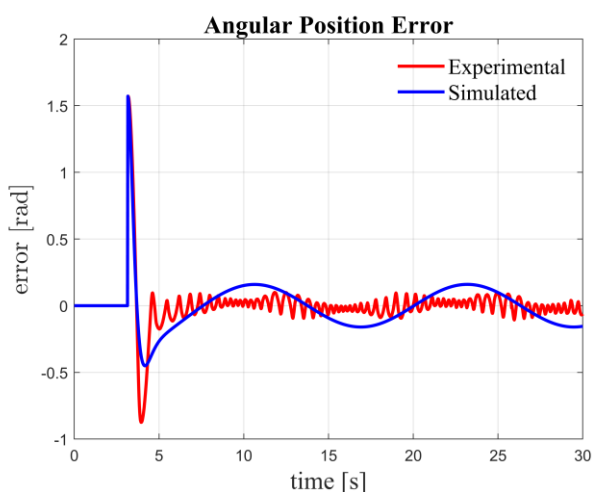


Fig. 8: Angular position's errors for Case 2.

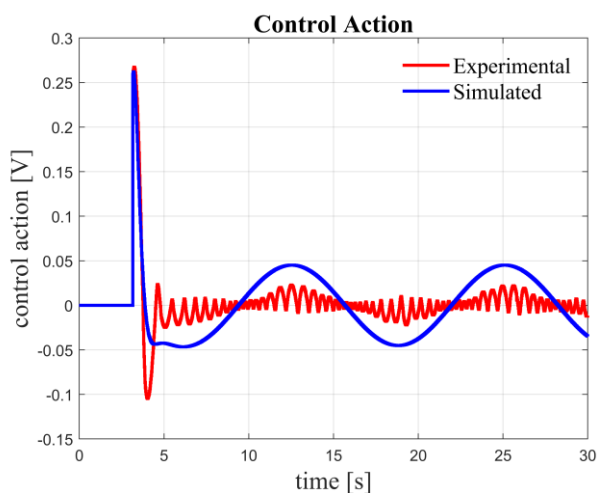


Fig. 9: Control actions for Case 2.

V. CONCLUSION

This work presented the details about the implementation of a discrete PI controller in a real experimental system, which has a set of a DC motor along with an arm (load), aiming to control the angular position of the motor's axis and load.

For the proposed methodology validation, the experimental results of the real system were compared with the simulation results, considering two distinct cases of reference signals (Case 1 and Case 2).

In Case 1, a step input was used and, although overshoot was noticed in all experiments, the proposed controller stabilized the arm (load), that is coupled on the motor's axis, in the desired position. The time indicators were similar in both simulation and real system experiments.

In Case 2, a sinusoidal reference signal was considered, the result from the real system presented a trajectory error smaller than the simulated experiment.

Based on the presented results in both of the analyzed cases, it can be conclude that the designed discrete PI controller was able to control the system's output in order to follow the desired references, leading the error practically to zero. In short, this work fulfilled its objective in the validation of the mathematical model precisely, so that other techniques based on this validated model can be applied.

In future works, we intend to implement other control strategies, as mentioned above, aiming the overshoot reduction and also analyze other performance metrics for the system in closed loop with the PI controller presented in this paper.

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