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Music Algebra: Harmonic Progressions Analysis and CAT (Cataldo Advanced Transformations)

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Abstract**—***In this article we formally introduce an original method, the purpose of which fundamentally lies in providing musicians with a reliable instrument that may effectively assist them in carrying out, as simply and intuitively as possible, the analysis of whatever chord progression, without resorting to the so-called "modal interchange". Net of a single exception (a routine named "structure reduction"), the whole method is based on a series of harmonic transformations. The above-mentioned transformations, named CAT (the acronym stands for Cataldo Advanced Transformations), turn out to be nothing but inverse chord substitutions characterized by specific conditions and restrictions. The method arises from the analysis of a considerable number of chord progressions, devoting particular (although not exclusive) attention to traditional jazz compositions: in this regard, it is worth highlighting how a significant improvement of CAT has been achieved by conducting an extremely thorough analysis of the so-called LEGO Bricks (public domain harmonic patterns).*

Keywords—Music Algebra, Chord Progressions, Chord Substitutions, Plagal Cadence, Perfect Cadence, Jazz, Harmonization, Reduction, Diminished Substitutions, Expansion, Tritone, Secondary Dominants, Diatonic Substitutions, CAT.

I. SHORT INTRODUCTION

The purpose of the method fundamentally lies in providing musicians with a reliable instrument that may effectively assist them in carrying out the harmonic progressions analysis. The method is primarily based upon the application, carried out by following a specific order, of a series of transformations, named *CAT* (Cataldo Advanced Transformations), by means of which whatever harmonic progression may be converted, within certain limits, into a mere sequence of Plagal and Perfect Cadences [1]. As far as jazz is concerned, a significant improvement of the method has been achieved by conducting an extremely thorough analysis of the so called *LEGO Bricks* (public domain harmonic patterns) [2] [3].

II. LIMITATIONS OF THE METHOD

The method is characterized by the following limitations:

The Key of any song must be considered as being major. Consequently, if the key of a song is manifestly minor, the analysis must be carried out by referring to the relative major key (for example, *C Major* instead of *A Minor*). It is worth specifying how a direct analysis of the songs written in minor key is obviously feasible: however, the procedure would require slight modifications concerning the conditions related to some transformations, herein not addressed in order not to weigh down the discussion.

Each Minor Major Seventh chord must be instantly replaced by a Minor Seventh one; similarly, each Augmented Major Seventh Chord must be instantly replaced by a Major Seventh. In other terms, the analysis is carried out by taking into consideration, exclusively, the first five kinds of Seventh Chords.

In the light of their extreme subjectivity, the (inverse) substitutions based on the so-called “Modal Interchange” are herein intentionally ignored. In fact, the outcomes usually obtained by resorting to the modal interchange can be alternatively deduced by exploiting the Quality (Dominant to Major) and Similitude Substitutions. [1]

The time signature must always be imagined as being equal to 4/4. For example, even if we deal with a 3/4, we have to consider four pulses per measure (four beats per bar): each beat, in this case, will be characterized by a duration equivalent to a dotted quaver (see *fig.1*)

Figure 1. Beats in ¾ (three-four time)

III. DESCRIPTION OF THE METHOD

The method consists of ten consecutive phases:

1. <u>Selection of the Key</u> (bearing in mind that the global tonal centre is herein regarded as necessarily major).

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2. The writing of the Ionian Scale Vector (the components of which coincide with the notes that constitute the Ionian Scale) [4] [5]. If we denote with *X* the Global Tonal Centre (the Key) and with *t* a whole tone interval, we have:

$$\boldsymbol{s}^{Ion}(X) = \left(X, X+t, X+2t, X+\frac{5}{2}t, X+\frac{7}{2}t, X+\frac{9}{2}t, X+\frac{11}{2}t\right) \quad (1)$$

For example, if we set $X = C$, we can banally write:

$$\boldsymbol{s}^{Ion}(C) = (C, D, E, F, G, A, B) \quad (2)$$

3. The writing of the Ionian Harmonization Vector. If we denote with $\boldsymbol{h}^{Ion}(X)$ the Ionian Harmonization Vector [4] [5] (the components of which are nothing but the seventh chords that arise from the harmonization of the Ionian Scale of *X*), with $\boldsymbol{M}^{Ion}(X)$ the Ionian Modal Tensor (of *X*) [4] [5], and with $\boldsymbol{d}^{1357} = (1,0,1,0,1,0,1)$ [4] [5] the so-called Seventh Chord Fundamental Vector, we have:

$$\boldsymbol{h}^{Ion}(X) = \boldsymbol{M}^{Ion}(X) \cdot \boldsymbol{d}^{1357} \quad (3)$$

For example, by setting $X = C$, we obtain:

$$\boldsymbol{h}^{Ion}(C) = \begin{bmatrix} C & D & E & F & G & A & B \\ D & E & F & G & A & B & C \\ E & F & G & A & B & C & D \\ F & G & A & B & C & D & E \\ G & A & B & C & D & E & F \\ A & B & C & D & E & F & G \\ B & C & D & E & F & G & A \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} Cmaj7 \\ Dm7 \\ Em7 \\ Fmaj7 \\ G7 \\ Am7 \\ Bm7b5 \end{bmatrix} \quad (4)$$

4. Structure Reduction (net of which a correct application of *CAT* would be de facto impossible). Very simply, the number of bars, as well as the duration of the chords, must be iteratively halved. The procedure is stopped the moment in which even a single chord characterized by a duration equal to a beat appears. Actually, the structure reduction should be applied every time it is possible, so as to obtain the highest simplification level.

5. Diminished Chords Elimination. These chords are herein regarded as deriving from Dominant Seventh Chords subjected to Diminished Substitution. [6] [7] [8]

6. Elimination of Chords that arise from Expansion Substitutions. [6] [7] Actually, the procedure should be applied every time it is possible, so as to obtain the highest simplification level.

7. Transformation of Extraneous Chords (not related to the key of the analysed song) into Diatonic Chords. This phase consists in applying Quality, Similitude, Tritone and Secondary Dominants Inverse Substitutions. [6] [7]

8. Diatonic Transformations of the remaining chords. The analysis of a significant number of traditional jazz compositions has allowed us to accurately determine some restrictions [1] concerning the Diatonic Substitutions. The above-mentioned restrictions are exclusively finalized to obtaining an outcome as simple and coherent as possible.

9. Optional Final Elimination of Chords that arise from Expansion Substitutions [6] [7] (for obvious reasons, this phase exclusively involves chords that coincide with the second component of the Ionian Harmonization Vector).

10. Elimination of the Reduction(s) and Final Outcome. Number of bars and duration of chords must recover their original values.

IV. TRANSFORMATIONS

Inverse Diminished Substitutions

Diminished Chords followed by Dominant Seventh Chords

If we denote with *Z* a generic note, with a_n and a_{n+1}, respectively, the n-th examined chord and the subsequent one, and with sub^{dim} *[dom. chord]* the set constituted by the Diminished Chords (four altogether, net of the enharmonic equivalences) arising from the Diminished Substitution of the Dominant Seventh Chord in square brackets, we have:

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}[Z7] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} Z7 \quad (5)$$

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}\left[\langle Z+\frac{5}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z+\frac{5}{2}t\rangle 7 \quad (6)$$

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}\left[\langle Z+\frac{7}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z+\frac{7}{2}t\rangle 7 \quad (7)$$

In order to explain how to interpret the notation we have been resorting to, the last relation is equivalent to the following assertion: if a Diminished Chord, denoted by a_n, is followed by a Dominant Seventh Chord, denoted by *Z7*, and if a_n, concurrently, belongs to the set of the Diminished Chords that can be obtained by applying a Diminished Substitution to the Dominant Seventh Chord distant an ascending perfect fifth from *Z7*, a_n must be replaced exactly by this chord (a_n must be regarded as deriving from a Diminished Substitution applied exactly to this chord).

Diminished Chords followed by Minor Seventh Chords

According to *CAT*, with obvious meaning of the notation, we have to consider the following transformations:

$$\begin{cases} a_{n+1} = Zm7 \\ a_n \in sub^{dim}\left[\langle Z+\frac{3}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z+\frac{3}{2}t\rangle 7 \quad (8)$$

$$\begin{cases} a_{n+1} = Zm7 \\ a_n \in sub^{dim}\left[\langle Z+\frac{5}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z+\frac{5}{2}t\rangle 7 \quad (9)$$

$$\begin{cases} a_{n+1} = Zm7 = h_i^{Ion}(X) \\ a_n \in sub^{dim}\left[\langle Z + \frac{7}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z + \frac{7}{2}t\rangle 7 \quad i = 2,3,6 \tag{10}$$

$$\begin{cases} a_{n+1} = Zm7 \neq h_i^{Ion}(X) \\ a_n \in sub^{dim}[\langle Z + 5t\rangle 7] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z + 5t\rangle 7 \quad i = 2,3,6 \tag{11}$$

Diminished Chords followed by Major Seventh Chords

According to *CAT*, we have:

$$\begin{cases} a_{n+1} = Zmaj7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{5}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z + \frac{5}{2}t\rangle 7 \tag{12}$$

$$\begin{cases} a_{n+1} = Zmaj7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{7}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z + \frac{7}{2}t\rangle 7 \tag{13}$$

$$\begin{cases} a_{n+1} = Zmaj7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{9}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \xleftarrow{dim.} \langle Z + \frac{9}{2}t\rangle 7 \tag{14}$$

Diminished Chords followed by Half-Diminished Chords

Albeit the case has never occurred during the analysis of more than 300 jazz harmonic progressions, we admit the possibility that a Diminished Chord may be followed by a Half-Diminished one. If this happens, the Diminished Chord cannot be immediately replaced by a Dominant Seventh: in this case, in fact, we have to necessarily wait for the Half-Diminished Chord to be subjected to an inverse substitution, so returning the analysis to one of the cases previously considered. [1]

Minor Seventh Chords and Half-Diminished Chords deriving from Expansion Substitutions

According to *CAT*, denoting with *Y* a generic note, with bar_k the k-th bar, with *T(chord)* and *beat(chord)*, respectively, the duration and the metric placement of the chord in round brackets, we have:

$$\begin{cases} a_n = Ym7, Ym7b5 \\ a_{n+1} = \langle Y + \frac{5}{2}t\rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n|a_{n+1} \xleftarrow{exp.} a_{n+1}|a_{n+1} \tag{15}$$

Quality (Dominant to Major) Inverse Substitutions

According to the method [1], if a Major Seventh Chord does not belong to the Harmonization Vector, it must be considered as deriving from a Quality Substitution (Dominant to Major). [6] [7] Consequently, we have:

$$a_n = Ymaj7 \neq h_1^{Ion}(X), h_4^{Ion}(X) \Longrightarrow a_n \xleftarrow{dom.\ to\ maj.} Y7 \tag{16}$$

Similitude Inverse Substitutions

According to the method [1], if a Minor Seventh or Half-Diminished Chord does not belong to the Harmonization Vector, it must be replaced by a Dominant Seventh chord distant an ascending perfect fourth. We can write:

$$a_n = Ym7 \neq h_i^{Ion}(X) \Longrightarrow a_n \xleftarrow{sim.} \langle X + \frac{5}{2}t\rangle 7 \quad i = 2,3,6 \tag{17}$$

$$a_n = Ym7b5 \neq h_7^{Ion}(X) \Longrightarrow a_n \xleftarrow{sim.} \langle X + \frac{5}{2}t\rangle 7 \tag{18}$$

Tritone (Inverse) Substitutions

$$\begin{cases} a_n = Y7 \\ Y \neq s_i^{Ion}(X) \end{cases} \Longrightarrow a_n \xleftarrow{tri.} \langle Y + 3t\rangle 7 \quad i = 1, \dots, 7 \tag{19}$$

$$\begin{cases} a_n = s_4^{Ion}7 \\ a_{n+1} = h_5^{Ion}(X), \langle s_5^{Ion}(X) + 3t\rangle 7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_7^{Ion}(X)7 \tag{20}$$

In order to clarify the last transformations, let's suppose we are dealing with a song in the key of *C*. The above-mentioned transformation simply requires that the chord *F7*, if preceded by *G7* or D^b7, must be regarded as deriving from a Tritone Substitution applied to *B7* (that, in turn, will be regarded, at a later time, as deriving from a Secondary Dominant Substitution [6] [7] applied to *Bm7b5*).

Secondary Dominants Inverse Substitutions

$$a_n = s_i^{Ion}(X)7 \Longrightarrow a_n \xleftarrow{sec.\ dom.} h_i^{Ion}(X) \quad i \neq 5 \tag{21}$$

Diatonic (Inverse) Substitutions

Transformations involving h_6

$$a_n = h_6^{Ion}(X) \Longrightarrow a_n \xleftarrow{dia.} h_1^{Ion}(X) \tag{22}$$

Transformations involving h_7

$$\begin{cases} a_n = h_7^{Ion}(X) \\ a_{n+1} = h_3^{Ion}(X), h_5^{Ion}(X), \\ a_n, a_{n+1} \in bar_k \\ beat(a_n) = on \end{cases} \Longrightarrow a_n \xleftarrow{dia.} h_2^{Ion}(X)$$

$$otherwise: a_n \xleftarrow{dia.} h_5^{Ion}(X) \tag{23}$$

Transformations involving h_3

$$\begin{cases} a_n = h_3^{Ion}(X) \\ a_{n+1} \neq h_2^{Ion}(X), h_4^{Ion}(X) \\ a_{n-1} = h_2^{Ion}(X), h_4^{Ion}(X) \\ a_{n-1}, a_n \in bar_k \end{cases} \Longrightarrow a_n \xleftarrow{dia.} h_5^{Ion}(X)$$

$$otherwise: a_n \xleftarrow{dia.} h_1^{Ion}(X) \tag{24}$$

Transformations involving h_2

$$\begin{cases} a_n = h_2^{Ion}(X) \\ a_{n+1} = h_1^{Ion}(X) \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_4^{Ion}(X) \qquad (25)$$

Transformations involving h_4

$$\begin{cases} a_n = h_4^{Ion}(X) \\ a_{n+1} \neq h_1^{Ion}(X) \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Ion}(X) \qquad (26)$$

V. FINAL REMARKS

Furthermore, all harmonic progressions could be further simplified by applying the Similitude Substitutions to h_2 and regarding the so-called Tonicization [6] [7] [8] [9] [10] as being a real substitution. Let's consider, for example, the Plagal Cadence *Fmaj7* | *Cma7* (we are in the key of *C*). *Fmaj* can be regarded as deriving from a Diatonic Substitution applied to *Dm7* that, in turn, may be regarded as deriving from a Similitude Substitution applied to *G7*. Finally, the Perfect Cadence so obtained may be considered as deriving from a single chord, *Cmaj7*, subjected to Tonicization. [11] [12] [13]

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