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# Shoaling-breaking Water Wave Modeling Using Velocity Potential Equation with Weighting Coefficient Extracted Analytically from The Dispersion Equation

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***Abstract*— A shoaling and breaking water wave model was developed in this study using the velocity potential of the solution of the Laplace equation. Formulation was carried out using Weighted Kinematic Free Surface Boundary Condition Equation, wave number conservation equation, and energy conservation equation. In the Weighted Kinematic Free Surface Boundary Condition equation, a weighting coefficient was used which needs to be determined. The value of the weighting coefficient was extracted from the dispersion equation. The dispersion equation was formulated using Weighted Kinematic Free Surface Boundary Condition and Weighted Momentum Equation to obtain weighting coefficient that meets the hydrodynamic equilibrium equation. By using the weighting coefficient, good shoaling and breaking results were obtained.*

## I. INTRODUCTION

This study is a follow up research ofHutahaean's (2021a) research. Previously, breaking equations were formulated, namely the equations to calculate the breaking wave height$H_b$, breaker wavelength $L_b$and breaker depth $h_b$. These breaking equations were formulated using weighted Kinematic Free Surface Boundary Condition Equation and the weighted momentum equation similar to this study. The weighting coefficient in that study was obtained by calibrating the breaker height$H_b$model with the breaker height from a number of empirical equations of breaker height index. Therefore, the value of the weighting coefficient was determined by the breaker index equations, instead of based on the hydrodynamic equation.

This study aims to obtain the value of the weighting coefficient analytically. The weighting coefficient was extracted from the dispersion equation formulated from the hydrodynamic equilibrium equations. Thus, it can be said that the weighting coefficient is obtained analytically and fulfills the law of hydrodynamic equilibrium.

## II. POTENTIAL VELOCITY EQUATION

The total velocity potential equation obtained from solving the Laplace equation with Variable Separation Method (Dean (1991)) is,

$$\phi(x,z,t) = G(\cos kx + \sin kx)\cosh k(h+z)\sin \sigma t \quad ..(1)$$

$x$ is the horizontal axis;$z$is the vertical axis; $t$ is time; $G$ and $k$ are wave constants, where $k$ is the wave number, where $L$ is wavelength $k = \frac{2\pi}{L}$, $\sigma$ is angular frequency, for a period of wave$T$, $\sigma = \frac{2\pi}{T}$.At (1) there are two wave constants for which the equation needs to be determined, $G$ and $k$.

There is a value of $kx$ where $\cos kx = \sin kx$. This point is called the characteristic point on the $x$-horizontal axis. The formulation of the wave constants$G$and$k$is carried out at the characteristic point, where the wave constants obtained apply to the*cosine*component and the *sine* component. At this characteristic point, (1) can be written,

$$\phi(x,z,t) = G\ \cos kx \cosh k(h+z) \sin \sigma t \qquad \ldots(2)$$

It is important to note that there is a double value of $G$ in (2).

## III. EQUATIONS OF CONSERVATION

There are conservation equations in velocity potential (1) and (2) (Hutahaean (2020)), namely

The wave number conservation equation,

$$\frac{dk\left(h+\frac{A}{2}\right)}{dx} = 0 \qquad \ldots..(3)$$

$h$is the water depth, $A$is thewave amplitude.In deep water, applies

$$\tanh k\left(h+\frac{A}{2}\right) = c_h \approx 1 \qquad \ldots\ldots(4)$$

$$k\left(h+\frac{A}{2}\right) = \theta\pi \ldots\ldots.(5)$$

$$\tanh \theta\pi = c_h \approx 1 \ldots.(6)$$

$\theta$ is a positive number greater than one. The value$\theta$is determined in the shoaling-breaking condition, where$\theta$affects the breaker depth.

Keeping in mind (3), then (4), (5) and (6) apply to all domains, both in deep water and in shallow water.

The next conservation equation is the energy conservation equation, namely,

$$G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x} = 0 \qquad \ldots\ldots(7)$$

And

$$\frac{\partial^2 G}{\partial x^2} = 0 \qquad \ldots\ldots(8)$$

Equation (8) shows that the wave constant $G$ changes linearly with respect to the $x$-horizontal axis. While, the water depth changes with respect to the $x$-horizontal axis, thenthe wave constant$G$also changes linearly with respect to water depth. Furthermore, (7) and (8) indicate that the wave number$k$also changes linearly with water depth.

## IV. WAVE AMPLITUDE FUNCTION

Wave amplitude function is an equation that expresses the relationship between $G$, $k$ and $A$. This equation is formulated using the weighted kinematic free surface boundary condition (Hutahaean (2021b)), which takes the following form,

$$\gamma\frac{\partial \eta}{\partial t} = w_\eta - u_\eta\frac{\partial \eta}{\partial x} \qquad \ldots.(9)$$

$\eta = \eta(x,t)$ is water surface elevation equation to the still water level; $w_\eta$ is vertical water surface particle velocity; $u_\eta$ is horizontal water surface particle velocity; $\gamma$ is weighting coefficient greater than one whose value will be determined.

From (2), the vertical particle velocity is,

$$w(x,z,t) = -\frac{\partial \phi}{\partial z} = -Gk\cos kx \sinh k(h+z)\sin\sigma t \qquad \ldots\ldots.(10)$$

Vertical water surface particle velocity at $z=\eta$ is,

$$w_\eta = -Gk\cos kx \sinh k(h+\eta)\sin\sigma t \qquad \ldots(11)$$

Horizontal particle velocityis

$$u(x,z,t) = -\frac{\partial \phi}{\partial x} = Gk\sin kx \cosh k(h+z)\sin\sigma t \qquad \ldots\ldots.(12)$$

Horizontal water surface particle velocity at $z=\eta$ is,

$$u_\eta = Gk\sin kx \cosh k(h+\eta)\sin\sigma t \qquad \ldots\ldots.(13)$$

Substitution (11) and (13) to (9),

$$\gamma\frac{\partial \eta}{\partial t} = -Gk\cos kx \sinh k(h+\eta)\sin\sigma t - Gk\sin kx\cosh k(h+\eta)\sin\sigma t\frac{\partial \eta}{\partial x}$$

At the characteristic point where $\cos kx = \sin kx$

$$\gamma\frac{\partial \eta}{\partial t} = -Gk\left(\sinh k(h+\eta) + \cosh k(h+\eta)\frac{\partial \eta}{\partial x}\right)\cos kx\sin\sigma t$$

Or,

$$\gamma\frac{\partial \eta}{\partial t} = -Gk\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)\cos kx\sin\sigma t \qquad \ldots\ldots.(14)$$

As a periodic function, then,

$$Gk\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right) = constant$$

Thus, (14) can be integrated with respect to time $t$ by integrating $\sin\sigma t$ only.

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)\cos kx\cos\sigma t$$

For

$$A = \frac{Gk}{\gamma\sigma}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right) \qquad \ldots(15)$$

The water level equation becomes

$\eta(x,t) = A\cos kx \cos \sigma t$ .....(16)

The characteristic point in the time domain$t$is the value of $\sigma t$ where, $\cos\sigma t = \sin\sigma t$. Then, at the characteristic point in the $x$ ddomain and in the$t$ domain,

$\eta = \frac{A}{2}$ .....(17)

$\frac{\partial\eta}{\partial x} = -\frac{kA}{2}$ ......(18)

Substitution (17) and (18) to (15),

$$A = \frac{Gk}{\gamma\sigma}\cosh k\left(h+\frac{A}{2}\right)\left(\tanh k\left(h+\frac{A}{2}\right)-\frac{kA}{2}\right)$$

Substitution (5) and (6),

$$A = \frac{Gk}{\gamma\sigma}\cosh\theta\pi\left(c_h-\frac{kA}{2}\right)$$

Given that there is a double value in $G$,

$A = \frac{Gk}{2\gamma\sigma}\cosh\theta\pi\left(c_h-\frac{kA}{2}\right)$ .....(19)

This equation is the wave amplitude function, which can be written as the equation for $G$,

$G = \frac{2\gamma\sigma A}{k\cosh\theta\pi\left(c_h-\frac{kA}{2}\right)}$ ......(20)

## V. DISPERSION EQUATION

Surface momentum equation (Hutahaean (2021b)) ignoring convective acceleration,

$\gamma^2\frac{\partial u_\eta}{\partial t} = -g\frac{\partial\eta}{\partial x}$ .......(21)

Substitutions (13) and (16). The equation is done at the characteristic point,

$$\gamma^2\sigma\, Gk\cosh k\left(h+\frac{A}{2}\right) = gkA$$

$k$on the left and right sides cancel each other out, and thewave amplitude $A$on the right side is substituted by (19),

$$\gamma^2\sigma G\cosh k\left(h+\frac{A}{2}\right) = g\frac{Gk}{2\gamma\sigma}\cosh\theta\pi\left(c_h-\frac{kA}{2}\right)$$

The $G$on the left and right sides of the equation cancel each other out. Keeping in mind (5), the terms of the hyperbolic function on the left and right sides also cancel each other out,

$\gamma^2\sigma = \frac{gk}{2\gamma\sigma}\left(c_h-\frac{kA}{2}\right)$ .....(22)

This equation can be written as,

$\frac{gA}{4}k^2 - \frac{gc_h}{2}k + \gamma^3\sigma^2 = 0$ ......(23)

$a = \frac{gA}{4}$ ; $b = -\frac{gc_h}{2}$ ; $c = \gamma^3\sigma^2$ ; $d = b^2 - 4\,ac$

$$k = \frac{-b-\sqrt{d}}{2a}$$

The minus sign used at$\sqrt{d}$ is to make the obtained wavelength not too short.

Table (1) presents the results of the calculation of the wavelength with (23), where the wave height $H$ from the Wiegel equation (1949,1964)is,

$H = \frac{gT^2}{15.6^2}$ .......(24)

which is the maximum wave height in a wave period in deep water. Assuming a sinusoidal wave, the wave amplitude is half of the wave height.

The calculation was carried out using $\gamma = 1.342$, the determination of $\gamma$ is aimed at getting wave steepness $\frac{H}{L} = 0.17$ which is following the criteria of criticalwave steepness from Toffoli et al (2010), where according to Toffoli et al, the critical wave steepness can actually reach more of 0.2.

*Table (1) Wavelength calculation results using (23)*

| $T$ (sec) | $H$ (m) | $L$ (m) | $\frac{H}{L}$ |
|---|---|---|---|
| 6 | 1.451 | 8.516 | 0.17 |
| 7 | 1.975 | 11.591 | 0.17 |
| 8 | 2.579 | 15.139 | 0.17 |
| 9 | 3.265 | 19.161 | 0.17 |
| 10 | 4.03 | 23.655 | 0.17 |
| 11 | 4.877 | 28.623 | 0.17 |
| 12 | 5.804 | 34.064 | 0.17 |
| 13 | 6.811 | 39.978 | 0.17 |
| 14 | 7.899 | 46.365 | 0.17 |
| 15 | 9.068 | 53.225 | 0.17 |

Calculation of the wavelength in Table (1) was carried out using the value of $\theta = 1.95$, where the value of $\tanh\theta\pi = 0.999990$. However, the value of $\theta$ does not really affect the wavelength. The effect of $\theta$ is on the breaker depth, which is discussed in section (7).

## VI. DETERMINATION OF THE VALUE OF THE WEIGHTING COEFFICIENT $\gamma$

Hutahaean (2021b) obtained the value of the weighting coefficient$\gamma$ by usingTaylor series on the function in the form (1), where the value of the weighting coefficient $\gamma = 3$. This coefficient is not yet a product of the hydrodynamic equilibrium equation.

In the previous section, it has been shown that the value of the weighting coefficient $\gamma$ can also be obtained by adjusting the obtained wavelength with the critical wave steepness. However, the value of the weighting coefficient is the result of coercion, not naturally contained in the hydrodynamic equilibrium law.

In this section, the value of the weighting coefficient $\gamma$ is determined analytically, extracted from the dispersion equation (23). This equation is derived from the hydrodynamic equilibrium equation. Hence, it can be said that it is another form of the hydrodynamic equilibrium equation. By extracting the weighting coefficient $\gamma$ from (23), the value of the weighting coefficient$\gamma$ is the product of the hydrodynamic equation.

The determinant $d$in(23)must be greater than or equal to zero.

$$d = \frac{g^2 c_h^2}{4} - 4\frac{gA}{4}\gamma^3\sigma^2 \geq 0$$

The amplitude of wave$A$is known variable which accordingly taking the equals sign will get the value of $\gamma_{max}$.

$$\gamma_{max}^3 = \frac{g c_h^2}{4\sigma^2 A} \quad \text{.....(25)}$$

If$\gamma$from (25) is used, where the determinant $d$=0, then the wave number equation becomes,

$$k = \frac{c_h}{A} \quad \text{...........(26)}$$

In (26), there is no longer a value of $\gamma$, but the equation is formulated based on the condition of the determinant value $d = 0$, then the value of $\gamma$ in (26) is $\gamma_{max}$. It was found that in deep water, the wave-number is only determined by the wave amplitude. Meanwhile, the value of $c_h$ is always close to or equal to one. Moreover, it is also found that the wave number is not determined by the wave period $T$.

Table (2) presents the results of the calculation of $\gamma$ using (25), wave number $k$is calculated by (26) wave height $H$ is calculated by (24), which assuming a sinusoidal wave, the wave amplitude $A = 0.5H$.

*Table (2) Wavelength is calculated by (26), $\gamma$ is calculated by (25)*

| $T$ (sec) | $\gamma$ | $H$ (m) | $L$ (m) | $\frac{H}{L}$ |
|---|---|---|---|---|
| 6 | 1.455 | 1.448 | 4.55 | 0.318 |
| 7 | 1.455 | 1.971 | 6.193 | 0.318 |
| 8 | 1.455 | 2.575 | 8.089 | 0.318 |
| 9 | 1.455 | 3.259 | 10.237 | 0.318 |
| 10 | 1.455 | 4.023 | 12.639 | 0.318 |
| 11 | 1.455 | 4.868 | 15.293 | 0.318 |
| 12 | 1.455 | 5.793 | 18.2 | 0.318 |
| 13 | 1.455 | 6.799 | 21.359 | 0.318 |
| 14 | 1.455 | 7.885 | 24.772 | 0.318 |
| 15 | 1.455 | 9.052 | 28.437 | 0.318 |

Table (2) shows that value of the weighting coefficient $\gamma$ is constant with respect to the wave period of $\gamma = 1.455$. The obtained wavelength is shorter than the previous calculation results because a larger weighting coefficient is used. the wave steepness obtained is constant for all wave periods with a value of $\frac{H}{L} = 0.318$, the value of this wave steepness is much greater than the critical wave steepness value of Toffoli et al. However, the critical wave steepness value is the natural value contained in (23).

Equation (26) obtained by working on the weighting coefficient of$\gamma_{max}$on (23). Therefore, the calculation of the wavelength with (26) at various wave amplitudes produce a wave steepness $\frac{H}{L}$ of 0.318 as presented in Table (3).

*Table (3) Wave steepness of (26)*

| $A$ (m) | $H$ (m) | $L$ (m) | $\frac{H}{L}$ |
|---|---|---|---|
| 0.1 | 0.2 | 0.628 | 0.318 |
| 0.2 | 0.4 | 1.257 | 0.318 |
| 0.3 | 0.6 | 1.885 | 0.318 |
| 0.4 | 0.8 | 2.513 | 0.318 |
| 0.5 | 1 | 3.142 | 0.318 |
| 0.6 | 1.2 | 3.77 | 0.318 |
| 0.7 | 1.4 | 4.398 | 0.318 |
| 0.8 | 1.6 | 5.027 | 0.318 |
| 0.9 | 1.8 | 5.655 | 0.318 |
| 1 | 2 | 6.283 | 0.318 |

## VII. FORMULATION OF SHOALING-BREAKING

**Equation**

The shoaling and breaking equations are formulated using the conservation equations contained in (1), (Hutahaean (2020)) namely,

a.The energy conservation equation (7) which can be written as

$$k\frac{\partial G}{\partial x}=-\frac{G}{2}\frac{\partial k}{\partial x} \quad \ldots\ldots.(27)$$

b.The wave number conservation equation (3), which can be written as

$$\left(h+\frac{A}{2}\right)\frac{\partial k}{\partial x}+\frac{k}{2}\frac{\partial A}{\partial x}=-k\frac{\partial h}{\partial x} \quad \ldots.(28)$$

Equation (19) is differentiated about the $x$-horizontal axis and (27) is substituted for the product of the differential,

$$\frac{\partial A}{\partial x}=\frac{G\cosh\theta\pi}{4\gamma\sigma}\left(c_h-\frac{kA}{2}\right)\frac{\partial k}{\partial x} \quad \ldots..(29)$$

To make writing easier, defined

$$\alpha=\frac{G\cosh\theta\pi}{4\gamma\sigma}\left(c_h-\frac{kA}{2}\right) \quad \ldots.(30)$$

Thus, (29) becomes

$$\frac{\partial A}{\partial x}=\alpha\frac{\partial k}{\partial x} \quad \ldots...(31)$$

Substituting this equation into (28), it is obtained,

$$\left(\left(h+\frac{A}{2}\right)+\frac{k}{2}\alpha\right)\frac{\partial k}{\partial x}=-k\frac{\partial h}{\partial x} \quad \ldots.(32)$$

This equation is used to calculate $\frac{\partial k}{\partial x}$. After $\frac{\partial k}{\partial x}$ is obtained, $\frac{\partial A}{\partial x}$can be calculated using (29).

Calculation steps:

1.At a point$x=x$, given$h_x$, $A_x$, $k_x$, $G_x$, will calculate $A_{x+\delta x}$, $k_{x+\delta x}$and $G_{x+\delta x}$, at point$x=x+\delta x$

2.Calculate :$\alpha=\frac{G_x\beta}{4\gamma^2\sigma}\left(c_h-\frac{k_xA_x}{2}\right)$

3. Calculate $\frac{\partial k}{\partial x}$ with the following equation :

$$\left(\left(h_x+\frac{A_x}{2}\right)+\frac{k_x}{2}\alpha\right)\frac{\partial k}{\partial x}=-k_x\frac{\partial h}{\partial x}$$

4. Calculate $\frac{\partial A}{\partial x}$with the equation: $\frac{\partial A}{\partial x}=\alpha\frac{\partial k}{\partial x}$

5. Calculate $k_{x+\delta x}$and$A_{x+\delta x}$ :

a. $k_{x+\delta x}=k_x+\delta x\frac{\partial k}{\partial x}$

b. $A_{x+\delta x}=A_x+\delta x\frac{\partial A}{\partial x}$

6. $G_{x+\delta x}$is calculated analytically by integrating (27),

$$G_{x+\delta x}=e^{\ln G_x-\frac{1}{2}(lnk_{x+\delta x}-lnk_x)}$$

At the starting point, namely in deep water, wave number$k$was calculated by (26), $G$was calculated by (20) and wave amplitude$A$was calculated by (24).

Fig (1) visualizes the results of the shoaling-breaking model for a wave period of 8 sec., the deep water wave height $H_0$ was calculated by (24) assuming a sinusoidal wave was $A_0=0.5H_0$. As a calculation parameter of$\gamma$(25) and the coefficient ofdeep water depth $\theta=1.95$ were used where $\tanh\theta\pi=0.999904$.The results of the calculation of $H_b$ are presented in Table (3), namely $H_b=2.987$ m while the results of the calculation using (33), namely the equation of Komar&Gaughan (1972), obtained $H_b=3.019$ m. Breaker depth index $\frac{H_b}{h_b}=0.78$, according to Mc Cowan's (1984) breaker depth index equation.

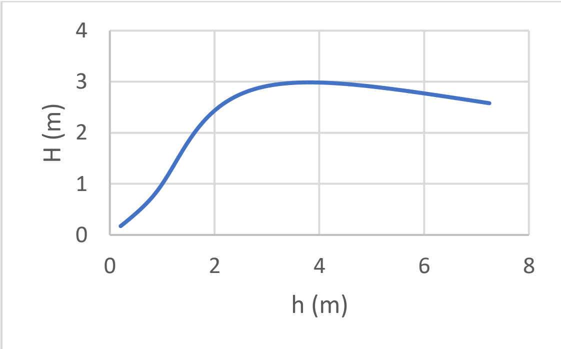


*Fig (1) Example of a shoaling-breaking model result*

The equation for breaking wave height $H_b$ from Komar & Gaughan (1972) is as follows:

$$H_b=0.39\,g^{1/5}(TH_0^2)^{2/5}\ (\mathrm{m}) \quad \ldots\ldots...(33)$$

This equation uses a primitive variable, namely the wave period $T$, making it very practical in its use.

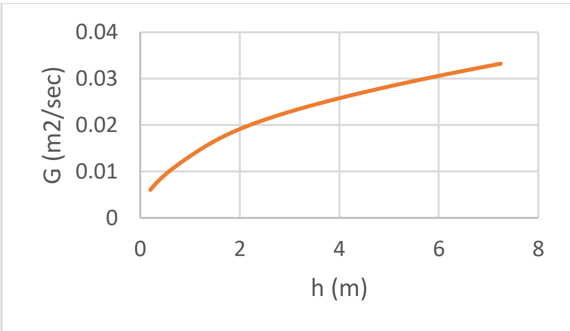


*Fig.(2). Change ofGwith water depth h*

(8) shows that the change in the wave constant $G$with respect to water depth is linear. In Fig (2), the linear

nature only occurs before breaking, while after breaking, it is nonlinear, especially in very shallow waters.

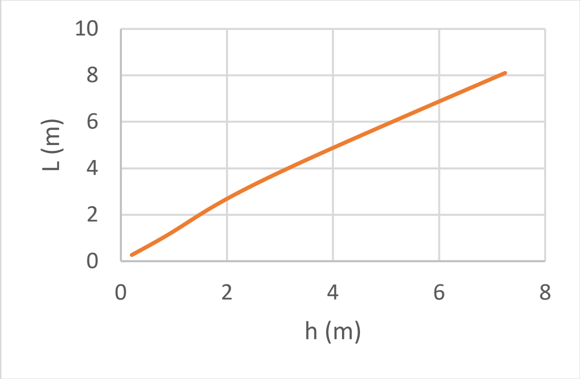


*Fig.(3). Change of wavelength Lwith water depth h*

Using (7) and (8), it was found that the change in wave number$k$on water depth is linear, the same with the one on wavelength$L$. Figure (3) shows that the change in wavelength$L$is linear, but slightly nonlinear in very shallow waters.With changes in $G$ and changes in $k$ as shown, further research and development is needed on wave conditions after breaking, especially in very shallow waters.

*Table (4) Results of the shoaling-breaking model for a number of wave periods*

| $T$ (sec) | $H_0$ (m) | $H_b$ (m) model | $H_b$ (m) K-G | $\frac{H_b}{L_b}$ | $\frac{H_b}{h_b}$ |
|---|---|---|---|---|---|
| 6 | 1.451 | 1.681 | 1.698 | 0.637 | 0.781 |
| 7 | 1.975 | 2.287 | 2.312 | 0.638 | 0.782 |
| 8 | 2.58 | 2.987 | 3.019 | 0.637 | 0.781 |
| 9 | 3.265 | 3.78 | 3.821 | 0.637 | 0.78 |
| 10 | 4.031 | 4.667 | 4.718 | 0.637 | 0.781 |
| 11 | 4.878 | 5.646 | 5.708 | 0.637 | 0.781 |
| 12 | 5.805 | 6.719 | 6.794 | 0.637 | 0.781 |
| 13 | 6.812 | 7.886 | 7.973 | 0.637 | 0.781 |
| 14 | 7.901 | 9.146 | 9.247 | 0.637 | 0.781 |
| 15 | 9.07 | 10.499 | 10.615 | 0.637 | 0.781 |

*Note : K-G, Komar&Gaughan*

The stability of breaking characteristics including $\frac{H_b}{L_b}$ and $\frac{H_b}{h_b}$ in the model on the wave period is shown in Table (4), indicating that both $\frac{H_b}{L_b}$ and $\frac{H_b}{h_b}$ can be said to be constant.The calculations in Table (4) were carried out using$\gamma$of (25), the coefficient ofdeep water depth $\theta = 1.95$.

To determine the effect of the deep water depth coefficient$\theta$, a calculation was carried out using $\theta = 1.85$ with $\gamma$ from (25).

Table (5) Results of the shoaling-breaking model with $\theta = 1.85$

| $T$ (sec) | $H_0$ (m) | $H_b$ (m) model | $H_b$ (m) K-G | $\frac{H_b}{L_b}$ | $\frac{H_b}{h_b}$ |
|---|---|---|---|---|---|
| 6 | 1.451 | 1.681 | 1.698 | 0.639 | 0.835 |
| 7 | 1.975 | 2.287 | 2.312 | 0.638 | 0.833 |
| 8 | 2.58 | 2.987 | 3.019 | 0.637 | 0.832 |
| 9 | 3.265 | 3.78 | 3.821 | 0.637 | 0.833 |
| 10 | 4.031 | 4.667 | 4.718 | 0.638 | 0.833 |
| 11 | 4.878 | 5.646 | 5.708 | 0.637 | 0.831 |
| 12 | 5.805 | 6.72 | 6.794 | 0.637 | 0.832 |
| 13 | 6.812 | 7.886 | 7.973 | 0.637 | 0.832 |
| 14 | 7.901 | 9.146 | 9.247 | 0.637 | 0.831 |
| 15 | 9.07 | 10.499 | 10.615 | 0.637 | 0.832 |

The calculation results in Table (5) present that there is no change in the breaking wave height$H_b$, even though the deep water depth coefficient $\theta$was reduced. The changes that occur were the enlargement of $\frac{H_b}{L_b}$ and $\frac{H_b}{h_b}$. This shows that the reduction of the deep water depth coefficient$\theta$causes the shallower breaker depth $h_b$.

## VIII. CONCLUSIONS

This study concludes that the weighting coefficient$\gamma$on theweighted kinematic free surface boundary condition and on the weighted momentum equation can be extracted from the dispersion equation.

The dispersion equation is formulated using hydrodynamic equilibrium equations. Accordingly, the value of the weighting coefficient obtained is to meet the laws of hydrodynamic equilibrium equations.

A good shoaling-breaking model is obtained using the weighting coefficient extracted from the dispersion equation. However, a large critical wave steepness is obtained both in deep waters and at the breaking point. Therefore, further research is still needed, both on the model and on the existing critical wave steepness criteria, both in deep water and at the breaking point.

In general, the results of the shoaling-breaking model provide good information regarding the height of the shoaling wave and the breaking wave height. However, further research is still needed on wave conditions after breaking, especially in very shallow waters.

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