

Modeling an experiment to measure the speed of gravity in short distances using vibrating masses: Frequency optimization

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Abstract— *In order to investigate the behavior of gravitational signals while travelling through a medium an experiment was designed, aimed at measuring the speed of these signals over short distances. The experiment contains 2 sapphire vibrating devices that emit a signal and one sapphire device that behave as a detector, which are suspended in vacuum and cooled down to 4.2 K. The amplitude of the detecting device is measured by an ultralow, phase-noise microwave signal that uses resonance in the whispering gallery modes. Since sapphire has a quite high mechanical Q, the detection band is expected to be small, thus reducing the detection sensitivity. A new shape for the detecting device is presented in this work, yielding a detection band of several hundred Hertz. With the aid of a Finite Element Program the normal mode frequencies of the detector are determined assuming the detector as a spring-mass system. The results show that the detection is achievable then the best operational frequency is determined.*

I. INTRODUCTION

The GRAVITON group is a Brazilian research group dedicated to the study of gravity, whose gravitational waves consist of its main area of interest. The announcement of the first direct detection of gravitational waves happened in 2016 (Abbot et al., 2016). The first attempts to directly detect gravitational waves date from the early 1960's, using resonant-mass gravitational wave detectors (Aguira, 2011).

GRAVITON's efforts for the direct detection of gravitational waves (GW) are concentrated on

SCHENBERG detector, whose main detection mass consists of a sphere with 0.65m in diameter, made of solid Cu6%Al alloy. Six transducers are connected to the sphere's surface in a semi-dodecahedron distribution. These mechanically amplify the motion and excite a membrane in a resonant cavity where microwaves signal is pumped. As these microwaves leave the cavity, they carry a sideband signal that contains information on the GW's amplitude. The direction of the incoming GW can also be found from the

analysis of the signals from the six transducers (Magalhaes, 1997A; Magalhaes, 1995; Magalhaes, 1997B).

Some of the research carried out in the GRAVITON group is presented in the references (Frajuca, 2002; Frajuca, 2004; Frajuca, Bortoli & Magalhaes, 2005; Frajuca, Bortoli & Magalhaes, 2006; Frajuca, 2008; Frajuca & Bortoli, 2006; Andrade, 2004; Ribeiro, 2004; Bortoli, 2010; Bortoli, 2016; Bortoli, 2019; Bortoli, 2020; Aguar, 2002) and the detector schematics is displayed in Fig. 1.

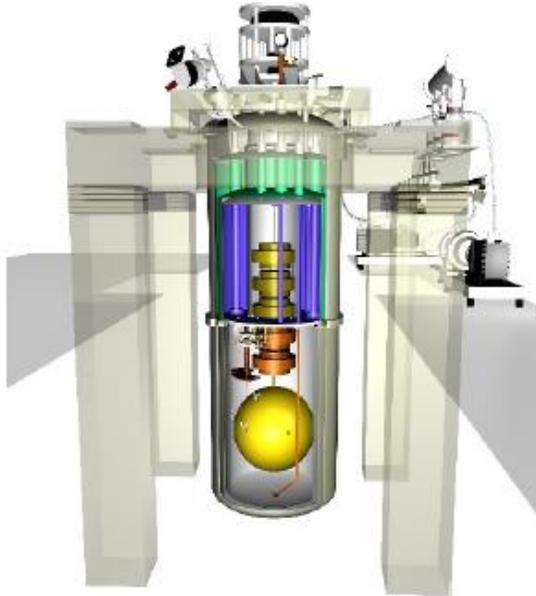


Fig. 1: The resonant-mass gravitational wave Brazilian detector SCHENBERG (Schematics by Xavier P. M. Gragens).

The expertise gained in the field of GW detection projecting the experiment entitled the group with knowledge to design an experiment to measure the speed of gravity. The measure of GW promises to determine the speed of gravity, nevertheless there will always be uncertainty if the signal coming from the GW is the correct one and multi messenger detections (GW and optical) seem to be quite rare.

In order to measure this speed a gravitational signal must be produced, but it is impractical to produce GW in the laboratory then a practical substitute is a gravitational tidal signal produced as a quadrupolar distribution of masses must rotate at a very high speed in a very stable motion, but this demands an engine operating at a very high and very stable rotational speed that could be very difficult. A substitute is to make masses oscillate by changing the

quadrupolar distribution of masses, as can be seen on the reference (Frajuca & Bortoli, 2019).

II. METHOD

Let's follow the procedures presented in (Frajuca & Bortoli, 2019) where the model for the artificial generation of gravitational signals is presented.

The experiment consists of three sapphire devices, first consider that these devices are sapphire bars (see Fig. 2). These devices would be apart for a distance x , (see Fig. 3), suspended as seen in Fig.2. The bars at the sides would emit gravitational signals due to vibrations, these vibrations will be excited by PZT systems, and those signals will excite the central sapphire. The amplitudes of this vibrations will be determined using the central bar itself as a microwave cavity excited by an ultra-low-noise microwaves (Fig. 3). The experiment would operate at 4.2 K in high vacuum.

Sapphire was chosen due to its physical properties such as mechanical and optical: mechanical quality factor of 10^9 and electrical quality factor for microwaves of 10^8 . The determination of the gravitational tidal amplitude force applied to the central bar (detector) is done using the model shown in Fig. 4 and is presented in Ref. (Frajuca, 2019). Also in this reference the viability of the experiment is proven, and the change proposed below, that changes the bars for sapphire devices and the frequency optimization improves those results.

The improvement is given by changing the detection bar for a more complex oscillator that will work as the detector as this shape has three vibration modes, they will have a detection band wider, as the detection band is given by the distance between the lower and the higher detection frequencies. These detection bands were simulated with the aid of a Finite Element Modeling (FEM) program (Solidworks software). In Fig. 5 the new experiment mounting is shown. Figures 6, 7 and 8 display the detector's vibrational modes, which occur at the frequencies of 4722.0 Hz, 5958.7 Hz and 7169.6 Hz (for this example of the simulation, it can be changed, changing the length of the individual bars).

For this example the bandwidth is of the order of 2500 Hz, what increases the bandwidth by a big factor. Original detector had a very narrow bandwidth as the mechanical quality factor is quite high and it had one one vibration mode.

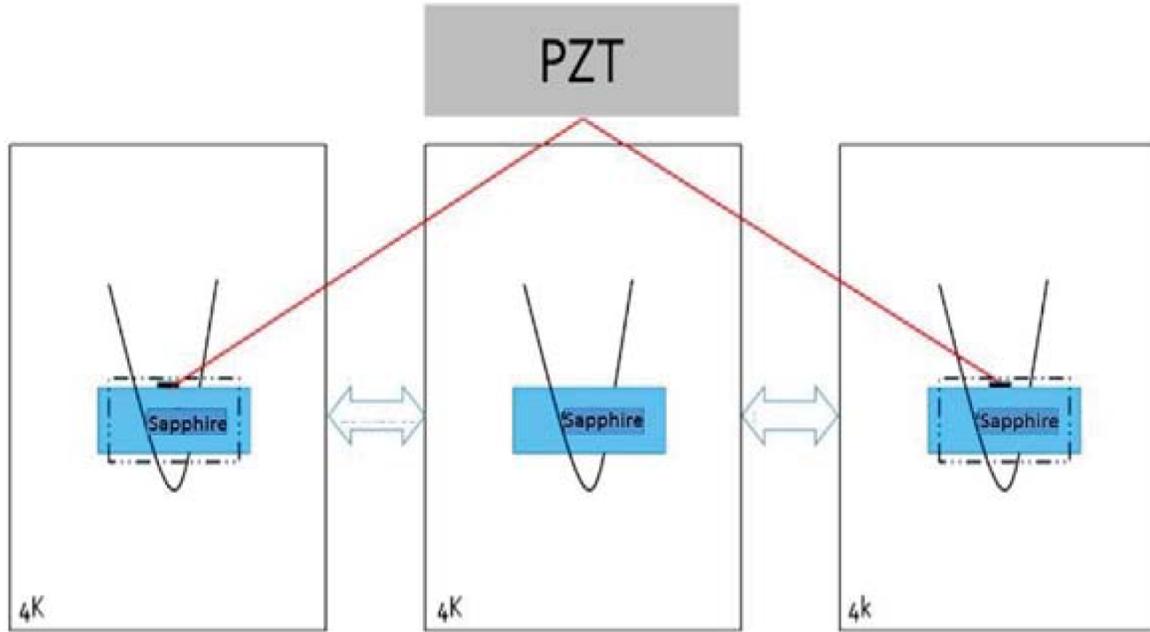


Fig. 2: Planned design of the project with PZT in phase signal. Distance x between emitters (in the sides of the experiments) and the detector (in the middle).

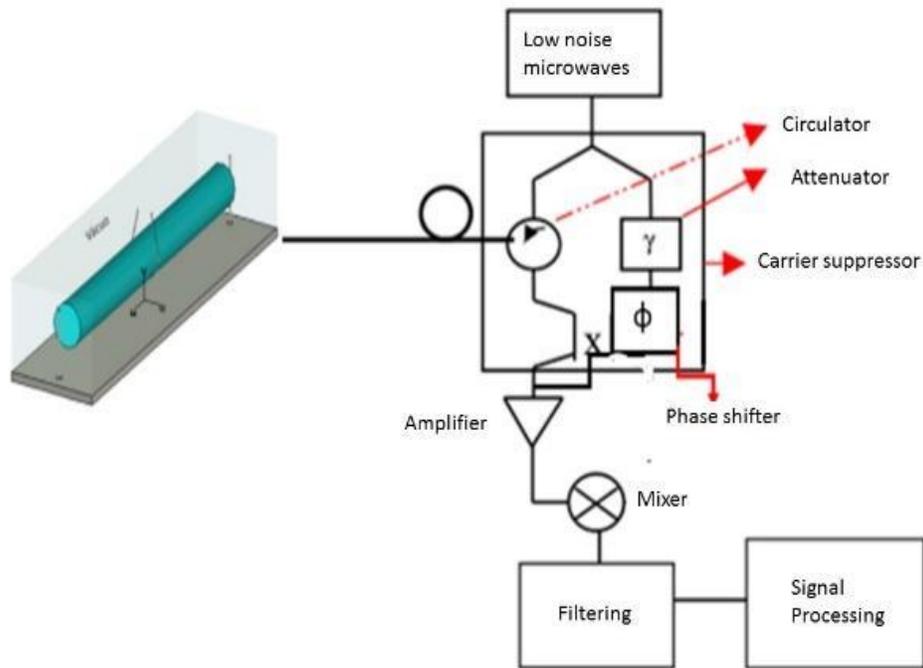


Fig. 4: Diagram of the experiment electronics using the low noise oscillator connected to the detecting central sapphire bar from the authors.

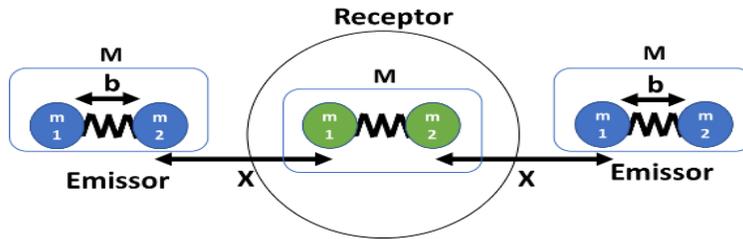


Fig. 4: Model for the detector and the emitters of oscillator tidal gravitational signals.(Figure from the authors)

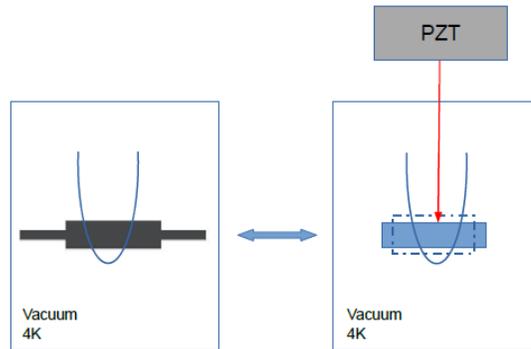


Fig. 5: The new experiment mounting of the planned detector. Figure from the authors

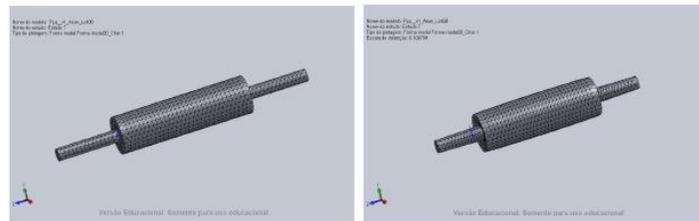


Fig. 6: Vibrational mode of the detector at 4722.0Hz. The authors.

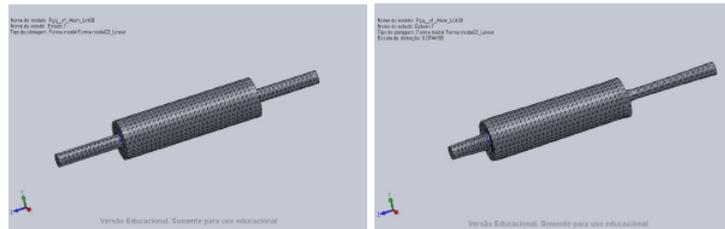


Fig. 7: Vibrational mode of the detector at 5958.7Hz. The authors.

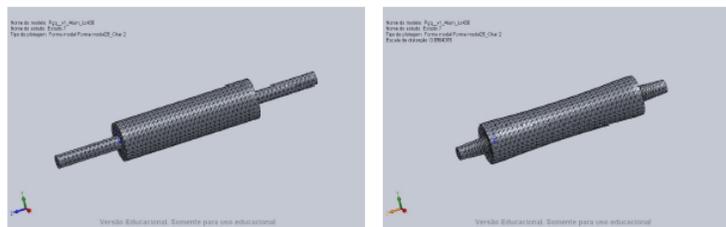


Fig. 8: Vibrational mode of the detector at 7169.6Hz. The authors.

III. RESULTS

In the calculations of the next subsection follows the same steps presented at (Frajuca, 2019) with the following parameters:

Oscillator phase noise: -160 dBc/Hz at 1 kHz;

$$M_{eff} = 1kg;$$

x = 1 m distance between the masses;

$$Q = 10^9;$$

$$G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2};$$

a = 10⁻⁴ (Vibration amplitude of the bars);

b = 1 m (Equivalent size of the bars);

x = 1.0 m (Distance between detector and emitter);

Frequency bandwidth (BW): 1000 Hz;

$$h = 6.626\ 069 \times 10^{-34} J \cdot s;$$

$$f = 10^3 Hz;$$

$$K = 1.38064852 \times 10^{-23} m^2 kg s^{-2} K^{-1};$$

df/dx = 2 x 10¹² Hz/m (Frequency sensitivity).

3.1 Signal amplitude

Following the work done in (Frajuca, 2019):

$$\Delta b = \frac{QGM_{eff}24ab^2}{w^2x^5} = 4 \cdot 10^{-12} m \quad (1)$$

3.2 Quantum limit

This corresponds to the minimum limit, when the number of phonons is 1. Therefore,

$$E = \hbar w. \quad (2)$$

$$hf = \frac{1}{2}A^2w^2m \Rightarrow hf = \frac{A^2w^2M_{eff}}{2}$$

$$\Delta b_{QL} = A = \sqrt{\frac{2\hbar}{\omega M_{eff}}} \quad (3)$$

$$\Delta b_{QL} = \sqrt{\frac{2\hbar}{M_{eff}2\pi f}} = 2 \cdot 10^{-19} m \quad (4)$$

3.3 Equipment sensitivity limit

$$S_x(f) = \left(\frac{df}{dx}\right)^{-2} S_\phi(f)f^2 \quad (5)$$

$$S_x = 0.5 \sqrt{10^{-34}} = 5 \times 10^{-18} \frac{m}{\sqrt{Hz}} \quad (6)$$

Using BW (bandwidth) = 1000 Hz:

$$\Delta b_{ES} = 1,6 \cdot 10^{-18} m \quad (7)$$

3.4 Thermal noise limit

$$\Delta b_{th} = \sqrt{\frac{KT}{2M_{eff}\omega Q (BW)}} \quad (8)$$

$$\Delta b_{th} = 2 \times 10^{-20} m \quad (9)$$

IV. DISCUSSION AND OPTIMIZATION FREQUENCY OPTIMIZATION

The results show that the detection is possible. Let's try to optimize the frequency.

The angular velocity of a harmonic oscillator is given by:

$$w^2 = k/m \quad (10)$$

and the energy is given by:

$$E = \frac{1}{2} k a^2 = 0.5 m w^2 a^2 \quad (11)$$

Then the vibration amplitude of the resonator is given by:

$$a = 1/w \sqrt{\frac{2E}{m}} \quad (12)$$

Using the parameters chosen in this work this energy is equal to 20 J.

Replacing this expression for a given energy of equation (12) in (1) and finding the same value for Δb given by the expression in (6), one can obtain the frequency where these values are the same and its value is 53 kHz. Then an operation frequency chosen of 1 kHz is reasonable in sensitivity.

V. CONCLUSION

The values for the frequency of 1 kHz for the signal and limit are:

Quantum limit: Δb_{QL} = 4 x 10⁻¹⁹ m;

Equipment sensitivity limit: Δb_{ES} = 1.6 x 10⁻¹⁸ m;

Thermal noise limit: Δb_{th} = 2 x 10⁻²⁰ m.

The signal amplitude is $\Delta b = 4 \times 10^{-12}$ m. The new shape for the detector improves the sensitivity of the because it lowers the sensitivity of the equipment noise sensitivity.

The frequency chosen makes the experiment possible at these parameters, nevertheless it is not determined yet if the phase can be obtained at this frequency and distance for the emitter and detector. The next step is to incorporate in the calculation the measurement of the phase difference between the emitted signal and the received one.

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