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Synthesis optimal guidance law for missile in cruise phases with terminal additional requires

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***Keywords*—Guidance law, optimal control, objective function.**

***Abstract*— *The article analyzes and proposes the synthesis of an optimal guidance method for missiles during the cruise phase in a combined remote–autonomous control system. The objective of the problem to be solved is to guide the missile to a non-fixed point in space according to a predefined rule, meeting specific requirements and criteria. In combined remote– control systems, at the moment of switching to guidance, the missile must satisfy several stringent requirements, such as: ensuring the target is within the seeker's field of view, determining the distance to the target, meeting the initial slip value h_0 at the moment of autonomous guidance, and considering the missile's maneuverability, among others. The article focuses on researching a synthesized optimal guidance law for the missile during the remote control phase while taking into account these requirements at the transition point to autonomous control at the end of the remote control phase.*

I. INTRODUCTION

A combined control system is a system capable of utilizing different control principles either in parallel or sequentially, incorporating various guidance methods (GMs). Each GM has its own advantages and limitations [1,3,6]. Combined control is employed to fully exploit the strengths of different GMs while minimizing their drawbacks [3].

The main advantages of a combined control system include the ability to control missiles at long or even very long ranges, expanding measurement range, assessing approach velocity and target line-of-sight angular rate, improving guidance accuracy, enhancing jamming resistance, and increasing control reliability.

The article discusses a form of combined remote–autonomous control for surface-to-air missiles (SAMs), where control is divided into two sequential phases: in the initial phase, the missile (M) is remotely controlled (cruise mode); in the final phase, it transitions to autonomous guidance until it intercepts the target (T) (autonomous mode).

The focus of the article is on analyzing the missile's trajectory and orientation to develop a guidance law during the remote control phase, ensuring that, at the transition point, all conditions for autonomous guidance are fully met. Thus, the objective of the guidance method (GM) in cruise mode is to create favorable conditions for the transition to autonomous guidance. These conditions include [7]:

- Transition timing that aligns with the seeker's detection range, considering autonomous guidance duration and the missile's maneuverability.

- Proper missile orientation angles, ensuring the target remains within the seeker's field of view, facilitating rapid detection, acquisition, and tracking within a limited timeframe.

- Acceptable initial miss distance (h_0) - the distance between the target's center of mass and its projection along the missile-target approach velocity vector [3].

When synthesizing the remote–autonomous combined control system, the following problems must be addressed [7]:

- Trajectory matching between the two control phases (remote and autonomous), ensuring smooth transitions, even if the target maneuvers.

- Seeker orientation towards the target, enabling rapid detection, acquisition, and tracking.

- Reliable transition from remote control to autonomous guidance, avoiding sudden disturbances in the closed-loop control system.

With this problem formulation, the article focuses on synthesizing an optimal missile guidance law for the remote control phase, ensuring it meets all autonomous guidance requirements at the transition point.

II. PROBLEM

2.1. Theoretical foundations

2.1.1 Synthesis of a Remote Guidance Method for Missiles Based on Optimal Control Theory

Modern control theory enables the implementation of various complex information processing and control algorithms [2]. To develop a two-point guidance method for remotely controlled surface-to-air missiles (SAMs), the problem can be formulated and solved based on optimal control theory as follows.

The equation of motion for a SAM is typically expressed as:

$$\dot{r}_P = V_p$$
$$\dot{V}_p = u_P$$

Where:

V_p – Missile velocity vector as the time derivative of range $\dot{r}_p$; u_P - Desired control vector.

With $x_1 = r_P$; $x_2 = V_p$; $u = u_P$, The missile's equation of motion is expressed in the following canonical form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \qquad t_0 \le t \le t_k \tag{1}$$

With initial conditions: $x_1(\mathrm{t}_0) = x_{10}$; $x_2(\mathrm{t}_0) = x_{20}$

The specific objective of the remote guidance process, without considering the autonomous guidance phase, is that the missile's final trajectory point must coincide with the target's final trajectory point. This means:

$$x_1(\mathrm{t}_k) = x_ц(\mathrm{t}_k) \tag{2}$$

The cost function of the control process is usually written in the form:

$$I = \frac{1}{2}\int_{t_0}^{t_k} u^T(\tau)u(\tau)d\tau \tag{3}$$

The problem is to find the control u(t) in such a way that the cost function (3) is minimized.

We consider the characteristics of the optimization process in its general form. The control system model is described by a differential equation:

$$\dot{x} = A(\mathrm{t})\,\mathrm{x} + \mathrm{B}(\mathrm{t})\,\mathrm{u} \qquad t_0 \le t \le t_k \tag{4}$$

Initial condition is $x(\mathrm{t}_0) = x_0$ with:

x – the state vector of size n;

u – control vector of size m;

A, **B** – The state transition matrix and the control matrix with their corresponding sizes are x_n and u_m.

The requirements for the system's state at the final time point can be set in the form:

$$y(\mathrm{t}_k) = Sx(t_k) \tag{5}$$

With $y(t_k)$ – The vector of pre-set requirements at time t_k with size l (l<n);

S – size matrix (lxn).

We consider a new phase vector at any given time t:

$$y(\mathrm{t}) = S\Phi(t_k, t)x(t) \tag{6}$$

Where $\Phi(t_k, t)$ - The basis matrix of a homogeneous system (4). At time t=t_k (6) convert to (5). At this point $\Phi(t_k, t_k) = \mathrm{E}$ (unit matrix).

Vetor y(t) follows differential equation:

$$\dot{y}(\mathrm{t}) = S\Phi(t_k, t)\,\mathrm{B(t)}u(t) = \mathrm{F(t)\,u(t)} \tag{7}$$

With $\mathrm{F(t)} = S\Phi(t_k, t)\,\mathrm{B(t)}$. Equation (7) Obtained when considering in the equation (4) và $\Phi(t_k, t)$ follows equation: $\dot{\Phi}_t = -\Phi(t_k, t)A(t)$.

From (7), we can see that to achieve the final state y(t_k), control must be applied to the vector y(t) and the state forecasts of this vector.

The necessary condition for optimization is that the implicit function has the form: $\frac{dH}{du} = 0$, where H is the Hamiltonian function.

From (3) and (7), we obtain:

$$H(\Psi, u, t) = \frac{1}{2}u^T u + \Psi^T F(\mathrm{t})\,\mathrm{u} \tag{8}$$

Ψ - The constraint vector satisfies the equation: $\dot{\Psi} = -(\frac{dH}{du})^T$

By satisfying the necessary condition (8), the optimization problem is transformed into solving the following system of differential equations:

$$\begin{aligned} &\dot{y} = F(\mathrm{t})\,\mathrm{u} \\ &\dot{\Psi} = 0 \\ &u = -F^T(\mathrm{t})\Psi \end{aligned} \tag{9}$$

With $\mathrm{y(t_0)} = \mathrm{y}_0;\ \mathrm{y(t}_k) = \mathrm{y}_k;$ the initial and boundary conditions for the vector y(t)y(t)y(t) are determined, while the vector Y(t)Y(t)Y(t) remains free.

Thus $\Psi(\mathrm{t}) = \Psi_0 = const$. To determine Ψ_0 we substitute the third equation of (9) into the first equation and integrate over the interval (t_0,t_k):

$$\mathrm{y(t}_k) = y(t_0) - \left[\int_{t_0}^{\mathrm{t}_k} F(\tau)\mathrm{F}^T(\tau)\,\mathrm{d}\tau\right]\Psi_0$$

$$\Psi_0 = \frac{[y(t_0) - \mathrm{y(t}_k)]}{M(\mathrm{t}_k, \mathrm{t}_0)} \tag{10}$$

where: $M(\mathrm{t}_k, \mathrm{t}_0) = \int_{t_0}^{\mathrm{t}_k} F(\tau)\mathrm{F}^T(\tau)\,\mathrm{d}\tau$

Substituting (10) into the third equation of (9), the optimal control vector is obtained as:

$$u = F^T(\mathrm{t})\frac{[y(t_k) - \mathrm{y(t}_0)]}{M(\mathrm{t}_k, \mathrm{t}_0)} \tag{11}$$

This is the optimal control vector uuu that we need to determine. This control vector acts on the system and transitions it from the known initial state y(t0) to the required final state y(tk) while minimizing the objective function (3).

The initial time t_0 can be chosen as any moment corresponding to a known state y(t). The control law (11) can be interpreted as a feedback control process based on the predicted final state. The transition from t0 to t using the control law (11) is given by:

$$u\left[\mathrm{y(t)}\right] = F^T(\mathrm{t})\frac{\left[y(t_k) - \mathrm{y(t)}\right]}{M(\mathrm{t}_k, \mathrm{t}_0)} \tag{12}$$

From (12), we observe that the instantaneous control parameter corresponds to the difference between the required final state and its predicted value at the current moment. Expanding M:

$$M(\mathrm{t}_k, \mathrm{t}) = \int_t^{\mathrm{t}_k} S\Phi(\mathrm{t}_k, \tau)B(\tau)B^T(\tau)\Phi^T(\mathrm{t}_k, \tau)\mathrm{S}^T \mathrm{d}\,\tau$$

From this, we can propose a solution to formulate the optimal control law for the missile when the conditions at the interception point are predetermined. Substituting (1) and (2) into the system (3), we obtain:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\ ;\ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\ ;\ S = \left(1, 0\right)$$

The fundamental matrix of system (1) has the form:

$$\Phi(t_k, t) = \begin{pmatrix} 1 & t_k - t \\ 0 & 1 \end{pmatrix}$$

From (7), we derive:

$$\mathrm{F}(t) = \left(1, 0\right)\begin{pmatrix} 1 & t_k - t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = t_k - t = t_*$$

In which t_* - is the remaining time for the missile to reach the target interception point.

Thus, we have:

$$M(\mathrm{t}_k, \mathrm{t}) = \int_t^{\mathrm{t}_k} (\mathrm{t}_k - \tau)^2\, \mathrm{d}\,\tau = \frac{1}{3}t_*^3$$

If $t_* = 0$, deducing from (2), we have: $y(\mathrm{t}_k) = x_1(\mathrm{t}_k) = x_ц(\mathrm{t}_k)$

From (6), we have: $$y(\mathrm{t}) = S\Phi(t_k, t)x(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & t_k - t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y(\mathrm{t}) = x_1 + x_2(t_k - t) = x_1 + x_2 t_*$$

From (12), we see that the optimal control vector ultimately takes the form:

$$u(\mathrm{t}) = \frac{3}{t_*^2}\left[x_ц(\mathrm{t}_k) - \left\{x_1 + x_2 t_*\right\}\right]$$

$$u(\mathrm{t}) = \frac{3}{t_*^2}\left[r_ц(\mathrm{t}_k) - r_p - V_p t_*\right] \tag{13}$$

If the control is time-varying in three-dimensional space, then r_p and V_p will be vectors in space.

The motion parameters of the target can be measured by ground-based devices or estimated using modern filtering and extrapolation algorithms. The main issue here is the ability to obtain information about the target's relative position vector

$r_ц(t_k)$, the predicted position of the target at time t_k. This information needs to be determined at any given time t, considering various hypotheses about the target's motion.

Typically, during the cruise phase, due to the large distance between the missile (TL) and the target (MT), the assumption of uniform straight-line motion of the target is often used over the interval (t, t_k);

This means that:
$$r_ц(t_k \mid t) = r_ц(t) + V_ц(t).t_* \tag{14}$$

Substituting (14) into (13), we obtain the control law:

$$u_p = \frac{3}{t^2_*}\left[r_ц(t) - r_p(t) + \left\{ V_ц(t) - V_p(t) \right\} t_* \right] \tag{15}$$

Using the control law (15) still depends on the assumption about the target's motion.

Thus, theoretically, the problem of constructing an optimal guidance law entirely from a distance for the missile (TL) has a solution, under the assumption that there is no self-guidance phase.

2.1.2. Synthesizing the optimal long-range guidance law for the missile, taking into account the requirements at the point of transition to autonomous guidance

As analyzed when defining the research problem, the condition for transitioning to self-guidance also requires determining the spatial orientation of the missile (TL) so that the target (MT) is within the field of view of the seeker at the time t_k the end of the long-range guidance phase.

The ability to meet this requirement can be achieved by solving special boundary conditions of the phase vector. The transition time to self-guidance is determined by several factors characteristic of the seeker's operational capabilities. Typically, the factor characteristic of the transition to self-guidance is the self-guidance distance (or the remaining distance to the target (MT) from the long-range guidance). Regarding control, we need to add pre-defined boundary conditions to the optimization problem. This condition is given as:

$$r_p(t_k \mid t) = r_ц(t_k \mid t) + D_{td} \ .$$

With D_{td} being the pre-defined self-guidance distance vector, its magnitude depends on the target detection range of the seeker. The direction of the vector D_{td} is opposite to the direction of the target's velocity vector in space.

On the other hand, the missile (TL) and the seeker must be oriented towards the target, such that the initial miss distance h_0 of the missile, at the moment of transition, is smaller than a pre-defined value. The upper limit of the initial miss distance h_0 is determined by the long-range guidance error and the missile's ability to generate force and torque when it transitions to self-guidance

According to the definition of instantaneous miss distance in the literature [3], h0 primarily depends on the magnitude and direction of the velocity vectors of the missile (TL) and the target (MT), specifically the relative velocity vector ($\vec{V}_{tc} = \vec{V}_p - \vec{V}_ц$).

To ensure that the instantaneous miss distance h_0 approaches 0 and that the target (MT) is within the seeker's field of view, the optimal condition is that the direction of the missile's velocity vector must coincide with the direction of the target's velocity vector. In this case, there will be two scenarios that satisfy this assumption:

Trường hợp bắn đuổi:

$$\begin{aligned} \theta_p &= \theta_ц \\ \psi_p &= \psi_ц \end{aligned} \tag{16}$$

Chasing case:

$$\begin{aligned} \theta_p &= \theta_ц - 180^0 \\ \psi_p &= \psi_ц - 180^0 \end{aligned} \tag{17}$$

In both of these cases, the boundary control is presented in the form of:

$$y(\mathrm{t}_k)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}r_p(\mathrm{t}_k)\\ V_p(\mathrm{t}_k)\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}r_ц(\mathrm{t}_k)+D_{td}\\ V_p^*\end{pmatrix} \quad (18)$$

In which V_p^* is the velocity vector of the missile at the end of the long-range guidance phase, and its magnitude and direction must satisfy the self-guidance condition. V_p^* has a magnitude within the limits that the missile's propulsion system can ensure, with a direction that satisfies the conditions of the equations (16), (17).

The necessary calculations according to the requirements of the optimal control method are provided correspondingly for system (1). We have:

$$\mathrm{F}(t)=\begin{pmatrix}1 & t_*\\ 0 & 1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}t_*\\ 1\end{pmatrix} \quad (19)$$

$$M(\mathrm{t})=\int_t^{\mathrm{t}_k}\begin{pmatrix}\mathrm{t}_k-\tau\\ 1\end{pmatrix}(\mathrm{t}_k-\tau,1)\,\mathrm{d}\,\tau=\begin{pmatrix}\frac{1}{3}t_*^3 & \frac{1}{2}t_*^2\\ \frac{1}{2}t_*^2 & t_*\end{pmatrix} \quad (20)$$

Note that M^{-1} exists when $t_* \neq 0$, meaning that:

$$\det M(\mathrm{t})=|ad-bc|=\frac{1}{12}t_*^4\neq 0$$

This is evident because $t_*\neq 0$. In this case:

$$F^T(\mathrm{t})M^{-1}(\mathrm{t})=(t_*,1)\frac{12}{t_*^4}\begin{pmatrix}t_* & -\frac{1}{2}t_*^2\\ -\frac{1}{2}t_*^2 & \frac{1}{3}t_*^3\end{pmatrix}=\frac{1}{t_*^2}(6-2t_*) \quad (21)$$

Finally:

$$u_p=\frac{(6-2t_*)}{t_*^2}\left[\begin{pmatrix}r_ц(\mathrm{t}_k)+D_{td}\\ V_p^*\end{pmatrix}-\begin{pmatrix}1 & t_*\\ 0 & 1\end{pmatrix}\begin{pmatrix}r_p(\mathrm{t})\\ V_p(\mathrm{t})\end{pmatrix}\right]$$

$$u_p=\frac{1}{t_*^2}\left[6\left[r_ц(\mathrm{t}_k)+D_{td}-\{r_p+V_p(\mathrm{t})t_*\}\right]-2t_*\{V_p^*-V_p(\mathrm{t})\}\right] \quad (22)$$

Equation (22) describes the control law equation, taking into account the boundary condition (18). Using the assumption about the target's motion, as presented in section 2, substituting (14) into (22), we get:

$$u_p=\frac{6}{t_*^2}\left[r_ц(\mathrm{t})-r_p(\mathrm{t})+D_{td}+\left\{V_ц(\mathrm{t})-V_p(\mathrm{t})\right\}t_*\right]-\frac{2}{t_*}\{V_p^*-V_p(\mathrm{t})\} \quad (23)$$

Let: $$r_ц^*(\mathrm{t})=r_ц(\mathrm{t})+D_{td} \quad (24)$$

From equation (23), we obtain

$$u_p=\frac{6}{t_*^2}\left[r_ц^*(\mathrm{t})-r_p(\mathrm{t})+\left\{V_ц(\mathrm{t})-V_p(\mathrm{t})\right\}t_*\right]-\frac{2}{t_*}\{V_p^*-V_p(\mathrm{t})\} \quad (25)$$

Equation (25) is the optimal guidance law to be determined based on the transition conditions, as analyzed. It can be seen that the guidance law (25) allows the missile (TL) to lead to a predicted point $Ц^*$ (the assumed target) at time ttt, which is not fixed and has a correlation with the target's motion, as expressed in equation (24).

To obtain complete control information, we need to find the value of t∗. This is an unmeasurable quantity and can only be estimated using certain methods. The accuracy of these estimates affects the quality of the control process. Below is one of these methods.

From [3,8], we have the concept of instantaneous miss distance when guiding towards the point Ц*:

$$h = r_{ц}^{*}(\mathrm{t}) - r_p(\mathrm{t}) + \left\{V_{ц}(\mathrm{t}) - V_p(\mathrm{t})\right\} t_* \tag{26}$$

Write in another form: $h = D_{td}(\mathrm{t}) + V_{tc}(\mathrm{t}) t_*$ với $D_{td}(\mathrm{t})$ is the relative distance vector TL - Ц*. Thus, we have the value of the vector that exists in the control equation u(t).

Taking the dot product of both sides of the equation of the instantaneous miss vector with the relative velocity vector, we get:

$$(h.V_{tc}) = 0 = V_{tc}.D_{td} + V_{tc}^2 t_* \ .$$

It follows that: $$t_* = -\frac{(V_{tc}.D_{td})}{V_{tc}^2} = -\frac{D_{td}\dot{D}_{td}}{V_{tc}^2} = \frac{D_{td}\left|\dot{D}_{td}\right|}{V_{tc}^2}$$

Here $\dot{D}_{td}$ is the projection of the relative velocity vector V_{tc} onto the line of sight. TL- Ц* và $\dot{D}_{td} < 0$. According to the initial conditions of the self-guidance phase $h << D_{td}$, we can consider $\left|V_{tc}\right| \cong \left|\dot{D}_{td}\right|$, in this case:

$$t_* = \frac{D_{td}}{\left|\dot{D}_{td}\right|} \tag{27}$$

On the other hand, from the geometric dynamics relation, we have:

$$h = \frac{D_{td}^2}{\left|\dot{D}_{td}\right|}\omega_L \tag{28}$$

With ω_L being the angular velocity of the missile-target line of sight TL- Ц*.

Substitute (27) and (28) into (25), taking (26) into account, we get:

$$u_p = 6\left|\dot{D}_{td}\right|\omega_L - 2\frac{\left|\dot{D}_{td}\right|}{D_{td}}\{V_p^* - V_p(\mathrm{t})\}$$

$$u_p = 6V_{tc}\omega_L - 2\frac{V_{tc}}{D_{td}}(V_p^* - V_p) \tag{29}$$

Thus, (29) is the equation of the optimal far-field guidance law constructed for system (1) with the objective function (3), taking into account the transition condition to autonomous guidance (18). This guidance law allows the missile to be directed to a predefined point, relative to the target, and at that point, the missile's velocity vector has the desired value and direction.

2.2. Experiment preparation

2.2.1. Instrumentation and Experimental materials

The examination of the new guidance law using the simulation method allows us to analyze and evaluate its effectiveness. The assumed survey conditions are as follows:

The target flies in a straight $\theta_{ц} = 0^0;$ and uniform motion, heading toward the control station, with a speed of 600 m/s and an initial distance of 60 km. The missile is launched from the control station with an average speed of 900 m/s. The relative distance at which the transition to autonomous guidance occurs is D_{td}=10km.

2.2.2. Target scenarios:

Scenario 1: Altitude of the target $H_{ц}$=8km, with the flight direction: $\psi_{ц} = 40^{0}$;

Scenario 2: Altitude of the target $H_{ц}$=10km, with the flight direction: $\psi_{ц} = 20^{0}$;

Scenario 3: Altitude of the target $H_{ц}$=12km, with the flight direction: $\psi_{ц} = -10^{0}$;

Scenario 4: Altitude of the target $H_{ц}$=14km, with the flight direction: $\psi_{ц} = -30^{0}$;

III. RESULTS AND DISCUSSION

3.1. Simulation results and comments

The survey results for the four cases are reflected in a single graph, as shown in Figures 1, 2, 3 and 4 below:

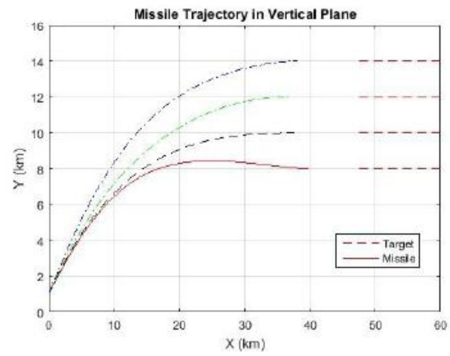

Figure 1: Missile-Target Trajectory in the Horizontal Plane

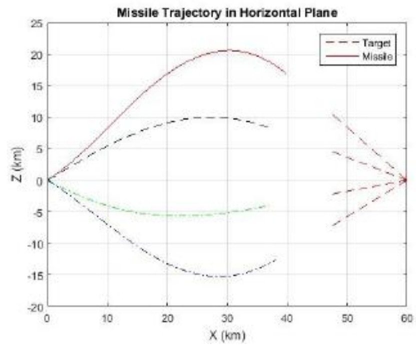

Figure 2: Missile-Target Trajectory in the Vertical Plane

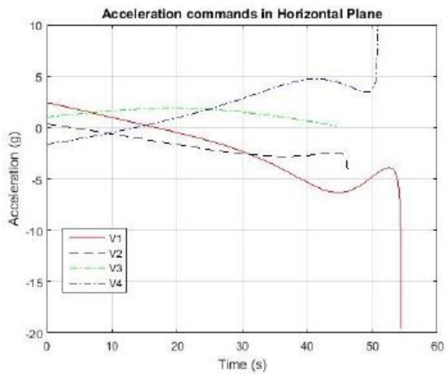

Figure 3: Control Overload in the Horizontal Plane

e

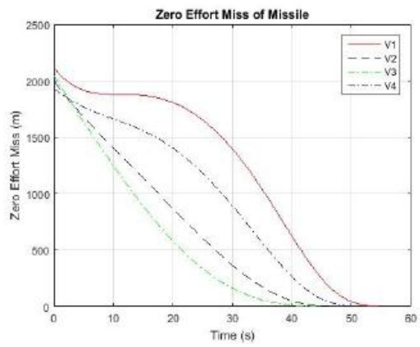

Figure 4: Instantaneous Slip

We draw the following observations:

- The missile-target trajectory in the vertical plane has a rainbow-like shape, which is energy-optimal (Figure 1). Guidance law (29) directs the missile to the final point of the cruise phase with its trajectory aligned with the target's motion direction, approximately 10 km from the target, ensuring the required distance and orientation for the transition to autonomous guidance. Even when engaging low-altitude targets, the missile's trajectory follows a top-down approach (Case 1).

- The missile-target trajectory in the horizontal plane has a parabolic shape, corresponding to the different approach directions of the target at the end of the cruise phase. The velocity vectors of the missile and the target lie on the same straight line (Figure 2). Simulation results also show that the constructed guidance law ensures the required distance D_{td} as specified.

- The control overload in the horizontal plane is smooth and uniform in the initial phase, then changes direction and increases significantly in the final phase due to the need to maneuver the missile's trajectory to achieve the desired approach angle (Figure 3).

- In all four cases studied, the instantaneous slip parameter approaches 0 before reaching the guidance point.

This ensures accuracy when directing the missile to a predefined point and also demonstrates the precision of the synthesized guidance method (Figure 4).

Thus, the simulation results have shown that the newly synthesized guidance law (29) can control the missile during the cruise phase to reach the predicted point, ensuring the required autonomous guidance distance and trajectory orientation. The simulation results also confirm the effectiveness of the synthesized guidance law, validating the analyses as accurate.

IV. CONCLUSIONS

From the approach of considering the guidance law as a state-space function and selecting the quality criterion function, we can see that if the control vector u(t)u(t)u(t) is proportional to the control overload, the quality criterion represents the energy required throughout the entire control process. Therefore, the trajectory executed by guidance law will be energy-optimal when the optimization criterion is applied. By constructing the guidance law as described, we can definitively determine the transition point to autonomous guidance by incorporating the required condition for the target distance vector at the final time into the guidance law. This is particularly important when transitioning from the cruise phase to autonomous guidance.

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