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The Relationship Between Wave Period, Deep Water Wave and Breaking Wave Heights, Formulated Using the Wave Amplitude Function

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***Keywords*— Wave amplitude function, deep water and breaking wave height.**

***Abstract*— The wave amplitude function is a relational equation that links wave amplitude with various water wave parameters, such as wave number, wave angular frequency, and wave constant. This function is derived by integrating the Kinematic Free Surface Boundary Condition over time. The wave amplitude function incorporates breaking characteristics, allowing for the extraction of breaking parameters, including breaking wave height, breaking wave length, and breaking water depth, as functions of the wave period. By combining the Euler momentum conservation equation with the wave amplitude function, a dispersion equation is obtained. This dispersion equation elucidates the relationships between deep water wave height, deep water wave length, and deep water depth in relation to the wave period. The results obtained for both deep water wave height and breaking wave height are consistent with previous research.*

I. INTRODUCTION

Water waves are commonly characterized by two key parameters: wave period and wave height. In the context of short waves, these parameters exhibit a one-to-one correspondence, meaning that a specific wave period is directly associated with a corresponding wave height. This correlation extends not only to deep water wave heights but also to breaking wave heights.

In this research, the correlation between deep water wave height and wave period, as well as between breaking wave height and wave period, has been systematically analyzed. The results are presented in the form of equations, tables, and graphical representations.

The relationship identified between deep water wave height and wave period aligns with the findings of Wiegel (1949, 1964), while the relationship between breaking wave height and wave period is consistent with the research of Komar and Gaughan (1972).

These relationships are valuable for both practical engineering applications and research purposes. Practically, they enable quick estimation of deep water wave height and breaking wave height based solely on the wave period. In the research domain, the ongoing development of water wave transformation models, including processes like shoaling and breaking, necessitates reliable guidelines or estimates regarding the relationship between breaking wave height and wave period.

II. WEIGHTED TAYLOR SERIES AND WEIGHTING COEFFICIENT

In this article, a specialized equation and parameters are introduced that may not be widely recognized, namely the weighted Taylor series and the associated weighting coefficients.

The weighted Taylor series is a modified form of the traditional Taylor series, truncated to a first-order approximation. In this formulation, the contributions of higher-order terms are accounted for by incorporating weighting coefficients into the first-order differential term.

Weighted Taylor series for function $f(x,t)$,

$$f(x+\delta x,t+\delta t)=f(x,t)+\gamma_{t,2}\,\delta t\frac{\partial f}{\partial t}+\gamma_x\,\delta x\frac{\partial f}{\partial x}$$

Weighted Taylor series for function $f(x,t)$,

$$f(x+\delta x,z+\delta z,t+\delta t)=f(x,z,t)+\gamma_{t,3}\,\delta t\frac{\partial f}{\partial t}+\gamma_x\,\delta x\frac{\partial f}{\partial x}+\gamma_z\,\delta z\frac{\partial f}{\partial z}$$

The weighting coefficients $\gamma_{t,2}$, $\gamma_{t,3}$, γ_x and γ_z are assigned base values of $\gamma_{t,2}=2.0$, $\gamma_{t,3}=3.0$, $\gamma_x=1.0$ and γ_z=1.0. There is no distinction between γ_x in the function $f(x,t)$ and $\gamma_x$in the function $f(x,z,t)$. The adjusted values of the weighting coefficients are functions of ε, the optimization coefficient, as detailed in Table 1. The method for calculating these weighting coefficients is described in Hutahaean (2023). For the purposes of this research, ε is set to 0.01.

Table 1. Corrected weighting coefficients values.

ε	$\gamma_{t,2}$	$\gamma_{t,3}$	γ_x	γ_z
0.010	1.9998	3.00465	0.99879	1.01093
0.011	1.99975	3.00563	0.99854	1.01325
0.012	1.99971	3.00671	0.99826	1.0158
0.013	1.99966	3.00788	0.99795	1.01858
0.014	1.99960	3.00915	0.99763	1.02159
0.015	1.99954	3.01052	0.99727	1.02484
0.016	1.99948	3.01198	0.9969	1.02832
0.017	1.99941	3.01355	0.99649	1.03205
0.018	1.99934	3.01521	0.99607	1.03601
0.019	1.99926	3.01697	0.99561	1.04022
0.020	1.99918	3.01883	0.99514	1.04468

In this work, a Cartesian coordinate system is employed, with x representing the horizontal axis and z denoting the vertical axis.

III. RESEARCH ON WAVE AMPLITUDE FUNCTION INTEGRATION RESULTS OF ORDER 0

Hutahaean (2024) integrated the Kinematic Free Surface Boundary Condition with respect to time using three levels of accuracy: 0th order, 2nd order, and 3rd order accuracy. The 0th order accuracy implies that the differential of the water surface elevation is absent in the integration results, represented as $\frac{\partial^0\eta}{\partial t^0}$, $\eta(x,t)$ is the water surface elevation function. The 2nd order accuracy includes the term $\frac{\partial^2\eta}{\partial t^2}$, and the 3rd order accuracy incorporates the term $\frac{\partial^3\eta}{\partial t^3}$.

The outcome of this integration is referred to as the wave amplitude function, which relates several wave parameters: wave amplitude A, wave number k, wave period T (expressed through the wave angular frequency $\sigma=\frac{2\pi}{T}$ and the wave constant G.

In the initial phase of this research, the focus is on the wave amplitude function derived from the 0th order accuracy integration. This research examines various aspects of wave parameters, including deep water wave height H_0, deep water wavelength, critical deep water wave steepness, breaking wave height, breaking wavelength, and breaking water depth. The wave amplitude function resulting from the 0th order accuracy integration is as follows:

$$A=\frac{2Gk}{\gamma_{t,2}\sigma}\cosh\theta\pi\left(\frac{\tanh\theta\pi}{\sqrt{\gamma_z}}-\frac{kA}{2}\right) \quad \text{........(1)}$$

A : wave amplitude

G : wave constant

σ: angular frequency, $\sigma=\frac{2\pi}{T}$, T is wave period.

k: is wave number

$\gamma_{t,2}$ and γ_z: are weighting coefficients

θ: deep water coefficient, where $\tanh\theta\pi\approx 1$.

The dispersion equation in deep water is derived from Euler's momentum conservation equation, under the assumption of negligible convective acceleration, and utilizing the wave amplitude function (Hutahaean, 2024). The resulting dispersion equation is as follows:

$$\frac{gA}{2}k^2-\frac{g\tanh\theta\pi}{\sqrt{\gamma_z}}k+\gamma_{t,2}\gamma_{t,3}\sigma^2=0 \quad \text{......(2)}$$

g gravitational acceleration and k wave number.

3.1. The relationship between wave period and deep water wave height

The relationship between wave period and deep water wave height, deep water wavelength and critical deep water wave steepness is formulated using (2).

The determinant value of (2) is,

$$d=\frac{g^2\tanh^2\theta\pi}{\gamma_z}-2\,gA\gamma_{t,2}\gamma_{t,3}\sigma^2$$

Given that the variable in the determinant equation is the wave amplitude A, and the wave period remains constant, there exists a maximum value of wave amplitude at which the determinant equals zero.

$$\frac{g^2 \tanh^2 \theta\pi}{\gamma_z} - 2\,gA\gamma_{t,2}\gamma_{t,3}\sigma^2 = 0$$

Obtaining A_{max},

$$A_{max} = \frac{g \tanh^2 \theta\pi}{\gamma_z \gamma_{t,2}\gamma_{t,3}\sigma^2}$$

Or for $H = 2\,A$,

$$H_{max} = \frac{2\,g \tanh^2 \theta\pi}{\gamma_z \gamma_{t,2}\gamma_{t,3}\sigma^2}$$

This equation represents the relationship between wave height and wave period. For each wave period, there corresponds a unique wave height. Thus, it can be stated that for a given wave period, there is a specific wave height of:

$$H_0 = \frac{2\,g \tanh^2 \theta\pi}{\gamma_z \gamma_{t,2}\gamma_{t,3}\sigma^2} \quad \text{……..(3)}$$

The index 0 on H indicates that the wave height refers to a condition unaffected by the sea bottom, representing the wave height in deep water.

When the determinant value is zero, the wave number in deep water, which is the root of equation (2), is given by:

$$k_0 = \frac{2\,\tanh \theta\pi}{H_0\,\sqrt{\gamma_z}} \quad \text{…..(4)}$$

$$L_0 = \frac{\pi H_0 \sqrt{\gamma_z}}{\tanh \theta\pi} \quad \text{…..(5)}$$

Substituting H_0 from equations (4) and (5) with the value obtained from equation (3) yields a direct relationship between deep water wavelength and wave period.

The deep water depth h_0 can then be calculated using the wave number conservation equation as detailed in Hutahaean (2023):

$$k\left(h + \frac{A}{2}\right) = \theta\pi \quad \text{…..(6)}$$

Where h is water depth. Therefore, deep water depth h_0 is,

$$h_o = \frac{\theta\pi}{k_0} - \frac{H_0}{4}$$

The term "deep water" refers to the minimum water depth at which waves remain unaffected by the seabed. Both k_0 and H_0 are functions of the wave period, indicating that h_0 is likewise dependent on the wave period.

The calculated values of deep water wave height H_0 deep water wavelength L_0, and minimum deep water depth h_0 are presented in Table 2. The calculations were conducted using $\theta = 1.928$ and $\varepsilon = 0.01$. It is noted that the impact of the deep water coefficient θ on the deep water wave height is minimal and can therefore be disregarded. The selection of $\theta = 1.94$ is associated with the breaking water depth, which will be explored in a subsequent section.

Since H_0 represents the maximum wave height, the wave steepness defined as $\frac{H_0}{L_0}$ can be referred to as the critical wave steepness. The calculation results indicate that $\frac{H_0}{L_0}$ is consistent across all wave periods, yielding a value of $\frac{H_0}{L_0} = 0.315$. In comparison, the critical wave steepness reported by Toffoli et al. (2010), which is $\frac{H_0}{L_0} = 0.170$ suggests that the critical wave steepness derived in this research is relatively high, although Toffoli et al. indicate that it can reach up to 0.20.

Table 2. Deep water wave parameter.

T (sec)	H_0 (m)	L_0 (m)	h_0 (m)	$\frac{H_0}{L_0}$
4	0.646	2.052	1.839	0.315
5	1.009	3.206	2.874	0.315
6	1.453	4.617	4.138	0.315
7	1.978	6.284	5.633	0.315
8	2.583	8.208	7.357	0.315
9	3.269	10.388	9.311	0.315
10	4.036	12.825	11.495	0.315
11	4.884	15.518	13.909	0.315
12	5.812	18.468	16.553	0.315
13	6.821	21.674	19.427	0.315
14	7.911	25.137	22.531	0.315
15	9.082	28.856	25.865	0.315
16	10.333	32.832	29.428	0.315
17	11.665	37.064	33.222	0.315
18	13.078	41.553	37.245	0.315

Wiegel (1949, 1964) formulated an equation to describe the relationship between deep water wave height and wave period as follows:,

$$H_{0-Wieg} = \frac{gT^2}{15.6^2}\ \text{m.} \quad \text{….(7)}$$

The comparison with the Wiegel's equation is presented in Table 3 and illustrated in Figure 1, where the calculations were performed using the parameters $\theta = 1.928$ and $\varepsilon = 0.01$.

As shown in both Table 3 and Figure 1, the difference between the results is minimal, with a variance of only 0.131%. In Figure 1, the graphs representing H_0 and H_{0-Wieg} overlap.

Table 3. Comparison to Wiegel's Equation (1949,1964)

T (sec)	H_0 (m)	H_{0-Wieg} (m)	$\frac{H_0 - H_{0-Wieg}}{H_{0-Wieg}} x100\%$
4	0.646	0.645	0.131
5	1.009	1.008	0.131
6	1.453	1.451	0.131
7	1.978	1.975	0.131
8	2.583	2.58	0.131
9	3.269	3.265	0.131
10	4.036	4.031	0.131
11	4.884	4.878	0.131
12	5.812	5.805	0.131
13	6.821	6.812	0.131
14	7.911	7.901	0.131
15	9.082	9.07	0.131
16	10.333	10.32	0.131
17	11.665	11.65	0.131
18	13.078	13.061	0.131

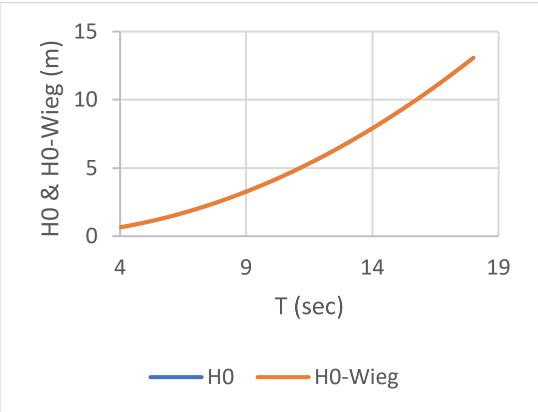

Fig.1: Comparison between H_0 to H_{0-Wieg}

The purpose of this comparison is to demonstrate that previous research findings exhibit similar characteristics to those observed in this research. It is important to note that this research does not aim to derive an equation that replicates the wave height-wave period relationship described by the Wiegel's equation.

3.1. Relationship between wave period and breaking wave height

The relationship between breaking wave height and deep water wave height can be described using the principle of energy relationship. The wave energy equation, as formulated by Dean (1991), for one wavelength is given by:

$$E = c_E \rho g \, H^2 L$$

c_E energy coefficient, based on linear wave theory (Dean (1991)), $c_E = \frac{1}{8}$. ρ water mass density.

The results of the shoaling-breaking research using a numerical model with the wave amplitude function of order 0 indicate a loss of wave energy as the wave travels from deep water to the breaking point. Therefore, the relationship between the wave energy at the breaking point and the wave energy in deep water is expressed as follows:

$$E_b = 0.776 E_0$$

$$H_b^2 L_b = 0.776 H_0^2 L_0$$

Substitute (5)

$$H_b^2 L_b = 0.776 H_0^3 \frac{\pi\sqrt{\gamma_z}}{\tanh\theta\pi} \qquad \text{.....(8)}$$

To determine L_b, the wave amplitude function equation (1) is used, which includes the breaking characteristic given by:

$$\frac{\tanh\theta\pi}{\sqrt{\gamma_z}} - \frac{kA}{2} = 0$$

Using the index b for breaking, thus,

$$\frac{k_b A_b}{2} = \frac{\tanh\theta\pi}{\pi\sqrt{\gamma_z}}$$

Given that $k_b = \frac{2\pi}{L_b}$ dan $H = 2\,A$, the relation is:

$$\frac{H_b}{L_b} = \frac{2\tanh\theta\pi}{\pi\sqrt{\gamma_z}} \qquad \text{....(9)}$$

Or,

$$L_b = \frac{H_b \pi\sqrt{\gamma_z}}{2\tanh\theta\pi}$$

Substituting into (8)

$$H_b^2 \frac{H_b \pi\sqrt{\gamma_z}}{2\tanh\theta\pi} = 0.776 H_0^3 \frac{\pi\sqrt{\gamma_z}}{\tanh\theta\pi}$$

$$H_b = H_0 \;\; 1.552^{\frac{1}{3}}$$

Substituting H_0 from (3)

$$H_b = \frac{2\,g\tanh^2\theta\pi \;\; 1.552^{\frac{1}{3}}}{\gamma_z \gamma_{t,2} \gamma_{t,3} \sigma^2}$$

The final equation establishes a direct relationship between the breaking wave height and the wave period.

To determine the breaking water depth, the wave number conservation equation (6) is employed at the breaking point. This calculation utilizes the breaking wave height as a key parameter.

$$h_b = \frac{\theta\pi}{k_b} - \frac{H_b}{4}$$

In the subsequent section, the results of the calculations for the breaking parameters are presented, utilizing the calculation parameters $\varepsilon = 0.01$ and $\theta = 1.928$. The selection of $\theta = 1.928$ is intended to achieve a value of $\frac{H_b}{h_b} = 0.78$, which is a widely accepted criterion established by McCowan (1894). Decreasing the value of θ increases $\frac{H_b}{h_b}$ whereas increasing θ reduces $\frac{H_b}{h_b}$. However, the effect of θ on both $H_0$and H_b is minimal, and it can be considered negligible. The value of $\frac{H_b}{L_b}$ can be calculated using equation (9), yielding a constant value of 0.633.

Table 4. Breaking parameter calculation results.

T (sec.)	H_0 (m)	H_b (m)	L_b (m)	h_b (m)	$\frac{H_b}{h_b}$
4	0.646	0.748	1.188	0.958	0.78
5	1.009	1.168	1.856	1.497	0.78
6	1.453	1.682	2.673	2.156	0.78
7	1.978	2.29	3.638	2.935	0.78
8	2.583	2.991	4.752	3.833	0.78
9	3.269	3.785	6.014	4.851	0.78
10	4.036	4.673	7.424	5.989	0.78
11	4.884	5.655	8.983	7.246	0.78
12	5.812	6.729	10.691	8.624	0.78
13	6.821	7.898	12.547	10.121	0.78
14	7.911	9.16	14.552	11.738	0.78
15	9.082	10.515	16.705	13.475	0.78
16	10.333	11.963	19.006	15.331	0.78
17	11.665	13.506	21.456	17.308	0.78
18	13.078	15.141	24.055	19.404	0.78

Subsequently, a comparative analysis was performed using the breaking wave height equation proposed by Komar and Gaughan (1972), which is given by:

$$H_{b-KG} = 0.39\, g^{1/5}(T_0 H_0^2)^{2/5} \quad \text{.....(10)}$$

The results of this comparison are detailed in Table (5) and illustrated in Figure (2). The analysis reveals a consistent relative difference of 1.047%.

Table 5. Comparison with the Breaking Wave Height Equation from Komar and Gaughan

T (sec)	H_0 (m)	H_b (m)	H_{b-KG} (m)	δ (%)
4	0.646	0.748	0.756	1.047
5	1.009	1.168	1.181	1.047
6	1.453	1.682	1.7	1.047
7	1.978	2.29	2.314	1.047
8	2.583	2.991	3.023	1.047
9	3.269	3.785	3.825	1.047
10	4.036	4.673	4.723	1.047
11	4.884	5.655	5.714	1.047
12	5.812	6.729	6.801	1.047
13	6.821	7.898	7.981	1.047
14	7.911	9.16	9.256	1.047
15	9.082	10.515	10.626	1.047
16	10.333	11.963	12.09	1.047
17	11.665	13.506	13.649	1.047
18	13.078	15.141	15.301	1.047

Note : $\delta = \frac{H_b - H_{b-KG}}{H_{b-KG}} x\ 100\ \%$

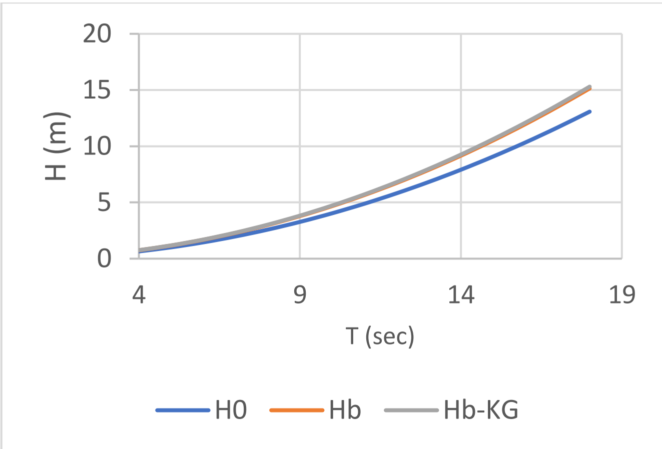

Fig.2: The comparison of breaking wave heights.

This research employs a zero-order wave amplitude function to derive simplified relationships between the wave period and various parameters: deep water wave height H_0, deep water wavelength L_0, deep water depth h_0, and the wave steepness criterion $\frac{H_0}{L_0}$. It also establishes relationships with breaking wave height H_b, breaking wavelength L_b, breaking water depth H_b, and breaking wave steepness $\frac{H_b}{L_b}$.

Comparisons with deep water wave height estimates from Wiegel and breaking wave height calculations from Komar and Gaughan reveal that the results of this research are consistent with, or closely approximate, those of previous research.

IV. RESEARCH ON WAVE AMPLITUDE FUNCTION RESULT OF 3RD ORDER INTEGRATION

The wave amplitude function, derived from integrating the Kinematic Free Surface Boundary Condition with third-order accuracy (Hutahaean, 2024), is given by:

$$A = \frac{2Gkcosh\,\theta\pi}{\sigma\gamma_{t,2}}\alpha_{kA} \qquad .....(11)$$

$$\alpha_{kA} = \frac{tanh\,\theta\pi}{\sqrt{\gamma_z}} + \left(\frac{3}{2\gamma_z} - \frac{3}{2}\right)kA - \frac{k^2A^2}{2\sqrt{\gamma_x}\sqrt{\gamma_z}}tanh\,\theta\pi + \left(\frac{1}{\gamma_z^2} - \frac{1}{\gamma_z}\right)\frac{k^3A^3}{8}$$

The dispersion equation of third order is expressed as

$$\left(\frac{1}{\gamma_z^2} - \frac{1}{\gamma_z}\right)\frac{A^3}{8}k^4 - \frac{tanh\,\theta\pi\,A^2}{2\sqrt{\gamma_x}\sqrt{\gamma_z}}k^3 + \left(\frac{1}{2\gamma_z} - \frac{3}{2}\right)Ak^2 + \frac{tanh\,\theta\pi}{\sqrt{\gamma_z}}\,k - \frac{\gamma_{t,3}\gamma_{t,3}\,\sigma^2}{g} = 0 \qquad ... .(12)$$

4.1. The calculation of deep water wave height H_0.

Equation (12) exhibits a characteristic where there is a maximum wave amplitude value, A_{max} beyond which any further increase in amplitude results in a wave number value that becomes zero or negative. This suggests that Equation (12) has a discontinuity at A_{max}.

To determine A_{max}, calculations are performed iteratively, starting from a small wave amplitude value and incrementally increasing it by a small amount. In this research, an increment of 0.0001 is used until A_{max}. is obtained.

Given that the equation is a fourth-degree polynomial, the Newton-Raphson method is employed for the iterative calculation. To initiate the iteration, the equation is simplified by assuming that k^3 and k^4 are negligibly small, reducing the problem to a second-degree polynomial, which is expressed as follows:

$$\left(\frac{1}{2\gamma_z} - \frac{3}{2}\right)Ak^2 + \frac{tanh\,\theta\pi}{\sqrt{\gamma_z}}\,k - \frac{\gamma_{t,3}\gamma_{t,3}\,\sigma^2}{g} = 0$$

In this equation, there are two values of k for each wave amplitude A and positive value is selected. Consequently, the calculation process involves two stages for each wave amplitude value. Initially, the wave number k is approximated using a simplified equation, which provides an estimated value of k. Following this approximation, the Newton-Raphson method is employed to refine the value of k by applying it to the complete dispersion equation. This two-stage approach enhances the accuracy of the calculated wave number k for the given wave amplitude A.

The calculation results using $\theta = 1.3395$, $\varepsilon = 0.01$ are presented in Table (6).

Table 6. Results of Wave Parameter Calculations in Deep Water.

T (sec)	H_0 (m)	L_0 (m)	h_0 (m)	$\frac{H_0}{L_0}$
4	0.68	2.735	1.661	0.249
5	1.063	4.262	2.588	0.249
6	1.53	6.129	3.721	0.25
7	2.083	8.291	5.03	0.251
8	2.721	10.85	6.584	0.251
9	3.443	13.746	8.342	0.25
10	4.251	16.935	10.275	0.251
11	5.144	20.46	12.412	0.251
12	6.122	24.326	14.756	0.252
13	7.184	28.538	17.31	0.252
14	8.332	33.088	20.069	0.252
15	9.565	37.941	23.01	0.252
16	10.883	43.246	26.232	0.252
17	12.287	48.746	29.564	0.252
18	13.775	54.682	33.166	0.252

The values of H_0 and L_0 calculated using the third-order dispersion equation are larger than those obtained from the zeroth-order dispersion equation. Additionally, the critical wave steepness $\frac{H_0}{L_0}$ is smaller for the third-order results, averaging 0.250, compared to the zeroth-order result of $\frac{H_0}{L_0} = 0.315$. Thus, the critical wave steepness derived from the third-order calculation is closer to the criteria proposed by Toffoli et al. (2010), which range from $\frac{H_0}{L_0}$ =0.17-0.2. However, it is important to note that this research does not aim to match the critical wave steepness criteria set by Toffoli et al.

Table 7. The Comparison between H_0 and H_{0-Wieg}.

T (sec)	H_0 (m)	H_{0-Wieg} (m)	$\frac{H_0 - H_{0-wieg}}{H_{0-wieg}} x100\%$
4	0.68	0.645	5.431
5	1.063	1.008	5.441
6	1.53	1.451	5.445
7	2.083	1.975	5.457
8	2.721	2.58	5.454
9	3.443	3.265	5.453
10	4.251	4.031	5.456
11	5.144	4.878	5.458
12	6.122	5.805	5.459
13	7.184	6.812	5.459
14	8.332	7.901	5.462
15	9.565	9.07	5.463
16	10.883	10.32	5.464
17	12.287	11.65	5.466
18	13.775	13.061	5.469

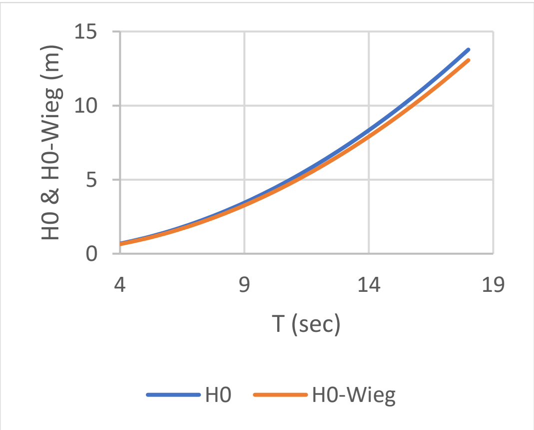

Fig.3: The Comparison between H_0 and H_{0-Wieg}

Next, the wave height H_0 is compared with the deep water wave height H_{0-Wieg} from Wiegel, as presented in Table 7 and Figure 3. The average difference between H_0 and H_{0-Wieg} is 5.45%, whereas the difference using the zeroth-order dispersion equation is only 0.131%. It is important to note that this research does not aim to replicate Wiegel's (1949, 1964) deep water wave height results precisely. The comparison serves to demonstrate that the findings are in general agreement with existing research.

4.2. Calculation of breaking parameters H_b, L_b and h_b

The calculation of breaking parameters is performed using the breaking characteristics outlined in Equation (11):

$$\frac{tanh\,\theta\pi}{\sqrt{\gamma_z}} + \left(\frac{3}{2\gamma_z} - \frac{3}{2}\right) kA - \frac{k^2A^2}{2\sqrt{\gamma_x}\sqrt{\gamma_z}} tanh\,\theta\pi + \left(\frac{1}{\gamma_z^2} - \frac{1}{\gamma_z}\right)\frac{k^3A^3}{8} = 0 \qquad ....(13)$$

This equation is a third-degree polynomial, which precludes the formulation of a simple equation for breaking wave steepness. However, the equation allows for the calculation of the kA value under breaking conditions. For example:

$k_bA_b = \alpha$

Therefore,

$$\frac{H_b}{L_b} = \frac{\alpha}{\pi}$$

Or,

$$L_b = \frac{\pi H_b}{\alpha} \qquad .....(14)$$

The calculation process is carried out in stages. In the first stage, a second-degree polynomial approximation is applied by neglecting the term containing k^3A^3 in equation (13), under the assumption that this term is negligible.

$$\frac{tanh\,\theta\pi}{\sqrt{\gamma_z}} + \left(\frac{3}{2\gamma_z} - \frac{3}{2}\right) kA - \frac{k^2A^2}{2\sqrt{\gamma_x}\sqrt{\gamma_z}} tanh\,\theta\pi = 0$$

This equation has two roots, with the smallest kA value being selected. Using this initial result, further calculations are performed with equation (13) using the Newton-Raphson iteration method.

The results of the shoaling-breaking research using a numerical model with the wave amplitude function of order 3 indicate a loss of wave energy as the wave travels from deep water to the breaking point. Therefore, the relationship between the wave energy at the breaking point and the wave energy in deep water is expressed as follows:

$E_b = 0.8\, E_0$

Once the α value is determined, the breaking wave height Hb is calculated using the energy conservation equation:

$H_b^2 L_b = 0.8\, H_0^2 L_0$

Substituting L_b,

$$H_b^3 = \frac{0.8\alpha}{\pi} H_0^2 L_0$$

With the obtained H_b value, L_b can be calculated using equation (14), and the breaking water depth is determined with the wave number conservation equation:

$$h_b = \frac{\theta\pi}{k_b} - \frac{H_b}{4}$$

Table (8) presents the results of the breaking parameter calculations. These calculations were performed with $\varepsilon = 0.01$ dan $\theta = 1.3395$. This value of θ yields $\frac{H_b}{h_b} = 0.781$, which differs from the zero-order calculation, which requires $\theta = 1.928$ to achieve $\frac{H_b}{h_b} = 0.78$. The calculated breaking wave steepness is $\frac{H_b}{L_b} = 0.438$, which is smaller than the zero-order breaking wave steepness of $\frac{H_b}{L_b} = 0.633$.

Table 8. Results of Calculation of Breaking Parameters with Wave Amplitude Function Accuracy of Order 3

T (sec)	H_b (m)	L_b (m)	h_b (m)	$\frac{H_b}{L_b}$	$\frac{H_b}{h_b}$
4	0.68	0.762	0.976	0.438	0.781
5	1.063	1.19	1.524	0.438	0.781
6	1.53	1.713	2.193	0.438	0.781
7	2.083	2.327	2.979	0.438	0.781
8	2.721	3.041	3.894	0.438	0.781
9	3.443	3.85	4.93	0.438	0.781
10	4.251	4.75	6.082	0.438	0.781
11	5.144	5.744	7.356	0.438	0.781
12	6.122	6.834	8.751	0.438	0.781
13	7.184	8.019	10.269	0.438	0.781
14	8.332	9.298	11.906	0.438	0.781
15	9.565	10.67	13.664	0.438	0.781
16	10.883	12.149	15.558	0.438	0.781
17	12.287	13.709	17.555	0.438	0.781
18	13.775	15.374	19.687	0.438	0.781

Next, the breaking wave height is compared with the breaking wave height from the Komar-Gaughan equation (10). The results of this comparison are detailed in Table (9) and illustrated in Figure (4). The average difference between the two measurements is 3.5%, with H_b consistently smaller than H_{b-KG}.

Table 9. Comparison between H_b and the breaking height Komar-Gaughan H_{b-KG}.

T (sec)	H_0 (m)	H_b (m)	H_{b-KG} (m)	δ (%
4	0.68	0.762	0.787	3.214
5	1.063	1.19	1.231	3.301
6	1.53	1.713	1.772	3.342
7	2.083	2.327	2.412	3.541
8	2.721	3.041	3.15	3.48
9	3.443	3.85	3.987	3.446
10	4.251	4.75	4.923	3.513
11	5.144	5.744	5.956	3.562
12	6.122	6.834	7.089	3.592
13	7.184	8.019	8.319	3.604
14	8.332	9.298	9.649	3.637
15	9.565	10.67	11.076	3.667
16	10.883	12.149	12.603	3.595
17	12.287	13.709	14.227	3.644
18	13.775	15.374	15.951	3.617

Note : $\delta = \frac{H_b - H_{b-KG}}{H_{b-KG}} x100\%$

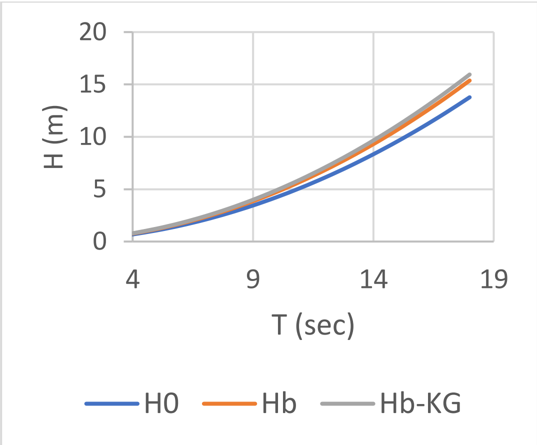

Fig.4: The comparison between H_b and H_{b-KG}

The research on breaking wave height, utilizing a third-order wave amplitude function, yielded results that differ from the breaking wave height values provided by Komar and Gaughan, with an average difference of 3.5%. Given this relatively small discrepancy, the relationship between breaking wave height and wave period can be effectively applied using either of the two methods.

V. CONCLUSIONS

The analysis concludes that the relationship between deep water wave height and wave period shows qualitative consistency across the three methods examined: the zero-order wave amplitude function, the third-order approximation, and the Wiegel's method. Despite this similarity in quality, the quantitative results suggest that one of these methods should be chosen for practical applications.

Between the zero-order and third-order approximations, the third-order method is preferred for its superior accuracy in representing critical deep water wave steepness, aligning more closely with other research findings.

For breaking wave height, the zero-order approximation, third-order method, and Komar-Gaughan approach are qualitatively equivalent. However, the selection of a method should be based on quantitative considerations.

In terms of breaking wave steepness, the third-order approximation provides more accurate results than the zero-order method.

For practical and engineering applications, the zero-order wave amplitude function is recommended due to its simplicity and ease of implementation.

The wave energy equation utilized in this study is based on a sinusoidal wave profile. However, given that the wave profile at larger wave heights tends to be cnoidal or solitary, it is essential to develop a wave energy equation tailored to these wave profiles.

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