

Principles of the Law of Universal Gravitation Applied to the Potential to Attract People

Alessandro Vinicios Schneider¹, Roberto Tuyoshi Hosokawa², Sebastião Cavalcanti Neto³

¹Doutor em Engenharia Florestal, Universidade Estadual do Paraná, Paranaguá, Brazil
e-mail: alessandro.schneider@unespar.edu.br

²Doutor *Honoris Causas*, Universidade Federal do Paraná, Curitiba, Brazil,
e-mail: rth@onda.com.br

³Doutor em Administração, Universidade Estadual do Paraná, Paranaguá, Brazil
e-mail: sebastiao.cavalcanti@unespar.edu.br

Abstract— *Principles of the law of universal gravitation applied to the potential to attract people. This work discusses the association of the Law of Universal Gravitation to estimate the potential to attract people by municipality, aiming to investigate regional economic dynamics. Such conceptions were applied in an experiment involving information of the flows of passengers in the intercity public transportation in the Second Paraná Plateau, State of Paraná, Brazil, besides economic factor of the regions. The flows observed in the intercity public transportation are proportional to the size of the population masses and inversely proportional to the distance that separates the municipalities. Therefore, according to the Law of Universal Gravitation, the larger the population in the municipality and the more centralized the force, the greater the potential to attract people. However, with the exhaustion of the production factor land in the central location, occurs greater use of this factor than other places, resulting in an increase in prices in areas adjacent to or closer it. Other associations are also presented in this paper, including applications of the law of universal gravitation in studies of regional economic dynamics.*

Keywords— *Central location, population masses, population distances, regional economy.*

I. INTRODUCTION

The idea of associating the Law of Universal Gravitation with travel distribution emerged with the American sociologist and economist Henry Charles Carey between 1858 and 1859 when he suggested that the phenomena of migration followed the laws of Newtonian physics. In which the number of people "attracted" or migrating to a given region (M_{ij}) is proportional to the product of the two populations P_i and P_j and inversely related to the distance between them.

However, it was with the theories of the Laws of Migration (1888/89) of the English geographer and

theorist Ernest Raveinstein (1834-1913) that the idea that migration was governed by the "push-pull process". It means unfavorable conditions in one place (lack of employment, few development opportunities, bad governments) pushed people to other places (chances of getting jobs, better living standards) and associated distance as an impedance factor for this process, so Carey's ideas began to gain more strength and meaning in the field of applied social sciences.

Raveinstein established the theory of human migration in the 1880s, which is still the basis for the theory of modern migration. He considered the implications of distance and different types of migrants, with women more likely than men to migrate within the country of their birth, but less likely than men to leave the country of their birth.

In 1931, William John Reyli published at the University of Texas the work "The Law of Retail Gravitation" in which applying Newton's theory explained the patterns of interaction of the retail trade. Some principles formulated by Reilly were thus denominated: a) the attraction of customers varies directly with the population of the area in which the retail is; b) the attraction of customers varies inversely with the square of the distance to be covered by them; c) a city of greater population attracts the consumption of a smaller locality, in the direct proportion of the number of inhabitants; d) a city with larger population attracts the consumption of a smaller locality, in the inverse proportion to the square of the distance between them.

In 1955, with the consolidation of traffic engineering with the work published by HJ Casey ("Applications to traffic engineering of the Law of retail gravitation") that adapting the researches of Reyli and of other scholars, analyzed the flow of purchases in the retail trade between several cities. Several further researches were carried out after Casey's work. However, the work developed by Voorhees showed that although the

principle of the Law of Universal Gravitation could be applied in the distribution of trips, the measure of attractiveness of a zone and the exponent of the distance factor varies with the purpose of the trip (BRUTON, 1979). Therefore the model proposed by Henry Carey was considered a very simplistic analogy with the Law of Universal Gravitation and there were soon improvements, falling in the form that is used today.

Then, anchored in the ideas of Ernest Raveinstein and the mathematical improvements of Voorhess and coming to understand the transit as a huge set of migratory flows (pendular movements). It is considered to apply the Gravitational Model to measure the number of trips attracted between two or more entities (neighborhoods, cities, states, regional, and towns) and with this tool can start urban and regional planning. It also opens the possibility of understanding the origin of social, economic and environmental problems, among others.

In this way, a new concept of planning arises, where the gravitational equation tends to be calibrated within the municipality in favor of being able to attract the population that today lives in pendular movement with the more developed cities where specialized products and services are offered. According to Richardson (1975), although several uses have been suggested for models of this nature, the vast majority of applications have been related to retail trend forecasts or intercity travel patterns. These Gravitational Models can provide a universal approach to the interpretation of nodal regions.

Gravitational models are related to the idea of pole of attraction or development. Thus, for Arango (2000) the location could be explained by the distance to a pole of attraction and its importance, which would function as a mass analogy in Newton's Universal Gravitation Model. In the case of space economy, this type of model suggests a strong emphasis on distance and market size.

"The gravitational model is a heuristically derived expression for synthesizing travel exchanges" (Hutchinson, 1979). According to Clemente (1994), the analogy called the gravitational model consists of considering the intensity of flows between two places instead of forces of attraction and replacing the concept of mass of bodies with some indicator of size of places.

According to Meyer (1971), for gravitational models in urban transport analysis, the transit between two points must be hypothetically positively related to the mass at each point and in a negative way to the friction. Friction can be measured in terms of distance, time, cost and various other factors. Likewise, mass has been defined as population, number of car owners.

According to Isard (1975), the working day and other travel patterns in a given region can develop a model of gravity. The author comments that in respect to

the working day, or "commuting fields," this set fits into the general class of space phenomena that involve movement and communication over space. In addition, there is a general pattern for many diverse and important communications and movements of populations.

According to Isard (1975), in this case, the predicted number of trips must correspond to the real numbers of trips. Assuming, measure the number of actual trips along the horizontal axis and predict the number of trips along the vertical axis. Then, for the pair of subareas, "A" and "B", you can draw on the figure that indicates both the actual number of trips originating from "A" and ending with "B" and the theoretical number predicted by the formula. For the author, "if the theory is good, the real and predicted numbers should be the same, or roughly."

Isard (1975) comments that this type of relationship has been found, in his studies, may be valid not only for transportation, but also for intrametropolitan rail travel, plane travel, and bus travel between pairs of cities, for movement of commodities, such as by rail express, by communication phenomena such as telephone numbers and telegrams between city pairs. His studies can even be valid for many other types of goods, people and movements of ideas, such as the dissemination of rumors, circulation newspapers, and the diffusion of innovations.

According to Isard (1975), as with gravitational phenomena in physics, we are not able to explain the spatial interaction phenomena of the journey to work and the relationship in the social world. We can only speculate on the "why" of them.

Isard (1975) argues that cities with high incomes and education generate more air travel, express train shipments, letters, telephone calls, telegrams, and other communications when compared to low-income, lower-education cities.

Several authors have used the gravitational model as a data analysis tool to explain their research. Portes and Rey (2005) and Head and Ries (2008) using the gravitational model observed that a large part of foreign direct investment (FDI) takes the form of mergers and acquisitions. For Vietze (2008), in social phenomena, the dependent variable is the interaction force between two social elements that would represent the "masses" in Newton's traditional model. These social elements could be: the population, the GDP, the quantities of beds available in hotels, the amount of food companies, educational institutions, and finally any kind of social element.

Družić, Anić and Sekur (2011) observed that the use of relatively simple and easy to obtain data, such as GDP and distance, made the gravitational model particularly popular in explaining the trade patterns of economies in transition, which lack more data

sophisticated and longer series on foreign trade. Arkolakis, Costinot and Rodríguez-Clare (2012) investigated how micro-level data have had a profound influence on trade research in the last ten years. Nijkamp (2013) addresses the validity of Newton's law of universal gravitation in the emerging world. Schneider et. Al. (2016) observed the potential of attracting people by municipalities. Allen, Arkolakis, and Takahashi (2017) propose a new strategy to estimate the gravity constant using an instrumental variable approach that is based on the overall equilibrium structure of the model. Chaney (2018) studied bilateral international trade.

Thus, through these various researches, the potentiality of applying the model to explain several aspects of the economic dynamics is evidenced. In this way, the purpose of this study was to associate the Law of Universal Gravitation to estimate the potential of attraction of people by municipality, aiming to investigate regional economic dynamics.

II. MATERIALS AND METHODS

2.1 SELECTION OF THE STUDY LOCATION

This research was developed in the region of Second Paraná Plateau, state of Paraná, Brazil. In the experiment we observed in particular the municipalities of Telêmaco Borba, Ortigueira, Sapopema, Curiúva, Figueira, São Jerônimo da Serra, Imbaú, Reserva, Tibagi and Ventania.

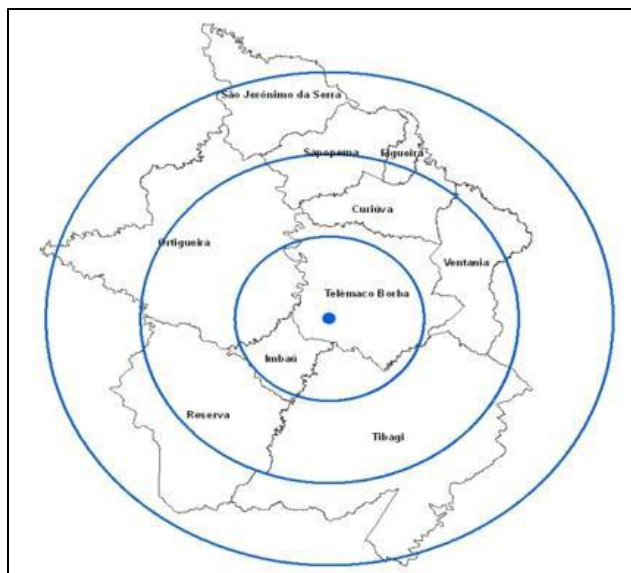


Fig.1: Municipalities of the region Second Plateau of Paraná.

Why was this region chosen in the state of Paraná? The main reason for the choice of this region is that the economy of the region presents itself in a diversified way, highlighting the timber sector, assuming that the municipality of Telêmaco Borba was initially driven by the activity of pulp and paper process wood,

located there, and later the Forest Activity was disseminated to the neighboring municipalities. In view of this context, it is worth mentioning Myrdal's brief quotation (1957), "from an initial agglomeration, a region in which economies of scale and technological development existed would attract new resources that would reinforce its expansion."

2.2 GRAVITATIONAL MODEL TO EXPLAIN THE POTENTIAL OF ATTRACTION IN MUNICIPALITIES

In the application of the model proposed by Isard (1975), the distances between the studied municipalities, their respective populations and the flows of passengers in the year 2010 in intermunicipal collective transport were used. The mass of the gravitational model in the present study corresponds to the variable population (P_i). The data come from the censuses conducted in the year 1991, 2000 and 2010 by the IBGE. To compose the original Gravitational Model, the values of the distance (d_{ij} , in kilometers) between the municipalities were provided by the Department of Roads and Drive of the State of Paraná (DER / PR, 2010).

The number of journeys made in the study area originated from the data observed by DER / PR (2010), and it corresponds to the number of intermunicipal trips made in intercity public transportation, among the municipalities of the region in 2010. Among the 90 possibilities of flows, 43 were obtained, since the rest did not occur, due to the lack of regular and direct lines of intermunicipal collective transportation among some municipalities. It was observed that on the sample was 309,079 displacements in the year 2010.

The study has the characteristics of being exploratory, with a quantitative approach from secondary data. For the application of the model proposed by Isard (1975), the distances between the municipalities, their respective populations and the flows of passengers observed in the intermunicipal collective transport were used.

Thus, according to Newton's proposals, the value of the force of attraction (F) is directly proportional to the masses of the two bodies and is inversely proportional to the square of the distance between them. The forces appear in pairs, that is, if one body attracts another, it is also attracted by the first. In the present study, the constant (G) represents a correction factor between the mass units and the distance that separates it.

With all the variables and constants obtained through these mathematical procedures, the model developed by Isard (1975) was applied to obtain a new flow estimated by the gravitational model of the displacements of the people through intermunicipal collective transportation. As such, it is expressed by:

$$I_{ij} = G \frac{P_i * P_j}{(d_{ij})^b} \quad (1)$$

In which: I_{ij} = Estimated flow; G = Correction factor; P_i = Population of i ; P_j = Population of j ; d_{ij} = distance between i and j ; b = exponential coefficient

In order to determine or estimate the Potential of Attraction (V_i) of the municipalities of the Second Parana Plateau, the Principle of Superposition was observed. This means that when two or more waves propagate simultaneously in the same medium and instant and in the same direction, it is said that there is a superposition of waves. The resulting wave is equal to the algebraic sum of the waves that each would produce individually. This principle can be applied to obtain a resulting scalar magnitude. Thus, considering passenger flows in inter-municipal collective transport, such as waves, the greater the volume and intensity of these flows to the same municipality, the greater its potential attraction (V_i).

According to Schneider et.al. (2016), two interpretations can be given to the concept of attraction potential (V_i). On the one hand, it is a measure of their influence or impact on point i , and the set of masses distributed in space i itself. On the other hand, it is a measure of accessibility of point i to the set of masses distributed in the space under study. In this way, the greater the intensity at point i , when compared to the other points, the greater its attraction potential (V_i).

In this context, the interaction (or flow) between points i and j estimated by the presented model can be calculated in order to obtain the interaction between i and all points j , which represents the attraction potential (V_i) of point i . It should be noted that the attraction potential (V_i) of point i is equal to its own mass (P_i), plus the masses of the remaining points, each corrected by its distance i , multiplied by a constant (G). As described, the interaction between i and j is expressed by:

$$V_i = 1 + (I_{i1} + I_{i2} + I_{i3} + \dots + I_{in}) \quad (2)$$

In which: V_i = Potential of Attraction of the municipality i ; I_{i1} = Estimated flow from municipality 1 to i ; I_{i2} = Estimated flow from municipality 2 to i ; I_{i3} = Estimated flow from municipality 3 to i ; I_{in} = Estimated flow of the municipality n to i .

The first approach was made using the original gravitational model by means of the original intermunicipal collective transport called the gravitational model 1, observing the data referring to the population in the year 1991 and the observed flows (I_{ij}) in the year 2010 and the respective distances between municipalities.

The gravitational model 2 was observed to the population in the year 2000 and the flows in the intermunicipal collective transport in the year of 2010. The gravitational model 3 was observed in the population in the year 2010 and the observed flows (I_{ij}) in the same period. In all the adjustments made it was estimated the potential of attraction (V_i) of people through the proposed method.

The adjusted models were evaluated using the coefficient of adjusted determination, F statistic and standard error in percentage.

III. RESULTS

When comparing the three situations, the gravitational models presented the same coefficient of determination (R^2) (TABLE 1). The gravitational model 3 presented the best R^2 (0,7287) and the gravitational model 1 the largest F (110,16).

The coefficient b_1 is directly related to the friction that the distance gives to the displacement of the people, be it friction of physical, psychological, behavioral order, among others. In the gravitational model 1 ($b_1 = -3,189$), the population presented greater elasticity in relation to the friction relative to the distance to be traveled in their movements or trips between the municipalities (TABLE 1). On the other hand, in the gravitational model 3 ($b_1 = -3,087$), it was less elastic to the displacements.

Table.1: Adjustment statistics of the models referring to the flows of displacements and distance between the municipalities of the region of the second Parana plateau

TEMPLATE	R^2	$S_{xy}\%$	F	b_0	b_1
Gravitational model1	0,7281	16	110,16	5,24	-3,189
Gravitational model2	0,7247	17	107,98	5,06	-3,092
Gravitational model3	0,7287	17	109,80	5,05	-3,087

Among the constants calculated in the three different situations, the gravitational model 1 obtained the highest constant k (1.71), that is, the largest mean displacement per individual (TABLE 2).

When the correction factor (G) is analyzed, it was observed that in the situation of the gravitational model 1, the correction factor had a higher value (1.63) when compared with the other situations under analysis (TABLE 2). This factor caused an increase in the

magnitude of the quotient between the product of the masses and the distance that separates them. This factor provided a better fit in the gravitational model so that the equality between the estimated flux quantities (I_{ij}), the magnitude of the quotient between the size of the masses and the distances separating them would occur.

Table.2: Constants of the gravitational model of passenger displacements in intermunicipalities collective transportation in the Second Plateau of Parana

TEMPLATES	K	Correction factor (G)
Gravitational model1	1,71	1,63
Gravitational model2	1,65	1,01
Gravitational model3	1,55	0,87

It was observed in gravitational model 1; the correction factor was higher when compared to the other situations. The correction factor (G) in the gravitational model 3 was 0.87, that is, this factor caused a decrease in the magnitude of the quotient between the size of the masses and the distance separating them. This factor provided the equality between the analyzed quantities. In this specific situation, it can be affirmed that the population increase (1991/2010), has made the factor, over time, decrease. In 1991, the correction factor was higher when compared to the other situations, because the estimated flows (I_{ij}) were greater than the quantity of the quotient between the size of the masses and the distance that separates them.

When the gravitational model 2 is observed, this factor was equal to 1.01, that is, in this situation the estimated flows (I_{ij}) were equal to the magnitude of the quotient between the size of the masses and the distance separating them. In this case, it is true to say that the flow of people in the intercity bus service of the region of the Second Parana plateau is directly proportional to the size of the populations of the respective municipalities and inversely proportional to the distance that separates them. In this model we can observe a perfect situation with regard to the law of physics, developed by Isaac Newton, where the force of attraction between two bodies is directly proportional to its size and inversely proportional to the square of the distance that separates them.

When the regression of the gravitational model 1 was performed, the equation obtained was:

$$\text{Log } (I_{ij}/T_{ij}) = 5,2371 - 3,1889 \text{ Log } (d_{ij}) \quad (3)$$

The estimated flows (I_{ij}) in the intercity bus service should occur in less quantity than was observed for the equilibrium between the two sides of the equation to occur. The correction factor provided an increase in the

value of the quantity of the quotient between the size of the masses and the distance that separates them, so that the mathematical equality occurred in the gravitational model.

The adjusted gravitational model can then be written as:

$$I_{ij} = 1,63 \frac{P_i P_j}{(d_{ij})^{3,18}} \quad (4)$$

With respect to the attraction potential (V_i), that is, an attraction capacity of each municipality in the region can be represented by the equation:

$$V_i = 1,63 * P_i + 1,63 * \frac{P_i * P_1}{(d_{i1})^{3,18}} + 1,63 * \frac{P_i * P_2}{(d_{i2})^{3,18}} + \dots + 1,63 * \frac{P_i * P_n}{(d_{in})^{3,18}} \quad (5)$$

When the regression of the gravitational model 2 was performed, the equation obtained was:

$$\text{Log } (I_{ij}/T_{ij}) = 5,0558 - 3,0921 \text{ Log } (d_{ij}) \quad (6)$$

In this specific case, the magnitude of the quotient between the size of the populations and the distance that separates them are in equilibrium or in equality with the estimated flows (I_{ij}) occurred in the municipality of the Second Plateau Paranaense.

The estimated flows (I_{ij}) in intermunicipal collective transport occur in quantities identical to what was observed, so it can be said that a balance exists between the two sides of the equation. The correction factor does not provide correction between the two sides of the mathematical equation, and it can be said that the estimated flows (I_{ij}) are directly proportional to the size of the masses (population) and inversely proportional to the distance that separates them.

The adjusted gravitational model can then be written as:

$$I_{ij} = 1,01 \frac{P_i P_j}{(d_{ij})^{3,09}} \quad (7)$$

With respect to the attraction potential (V_i), that is, the attractiveness of each municipality in the region can be represented by the equation:

$$V_i = 1,01 * P_i + 1,01 * \frac{P_i * P_1}{(d_{i1})^{3,09}} + 1,01 * \frac{P_i * P_2}{(d_{i2})^{3,09}} + \dots + 1,01 * \frac{P_i * P_n}{(d_{in})^{3,09}} \quad (8)$$

When the regression of the gravitational model 3 was performed, the equation obtained was:

$$\text{Log } (I_{ij}/T_{ij}) = 5,0505 - 3,087 \text{ Log } (d_{ij}) \quad (9)$$

The estimated flows I_{ij} in intermunicipal collective transport should occur in less quantity than was observed for the equilibrium between the two sides of the equation to occur. The correction factor provided an

increase in the value of the quantity of the quotient between the size of the masses and the distance that separates them so that the mathematical equality occurred in the gravitational model.

The adjusted gravitational model can then be written as:

$$I_{ij} = 0,87 \frac{P_i * P_j}{(d_{ij})^{3,08}} \quad (10)$$

With respect to the attraction potential (V_i), that is, the attractiveness of each municipality in the region can be represented by the equation:

$$V_i = 0,87 * P_i + 0,87 * \frac{P_i * P_1}{(d_{i1})^{3,08}} + 0,87 * \frac{P_i * P_2}{(d_{i2})^{3,08}} + \dots + 0,87 * \frac{P_i * P_n}{(d_{in})^{3,08}} \quad (11)$$

IV. DISCUSSION

Observing the configuration of the attraction potential distribution (V_i) of people in the municipalities of the Second Paraná Plateau, this refers to the theory proposed by Chirstaller (1966), where the author sought to understand the laws that determine the number, size and distribution of cities, which, according to him, are known as central places. In this case, the municipality of Telemaco Borba is characterized as the main nucleus and the neighboring municipalities as a complementary region. Therefore, a nodal or polarized region behavior is observed.

The spatial distribution of the municipalities of the Second Paraná Plateau results in an economic space organized around a main urban nucleus. Observed in this perspective, the spatial distribution of the population can be considered as being in a hierarchical system and with the most varied functional connections, providing a hierarchical system of the attraction potential (V_i) of people proportionally identical to the population distribution.

The region of the Second Parana plateau, Telêmaco Borba is the dominant center with the largest population, and in its surroundings gravitate flows of populations, goods and services, communication and traffic contemplating the complementary region. The proportional centrality of the population of Telemaco Borba determines the centrality of the attraction potential (V_i) of people, making this potential more intense in the regions closer to the center due to distance factor, when compared to municipalities more distant from the central place.

In this context, the theory proposed by Hirshman (1958) is emphasized, based on the assumption that economic progress does not occur everywhere at the same time and that, once it has occurred, certain forces provoke

a spatial concentration of economic growth, around the points where the process begins.

According to Schneider et. al. (2018), among the factors that make up the economic activities of cities, the Gross Value Added to industrial activity offers the best potential to attract people (V_i).

The municipality of Telêmaco Borba was initially driven by the activity of process wood for paper, and later for pulp. Due to the demand generated by this process, Forest Activity (forest plantations) was first disseminated in the municipality of Telêmaco Borba and later to the nearest municipalities.

This process of development of the central place began in the 1950s, and intensified in the 1980s, with APL installed in the municipality. Forest plantations for commercial purposes in Telêmaco Borba and neighboring municipalities (Imbaú, Curiúva and Ventania) increased as a result of fiscal incentives for reforestation in the 1970s, in order to increase the supply of wood.

The neighboring municipalities, or closer to the central place, when compared with the more distant municipalities, benefit because of the decrease of the friction regarding the distance between them and the central place. Thus, the greater the increase of the benefits pertinent to this potential and consequently an increase occurred in the population of these municipalities. The more distant ones lose the potential of attraction (V_i) of people to the central place, provided a diminution of that potential over time, when compared to those closest to the center of attraction. During this period, a policy of private forestry development was developed in the region of the Second Parana Plateau, called Fomento Florestal, with the purpose of increasing the supply of reforested wood in the region and, consequently, the development of the municipalities closest to Telêmaco Borba due to the lack of the "land" factor in the central place.

According to Richardson (1973), in his proposed theory, the movements of the factors of production may not be balanced, and the backward or less developed counties suffer from capital flight as well as emigration. From this point of view, the behavior of the attraction potential (V_i) of people from the most distant municipalities of the central place tended to lose strength due to the emigration occurring in the municipalities of Ortigueira, São Jerônimo da Serra, Figueira and Sapopema due to the decrease in opportunities for the population of these municipalities.

This characteristic of production factors movement is due to the opportunities generated in the central place. Thus, the greater the centrality of a place, in this case Telêmaco Borba, the greater the surroundings, or area of influence, and the neighboring municipalities depend on the central place. In addition, the more complex the services offered, the greater will be the region served

by this center. Thus, the larger the population, the more likely their growth rates and economic development levels are to be higher when compared to their peers.

In the region of Second Parana plateau it is noted the interdependence of economic growth between the municipalities and the central place. Telêmaco Borba has the best index regarding the number of companies, higher income per capita, more and better services, number of bank branches, number of beds, among other public or private services, generating a spatial concentration. The central place generates positive externalities for the nearest municipalities.

The spatial concentration can be understood as access to a larger market, with more abundant and more qualified labor supply, as well as the presence of commercial, banking, financial and legal facilities, among others. On the other hand, large concentrations result in pecuniary diseconomies such as elevated land values, higher rental costs, among other factors. Nevertheless, seldom do these diseconomies destroy the attraction potential (V_i) of the central place, leading to an increase in the agglomeration of the municipalities closest to the center of attraction, in this case Telêmaco Borba.

This assertion is justified because, in the municipalities of Ventania, Imbaú, Curiúva and Tibagi, population growth occurred between 1991 and 2010, and consequently an increase in the attraction potential (V_i) of people. This behavior was not observed in the most distant municipalities, and São Jerônimo da Serra, Sapopema, Figueira and Ortigueira lost part of their population in the period from 1991 to 2010, and as a consequence, part of their potential to attract (V_i) people.

Between 1991 and 2000, the region of the Second Paraná Plateau had the potential of attraction (V_i) of people greater than the total population of the region. Thus, it is assumed that during this period there was an intense use of the Capital (K) and Natural Resources

(land) production factor, increasing the total population in the region, due to the greater availability of opportunities in the municipalities, especially in Telêmaco Borba.

In the year 2000, there is a balance between the factors of production. This characteristic is linked to the meeting of the attraction potential (V_i) and the total population of the region.

In the period between 2000 and 2010, the potential of attraction (V_i) of people decreased, making it smaller than the population of the region. This feature may be associated with a decrease in the supply of factors of production, especially the "land" factor, providing a limit of growth and, consequently, stabilizing the centrality of the region.

Throughout the decades of 1991 to 2010, the potential of attraction (V_i) of people in the region of the Second Plateau Paranaense presents a line with negative slope due to the decrease of the potential of the municipalities more distant from the central place. In the year 1991, the potential had a value of 295,202. In the year 2000, this value decreased to 188,026 and in 2010 it increased to 174,236 (TABLE 03).

Differently, the total population of the region presents a positively inclined line, due to the increase in the population of Telêmaco Borba and the municipalities closest to the central place. Given this scenario, it was observed that there was a population increase in the municipalities closest to Telêmaco Borba, mainly in Ventania, Imbaú and Curiúva. This characteristic may be related to the supply of the "land" production factor. In the municipality of Telêmaco Borba the supply of this factor is limited, making it scarce when compared to other municipalities in the region. According to DERAL (2014), the price of mechanized land increased by 375%, from R \$ 4, 400.00 per hectare to R \$ 16, 500.00 between 2000 and 2010.

Table.3: Potential attraction of the municipalities of the second plateau of Paraná.

MUNICIPALITY	1991	(%)	2000	(%)	2010	(%)
Telêmaco Borba	94.984	32,18%	61.595	32,76%	61.085	35,06%
Reserva	39.029	13,22%	24.117	12,83%	22.007	12,63%
Ortigueira	44.914	15,21%	25.363	13,49%	20.440	11,73%
Tibagi	26.820	9,09%	18.543	9,86%	16.913	9,71%
Curiúva	17.152	5,81%	12.980	6,90%	12.173	6,99%
São Jerônimo da Serra	21.678	7,34%	11.819	6,29%	9.911	5,69%
Imbaú	13.038	4,42%	9.534	5,07%	9.861	5,66%
Ventania	10.347	3,51%	8.071	4,29%	8.705	5,00%
Figueira	15.653	5,30%	9.091	4,84%	7.251	4,16%
Sapopema	11.587	3,93%	6.913	3,68%	5.890	3,38%
TOTAL	295.202	100 %	188.026	100 %	174.236	100 %

With the public and private development policies implemented, there was an increase in the supply of raw materials, that is, with this increase of the policies, the use of the land factor in the municipalities around the central place, due to the scarcity of this resource in the municipality of Telêmaco Borba. Thus, it can be affirmed that there was an increase in the area of influence of the municipality of Telêmaco Borba, due to the increase in the radius of influence of the central place, causing an increase in the attraction potential (V_i) of people in the municipalities more near the central place. Consequently, the scarcity of the "land" factor of production, in the central place, provided new investments in the nearest municipalities.

At the outset, these investments were made in forest plantations for commercial purposes to supply the demand of the municipality of Telêmaco Borba. Moreover, in the second moment, in small and medium-sized companies, directed to forest-based products, in order to obtain the facilities and benefits from the local productive arrangement installed in Telêmaco Borba and the distance from the raw material, located there from these plantations.

In the more distant municipalities, Ortigueira, São Jerônimo da Serra, Figueira and Sapopema, there was a decrease in the total population over time. This feature is linked to the attraction potential (V_i) of people from Telêmaco Borba, causing these municipalities to lose population to the central place and to the nearest municipalities due to the rural-urban migration that occurred during the period in the region, mainly between the years 1991 and 2000. The rural population of the more distant municipalities migrated to the urban area of Telêmaco Borba and to the municipalities of Ventania, Imbaú, Curiúva.

In Reserva, an atypical situation was observed when compared with other municipalities. The municipality presented a stabilization in the population growth and, consequently, a stabilization in the potential of attraction (V_i) of people, being the second greater potential of the region. This characteristic may be related to the balance between the supply and demand of the factors of production available in the municipality, making it a "node" with less dependence on Telêmaco Borba, when compared to the nearest municipalities (Imbaú, Ventania and Curiúva) and more distant municipalities (Ortigueira, São Jerônimo da Serra, Sapopema, Figueira).

When observed the behavior of the potential attraction (V_i) of people in the region of the Second Plateau Paranaense and the individual behavior of each municipality that makes up the region, those that differed in relation to their peers were Imbaú, Ventania and Curiúva. In 1991, the municipality of Imbaú represented

4.4%, growing to 5.1% in 2000 and 5.7% in 2010. A trend of proportional potential growth has been observed in the last ten years, resulting in a positive trend when compared to the region. This feature is linked to the new investments made in the municipality, making it more dynamic with the increase of new business opportunities, services and jobs, providing a better quality of life for residents.

The municipality of Ventania obtained 3.5% of the attraction potential (V_i) of people of the region in the year of 1991. In the year 2000, the potential grew to 4.3% and in 2010 was 5%. The same growth trend occurred when compared to the municipality of Imbaú. This positive trend may be due to the significant increase in the total population between 1991 and 2010, due to new opportunities in the agricultural sector developed in the municipality. These municipalities were the ones that added more population in that period due to the potential of attraction (V_i) of people.

An inverse situation occurred with Ortigueira, which in 1991 represented 15.2%, in 2000 13.5%, being the second largest potential in this period. In 2010, the potential of attraction (V_i) of people in the municipality decreased to 11.7%, representing the third largest potential. Among the municipalities observed, Ortigueira showed the greatest proportionality oscillation of this index. This result is directly related to the fact that in the same period the municipality lost 14.99% of its total population, mainly between 1991 and 2000. When compared to its peers, the one lost more population in absolute numbers. This feature is linked to the reduction or non-emergence of new investments in the municipality, providing an emigration of the population to municipalities that presented new opportunities in this period.

V. FINAL CONSIDERATIONS

The flows observed in intermunicipal collective transport are directly proportional to the size of the masses and inversely proportional to the distance that separates them, and can be expressed by a central, attractive force, hierarchically organized by the size of the populations and the distances that separate them.

In gravitational modeling, the larger the population of the municipality, the greater its potential for attracting people. The municipality with the greatest potential for attracting people is Telêmaco Borba, followed by the municipality of Reserva. The municipalities highlighted in the study are Imbaú, Ventania and Curiúva, due to the greater proximity of their areas of the central region. There is an increase in the potential for attracting people over time and, consequently, an increase in the population of the respective municipalities.

In the more distant municipalities of the central area occurs a decrease of the population, due to the proportionality of the decrease of the potential of attraction of people. These municipalities are Ortigueira, Tibagi, Sapopema, São Jerônimo da Serra and Figueira.

The population variable alone does not fully explain the behavior of people in terms of displacement in intercity bus service. The model developed by Newton has as characteristic to be static and closed, demonstrating limitation in the explanation of the attractiveness of people between the municipalities. Hence, the importance in establishing the correction and weighting factors that were presented in this work.

By means of the "land" production factor, which has the characteristics of constraints, causing its exhaustion in the central place, a greater use of this factor causes a rise of the prices in the areas adjacent to or closer to the centrality, resulting in a significant increase of the potential of attracting people to these areas.

By means of the correct modifications in the gravitational model developed by Newton, presented in this work, good results are evidenced with respect to practical applications for the regional development that is, enabling to develop studies of regional economic development. Using the proposed model, future scenarios can be developed where the present scenario can be evidenced and the impact generated by new regional investments can be quantified.

In future research it is suggested to add the potential of attraction of information, replacing the flows in the intermunicipal collective transport by the flows of intermunicipal telephone calls, and, thus, to determine the degree of information generated by a certain municipality.

REFERENCES

- [1] ALLEN T., ARKOLAKIS C., e TAKAHASHI Y. 2017. Universal Gravity. Disponível em: <<http://www.econ.yale.edu/~ka265/research/UniversalGravity/AAT%20%20Universal%20Gravity%20-%20paper.pdf>>, Acesso em 10 de abril 2018.
- [2] ARANGO, J. 2000. Explaining Migration: A Critical View. *International Social Science Journal*, v. 52, p. 283-296.
- [3] ARKOLAKIS, C., COSTINOT, A., e RODRÍGUEZ-CLARE, A. 2012. New Trade Models, Same Old Gains? *American Economic Review*, 102(1): 94-130.
- [4] BRUTON, M. J. 1979. *Introdução ao Planejamento dos Transportes*. Interciência: Rio de Janeiro.
- [5] CLEMENTE, A. 1994. *Economia regional e urbana*. São Paulo: Atlas.
- [6] CHANEY T. 2018. "The Gravity Equation in International Trade: An Explanation," *Journal of Political Economy*, 126, no. 1: 150-177.
- [7] CHRISTALLER, W. 1966. *Central places in southern Germany*. New Jersey: Prentice-Hall.
- [8] Departamento de Estrada e Rodagem – DER. 2010. *Anuário estatístico do sistema de transporte coletivo regular intermunicipal de passageiros do estado do Paraná*. Curitiba.
- [9] Departamento de Economia Rural – DERAL. 2014. Disponível em: <http://www.agricultura.pr.gov.br/arquivos/File/deral/terras_pdf_publicacao.pdf>. Acesso em 28 de março de 2014.
- [10] DRUŽIĆ, I., ANIĆ, M., e SEKUR, T. 2011. Gravity Model of Croatian Regional Foreign Trade. *Proceedings of 8th International Conference: Economic Integration, Competition and Cooperation*, 6-9 April, Opatija, University of Rijeka – Faculty of Economics, CD with Full papers.
- [11] HEAD, K. e RIES, J. 2008. FDI as an outcome of the market for corporate control: Theory and evidence. *Journal of International Economics*, Elsevier, vol. 74(1), pages 2-20.
- [12] HIRSCHMAN, A.O. 1958. *The strategy of economic development*. New Haven: Yale University Press.
- [13] HUTCHINSON, B. G. 1979. *Princípios de Planejamento do Sistema de Transporte Urbano*. Rio de Janeiro: Guanabara Dois.
- [14] Instituto Brasileiro de Geografia e Estatística – IBGE. 2010. *Censo demográfico*. Rio de Janeiro.
- [15] ISARD, W. 1975. *Introduction to Regional Science*. Englewood Cliffs, New Jersey: Prentice-Hall.
- [16] MEYER, J. R. 1971. *Panorama da moderna teoria econômica*. São Paulo: Atlas.
- [17] MYRDAL, G. 1957. *Economic theory and under-developed regions*. London: Gerald Duckworth & CO. LTD.
- [18] NIJKAMP. 2013. The universal law of gravitation and the death of distance. *Romanian Journal of Regional Science*, vol. 7, issue 2, 1-10.
- [19] PORTES, R., REY, H. 2005. The determinants of cross-border equity flows. *Journal of International Economics*, vol 65, 269-296.
- [20] SCHNEIDER A. V., HOSOKAWA, R. T., ROCHADELLI, R., and BEHLING, A. 2018. "Potential to attract people weighted by economic factors", *International Journal of Development Research*, 8, (08), 22570-22676.
- [21] SCHNEIDER, A.V., HOSOKAWA, R.T., JUNIOR, R.T., BERGER, R. 2016. Potencial de atração de

pessoas na região do Segundo Planalto Paranaense. **Floresta**, vol 46, Nº 3, pag 379-386.

[22] RICHARDSON, H. W. **Elements of regional economics**. England: Penguin Books INC., 1969.

[23] RICHARDSON, H. W. 1975. **Economia regional: teoria da localização, estrutura urbana e crescimento regional**. Rio de Janeiro: Zahar Editores.

[24] VIETZE, C. 2008. Cultural effects on inbound tourism into the USA: A gravity approach. **Jena Economic Research papers**. Jena, Germany.