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Reliability analysis of reinforced concrete slabs designed according to NBR 6118

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***Abstract*— *NBR 6118 (2014) is the Brazilian standard that guides the design of reinforced concrete structures and adopts semi-probabilistic methods as a reference. These establish safety criteria that confront internal forces resulting from actions, increased by majoring coefficients, with the characteristic strengths of steel and concrete materials also reduced by minoring coefficients so that the former is equal to or less than the latter ($Sd \leq Rd$). However, unlike the Brazilian standard, the international standards determine the calibration of these coefficients through probabilistic methods. This calibration is a factor of paramount importance concerning the measurement of the risk of the structure. It is known that the material's properties present a certain level of dispersion. Depending on the workmanship quality, there are also uncertainties regarding the geometry of the structural parts. Furthermore, the actions in the structure show considerable variation throughout its useful life. In this context, one of the objectives of this work was to determine the reliability of reinforced concrete slabs designed according to NBR 6118 (2014), with loads determined by the recently updated standard NBR 6120 (2019), through a probabilistic analysis using a Finite Element numerical model and through a non-linear analysis. For this, the proposed study addresses the determination of resistance, represented by a theoretical distribution adjusted from simulations generated by the Monte Carlo Method using the ANSYS software. The reliability indices were obtained using the FORM method. As a result, it was possible to verify that most slabs are above the reliability indices indicated as acceptable by the American standard ACI 318 (2014). In addition, the significant influence of the variable loading on the results was confirmed due to its great variability.*

I. INTRODUCTION

It is necessary that Brazilian standards, like European and American standards, can be calibrated in the light of the Reliability Theory. However, it is known that there is a lack of studies that make this feasible.

Some studies point out that the behavior of reinforced concrete structures is complex due to its non-linearity, generating uncertainties in its approach in studies and designs. Thus, the probabilistic analysis presents an excellent way to investigate the safety margin of structures as a function of their failure probability [1].

Santiago (2019) presented a reliability-based calibration of the partial safety factors of Brazilian standards used in the design of steel and concrete structures. About reinforced concrete structures, the study addressed reinforced concrete beams subjected to bending, reinforced concrete beams subjected to shear, reinforced concrete columns subjected to normal bending-compression, and reinforced concrete slabs subjected to bending. The work contributed to statistically adjusting the main random variables of resistance and load associated with both metallic and reinforced concrete structures in Brazil. However, the authors emphasize the need for more work to support reviewing the safety coefficients in force [2].

The safety of a structure must be linked to the reliability that indicates its probability of failure - preferably low - taking into account the ultimate and service limit states. It can be said, then, that the Reliability Theory considers it essential to assess the uncertainty linked to all the variables involved in the safety and performance of the structure to obtain knowledge of the probability of failure corresponding to its limit states. [3].

Among the methods used for this type of study, the most accurate is the Finite Element Method (FEM), which presents the best prediction of behavior and failure for a reinforced concrete structure [4]. The FEM is the most used tool for engineering modeling and analyzing structures with non-linear behavior. The use of this type of analysis results, in contrast to experimental models, in the possibility of not having to use a large number of physical models, saving considerable financial and material resources [5].

The loading variables (actions) are divided into permanent and variable, and it is assumed that they must be present during all or part of the service life of the structures. It is important to predict the loads acting on a structure precisely. The loads' characteristics and variability are fundamental parameters in reliability analysis. That is, a reliable database conducts a good statistical analysis. [6]. In this sense, it is worth noting that the Brazilian standard NBR 6120 - Actions for the Calculation of Building Structures had its last revision in 2019 [7], so its evaluation from the perspective of the Reliability Theory should be desirable and necessary.

The purpose of this research is the numerical study of the reliability of reinforced concrete slabs subjected to bending designed according to the NBR 6118 [8], using a non-linear analysis employing the Finite Element Method and taking into account loadings recommended by NBR 6120, updated in 2019. The numerical model used was validated, and more information can be found in Viegas et al. [9].

II. METHODOLOGICAL STRATEGY

With the proper performance of this model, it is possible to obtain the resistant capacity of slabs designed according to the NBR 6118 (2014) standard. ANSYS has a handy platform called APDL (Ansys Parametric Design Language) so that the user can add routines - in a programming language similar to Fortran 77 - together with pre-existing computational models of the software. The used model was validated by comparing the model's rupture load with data from experimental slab tests.

The model was developed and used for rectangular slabs simply supported on the four edges. The slab strength statistics and distributions were determined by the Monte Carlo method, which is available in the ANSYS software through the Probabilistic Design System (PDS) tool. The main random variables related to geometry and material properties are considered in the process and represented by probability distributions [8].

For the reliability study, the FORM transformation method (First-Order Reliability Method) and the Monte Carlo simulation method were used, with the algorithms implemented in Python software. The resistance obtained as a function of the Ultimate Bending Limit State determines the model's safety margin. This analysis is accomplished using the numerical model, and the actions composed in each combination are determined through the Brazilian norms [7] and [10]. Finally, the reliability indices obtained in this work were analyzed with the target reliability indices indicated by international standards, in addition to a parametric study that stated the main design parameters which influenced the variation of reliability indices. The rupture model implemented was the one present in recent versions of ANSYS called Drucker Prager Rankine (DP-Rankine). For the reliability analysis, slabs with dimensions of 400x400cm, 500x500cm, 600x600cm, a minimum thickness of 10 cm and increased accordingly to design were used; and, for fck of 25, 50, and 70 MPa. The loading variation, qk/(gk+qk), will be 0.25, 0.5, and 0.75, where: qk is the characteristic variable loading, and gk is the characteristic permanent loading.

III. STRUCTURAL RELIABILITY

Structural reliability deals with the ability of a structure to fulfill the structural function for which it was designed, associated with a certain risk. For this, the so-called degree of confidence is used, measured through the probability of non-failure (1-Pf), where Pf is the failure probability.

Thus, each model developed to analyze structures must consider the structural behavior as accurately as possible through a specified set of basic variables. Among them, we can mention the weight of materials, dimensions, influences of loads, and environmental actions, as well as parameters of the model itself and other structural requirements. The fact is that most of these variables are more or less random depending on their nature, and thus it is almost impossible to create an exact model for them. This way, simplifications are used through probability distributions of some parameters, transforming the analysis result into a random variable [11].

This way, for structures to be designed to fulfill their predetermined functions throughout their useful life, they must meet safety requirements. At the same time, they must be economically viable. One of the ways used to achieve these requirements of a technical nature is the so-called Limit States method.

In this direction, for reinforced concrete elements, the design and analysis must be based on: Ultimate Limit States - which deal with the collapse conditions of the structure - and Service Limit States - which deal with their conditions of use involving durability, functionality, comfort, among others. Any of these limit states make the use of the structure unfeasible. [12].

In this way, the degree of confidence is measured considering the physical and design uncertainties, and, for this purpose, it uses, among others, physical, mathematical, and statistical models. Thus, the uncertainties in engineering projects can be classified as intrinsic when related to physical, chemical, and biological phenomena of nature; epistemic, when associated with the knowledge of system variables as well as situational processes; and human error, which, through training, can be avoided or reduced considerably. In the study of structure reliability, several efficient techniques exist to estimate these uncertainties [12].

In addition, it is necessary to specify the performance function for the safety and failure regions in the design variable space. Then, the probability distributions are integrated using numerical integration or simulation techniques. One of the possible methods for this calculation is the Monte Carlo method [13].

The Monte Carlo method was presented in 1949 through the article "The Monte Carlo Method," developed by mathematicians John Von Neumann and Stanislaw Ulam. The technique aims to simulate the response of functions of random variables through deterministic values of these variables in each simulation cycle [14].

IV. BASIC RELIABILITY PROBLEM

The reliability study combines all load and resistance distribution functions and a performance function that will characterize the safety and failure region. In this way, this is accomplished through the integration of the probability density function over the failure region.

According to [13], reliability considers a load effect, S, resisted by a resistance, R, where a probability distribution represents each, namely: f_S and f_R. This way, S can be determined from the applied load or set of resulting internal forces of structural analysis. A structural element fails when its strength R is less than the stress resulting from load S. So, the probability of failure is given by:

$$pf = P[\{S \geq R\}] \quad (1)$$

V. LIMIT STATE FUNCTIONS

According to [12], limit state functions, also called performance functions, constitute one of the first situations to be established in the scope of structural reliability and follow a "margin of safety" style approach involving two statically independent random variables of normal distribution. If (R) represents the resisting capacity and (S) represents the load, the performance function is a failure condition. Thus, the limit state function can be defined by Equation 2 and presented in Fig. 1.

$$G(R,S) = R - S \quad (2)$$

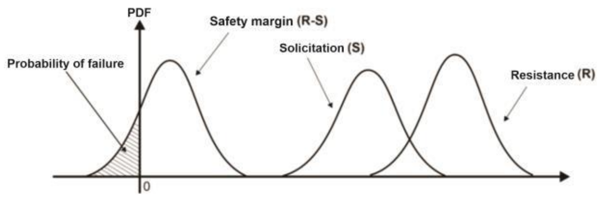

Fig.1: Function of request probability density, resistance, and safety margin. Adapted from [15]

The safety parameters related to the failure of the structure are directly linked to the Ultimate Limit State, where the load intensity (S) must always be below the resistance intensity (R). The probability of failure is equal to the likelihood of non-compliance with the analyzed Limit State and is given by Equation 3:

$$P[G(R,S) \leq 0] \quad (3)$$

Thus, if R and S are configured as random variables, each one has a probability function, all of which are configured as random variables. In Fig. 2, the equations are represented by the failure domain (hatched region) G < 0 = D, so that the failure probability can be described as

$$P_f = P(g(x) \leq 0) = P(R - S \leq 0) = \int_D \int f_{RS}(r,s)drds$$

(4)

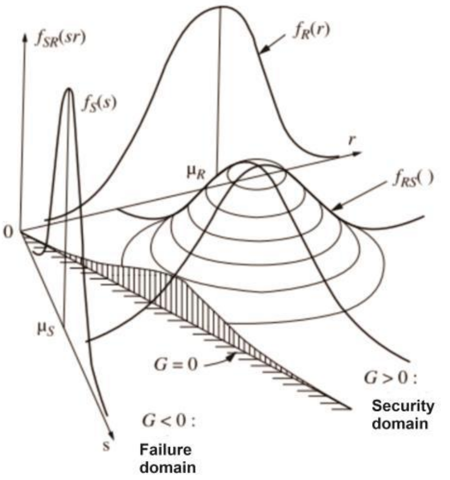

Fig.2: Space of two random variables (r,s) and the joint density function $f_{RS}(r,s)$, of the density functions f_R and f_S and a failure domain D given by G<0. Adapted from [16]

Target reliability index β_0

The target reliability index, β_0, is the reference index suggested in several standards to compare the index obtained in the reliability analysis. Thus, the target reliability index is the value indicated by different codes for each type of element and internal forces or simply for the Limit State.

Since NBR 6118 [8] does not present reliability studies or references for such, international codes must be adopted to be used as a reference to obtain a target reliability index. There are at least three critical standards that address the subject, namely: ACI 318 (2014), EUROCODE, and CEB-FIP/MC (2010) [16].

In this study, the reference value stipulated by the American standard ACI 318 [17] will be used as it is the only one to present values referring to the type of structural element analyzed, in this case, reinforced concrete slabs subjected to bending (Tab. 1).

Table 1: Acceptable values for the parameter (β). Adapted from [17]

Element	Acceptable β parameter
Pillars	3.8
Beams	3.3
Slabs subject to bending	2.5
Slabs subjected to punching	2.5 a 3.0

VI. FORM TRANSFORMATION METHOD

The first order analytical method FORM (First Order Reliability Method) is proposed as an evolution of the FOSM method (First Order Second Moment), where the restriction to the second moment of the variables is removed. The technique employs an idealization of a joint probability distribution function, transforming this distribution into a multivariate reduced normal [13]. One of the changes regarding the FOSM occurs due to the restriction of the second-moment method to only the normal probability distribution for the random variables. At the same time, the FORM can be integrated with other probability distribution analyses, as well as the linear correlation between the variables of the problem. The method approximates the failure surface in a reduced space at the design point as a truncated linear failure surface in the first order of the Taylor series [15].

The use and acceptance of the FORM as an efficient and effective method has been widely reported in the literature in general and recommended by the JCSS (Joint Committee on Structural Safety) [19].

The method is based on transforming a vector of random variables of a group X = (X_1, X_2,...X_n,) of a real space in a group of statistically independent, normalized, and standardized random variables represented by X'. And, still, they can be constituted by any probability functions, with or without correlation between them, and the accumulated probability function $F_{Xi}(x_i)$, para i=1,2,...,n,

Thus, it is shown that the minimum distance between the origin of the standardized coordinate system and the point with the highest failure probability on the tangent plane to the surface g(X')=0 corresponds to the reliability index. β (Fig. 3) [20].

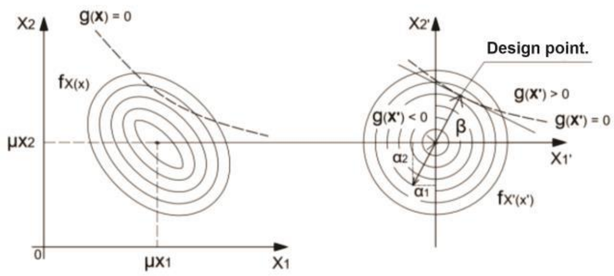

Fig.3: The reliability index and the uncorrelated standardized normal system transformation. Source: [16] adapted from [19]

VII. PDS AND THE MONTE CARLO METHOD

Ansys contains a module called PDS (Probabilistic Design System) for probabilistic studies. The Monte Carlo Simulation Method or the Response Surface can be chosen, where a parameterized model can be determined by defining a group of input variables with their probability distributions. [21].

The Monte Carlo method is defined by randomly generating a number N of values for input variables of the model from their respective theoretical probability distributions. Several distributions can be pre-established for variables such as Beta, Exponential, Gamma, Lognormal, Normal, Triangular, Uniform, and Weibull [21].

In addition, it is possible to work with the techniques of direct sampling, Latin-Hypercube sampling, and custom sampling. In direct sampling, it imitates natural processes given by the random generation of values according to their probability distributions. In this case, there is no control over the proximity of values. For Latin-Hypercube sampling, domains of variables are segmented into equiprobable intervals. Only one sample is generated for these intervals, not repeating the interval for the subsequent simulations. The statistical convergence of the results is accelerated using a "memory" of the generation of sample points, guaranteeing the non-generation of nearby points and covering the probability domain of the variable as a whole [22].

VIII. ALGORITHM FOR DETERMINING RELIABILITY

One of this work's objectives is to determine the reliability of the slabs studied; then, a parametric study was carried out. Thus, the FORM transformation method determined the reliability indices and the corresponding failure probability.

For implementing these methods, Python software was used through a computational routine to determine the reliability developed by [23] in open source, based on the model presented by [12]. The routine for use in Python is available for download in the domain https://github.com/mvreal/Reliability.

This routine is adapted from the algorithms of Hasofer and Lind, Rackwitz, and Fissler (HLRF), developed exclusively for solutions of optimization problems in structural reliability based on the approximation of a limit state by a hyperplane.

According to [12], solutions to non-linear reliability problems involving limit state equations converge to determine a design point. For this, any possibility can have the ability to find the design point. Concisely, a joint probability distribution function must be developed and perform the transformation to a multivariate normal distribution.

Basically, within the GitHub domain, it is possible to download the routines and some examples of application tests. The essential files for the routine execution consist of the *realpy.py* Python class and one of the *example.py* files containing the input routine.

The algorithm considers the possibility of random variables following the normal, uniform, lognormal, Gumbel, Fréchet, and Weibull distributions. This way, the routine was implemented using the Nataf transformation model because it is a practical method.

The model aims to transform the workspace from the design space (Fig. 4a) in three steps: Transforming distributions into equivalent normal probability distributions; introducing the equivalent normal correlation coefficients in a reduced correlated space (Fig. 4b), and finally, eliminating the correction between the variables, resulting in a reduced uncorrelated space (Fig. 4c) [24].

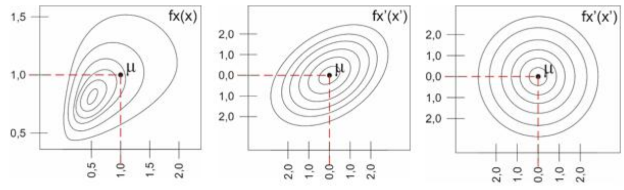

Fig.4: Space transformation by the Nataf model. Adapted from [24]

The principle of normal approximation for probability distributions was based on [25], which aims to find an equivalent normal distribution for the point x_i*, conserving the probability characteristics of the original distribution considering parameters of the equivalent mean (μ_{xeq}) and equivalent standard deviation (σ_{xeq}). To determine these equivalent parameters, it is necessary to solve a system of two equations for two unknowns (Equation 5 and 6), where [26] suggest that for the point x*, the probability function (FDP) and the accumulated probability function (FDPA) must have the same value.

$$\Phi\left(\frac{x^* - \mu_{xi}^{eq}}{\sigma_{xi}^{eq}}\right) = F_X(x^*) \tag{5}$$

$$\frac{1}{\sigma_{xi}^{eq}}\Phi\left(\frac{x^* - \mu_{xi}^{eq}}{\sigma_{xi}^{eq}}\right) = f_X(x^*) \tag{6}$$

From this, these equations in analytical format for the average and standard deviation of the equivalent normal distribution can be represented by:

$$\mu_X^{eq} = x^* - \Phi^{-1}[F_X(x^*)]\sigma_X^{eq} \tag{7}$$

$$\sigma_X^{eq} = \frac{\phi\{\Phi^{-1}[F_X(x^*)]\}}{f_X(x^*)} \quad (8)$$

One of the difficulties in implementing the algorithm is that the transformation procedure has to be performed individually for each of the marginal distributions, valid for a point x*. From this, it is necessary to verify the correlation coefficients between pairs of variables since, from the development of the normal approximation, random variables of normal joint distribution with original correlations are produced [24].

Thus, to correct the correlations of the variables, the model of Liu e Der Kiureghiam [25], where, through the implementation of the Nataf model to determine correlation adjustment factors (r) from non-normal to normal distributions (ρ to ρ_{eq}) [12] The transformation equation is:

$$r_{X,Y} = \frac{\rho_{X,Y}^{eq}}{\rho_{X,Y}} \quad (9)$$

To reach the uncorrelated reduced space, there are two ways: using the eigenvectors of the covariance matrix or Cholesky decomposition. In the algorithm in question, the second option was used.

Also, the transformation method uses an iterative process, where at each cycle, it is necessary to restructure the covariance matrix through the equation:

$$COV = \sigma^{eq}.\rho^{eq}\sigma^{eq} \quad (10)$$

By applying the Cholesky decomposition, the matrix is rewritten according to equation 11:

$$COV = L.L^T \quad (11)$$

Where Lis a lower triangular matrix.

Then, through Equation 12, there is the vector of uncorrelated reduced variables.

$$x' = L^{-1}.(x - \mu^{eq}) \quad (12)$$

Subsequently, through the results found for the mean and equivalent standard deviation, the procedures of the FOSM method (first order and second-moment method) are used. And, getting the new design point in the reduced space, it transforms from the reducing space to the design space through the equation:

$$x = \mu^{eq} + L.x' \quad (13)$$

Determining reliability (Hasofer, Lind, Rackwitz, and Fissler Algorithm)

The improved Hasofer, Lind, Rackwitz, and Fissler algorithm (iHLRF) was used to calculate the reliability index in the FORM method. Solutions of reliability problems can be developed through an optimization problem to determine the design point by approximating the limit state equation by a tangent hyperplane. (Fig. 5).

According to [12], HLRF presents some convergence problems in cases that are too non-linear. However, it is widely used due to its simplicity, although it does not obtain a guarantee of convergence.

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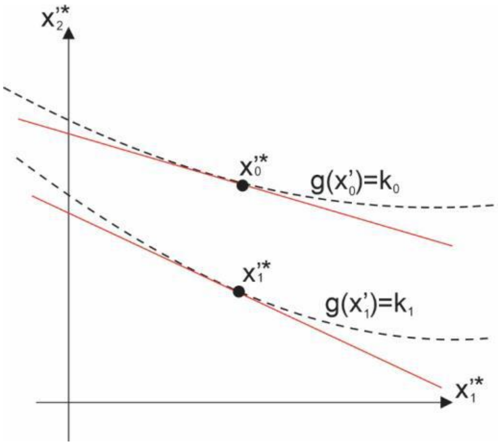

Fig.5: Iteration process that determines the design point. Adapted from [27]

For HLRF to be implemented, it is necessary to execute the recursive equation through x'_{k+1} by Equation 14. Where x'_k is the vector destined for iteration by checking in the reduced space where the iterative process presents convergence (without guarantees) at the point of $g(x'_k) \cong 0$ e $x'_{k+1} \cong x'_k$.

$$x'_{k+1} = \frac{\nabla g(xk')}{\|\nabla g(x_k)\|^2}[\nabla g(x'_k)^T.x'_k - g(x'_k)] \quad (14)$$

An improvement was suggested based on the HLRF algorithm by adding the letter "i" to the name "improved" (iHLRF). The central idea is to use the original algorithm to find an optimal step (λ_k), which minimizes a previously defined merit function in the direction indicated by the HLRF in Equation (15). and getting a new point by Equation (16) [28].

$$d_k = x'_{k+1} - x'_k = \frac{\nabla g(xk')}{\|\nabla g(x_k)\|^2}[\nabla g(x'_k)^T.x'_k - g(x'_k)] - x'_k \quad (15)$$

$$x'_{k+1} = x'_k + \lambda_k d_k \quad (16)$$

This function guarantees convergence by determining the value of penalties (c) of the merit function through the condition presented in Equation (17) and adopting γ = 2 (serves to meet the penalty condition) and δ the tolerance for $g(x_k') = 0$,,being x_k' the design point. The direction of d_k is the descent direction of the merit function [12; 28].

$$m(x_k') = \frac{1}{2}\|x_k'\|^2 + c|g(x_k')| \tag{17}$$

$$If\ g(x_k') \geq 0$$

$$c = \gamma max\left[\frac{\|y_k\|}{\|\nabla g(x'_k)\|'^2}, \frac{1}{2}\frac{\|y_k + d_k\|}{|g(x'_k)|}\right] \tag{18}$$

$$If\ g(x_k') < 0$$

$$c = \gamma\left[\frac{\|y_k\|}{\|\nabla g(x'_k)\|}\right]$$

Armijo's rule [29] is then used for the linear search of the optimal step (λ_k) through Equation (19). Typical values for these parameters are a=0.1; b=0.5 in addition to the already mentioned γ=2 [12].

$$\lambda_k = \max_{n\in\mathbb{N}}[b^n|m(x_k' + b^n.d_k) - m(x_k') \leq -a.b^n.\nabla m(x_k')^T.d_k] \tag{19}$$

The reliability index is obtained at the design point when $\beta = \|x^*\|$ in the moment that $x' = x'_{k+1}$.

IX. RESULTS AND PROBABILISTIC STUDY OF REINFORCED CONCRETE SLABS

Simulations were performed with nine (9) slabs, designed according to NBR 6118 (2014). For loading, the element's self-weight was adopted as a permanent load, in addition to a floor load of 1 kN/m^2 traditionally used in projects. As the variable loading, a fixed load of 6 kN/m^2 was used, stipulated by the NBR 6120 (2019) standard as the minimum for a room used as a library. This load was chosen because it is one of the largest of the standard in question. With the proper sizing of all slabs, it was possible, through the ANSYS software, to determine the rupture loads for each slab using the Monte Carlo simulation method divided into eight cycles of 50 simulations.

The nine slabs chosen for the analysis were named from L1 to L9 with variations of spans of 4x4, 5x5, and 6x6 meters. The diameter of the steel bars was fixed at 6.3mm. The spacing of the bars (esp) and thickness (h) of the slab varies according to the design according to NBR 6118 (2014). A summary of the design of these slabs is presented in Tab 2.

Table 2: Summary of slab design result.

					Direction X		Direction Y	
SLAB	fck(MPa)	Lx(m)	Ly(m)	h(m)	ϕ(mm)	S(cm)	ϕ(mm)	S(cm)
L1	25	4	4	0.1	6.3	10	6.3	10
L2	25	5	5	0.12	6.3	7	6.3	7
L3	25	6	6	0.15	6.3	6	6.3	6
L4	50	4	4	0.1	6.3	10	6.3	10
L5	50	5	5	0.1	6.3	6	6.3	6
L6	50	6	6	0.13	6.3	6	6.3	6
L7	70	4	4	0.1	6.3	10	6.3	10
L8	70	5	5	0.1	6.3	6	6.3	6
L9	70	6	6	0.13	6.3	6	6.3	6

From this, five random variables were previously determined, namely the compressive strength of the concrete (fc), the yield strength of the steel (fy), the spacing between bars (esp), the slab thickness (h), and the covering of the reinforcing bars (cobr).

With the code calibrated with the ANSYS PDS tool, the Monte Carlo simulation method was used for the slabs from L1 to L9.

Monte Carlo simulation

The Monte Carlo Method was limited to 400 simulations per slab divided into eight cycles of 50 simulations each. Still, as a result, it was possible to request the "print" of a vector referring to the rupture load of the structural element for each simulation, as well as the values used in the random variables in each simulation (Fig. 6)

ANALISEPROBLAJE

ITER	CYCL	LOOP	ERR	FC	FY	ESP
1	1	1	0	5.504749554e+00	4.336034481e+01	6.093570681e+00
1	1	2	0	6.029637797e+00	4.718144176e+01	6.052757049e+00
1	1	3	0	6.743018692e+00	4.678735236e+01	6.825335005e+00
1	1	4	0	7.217305913e+00	5.135050769e+01	6.457376383e+00
1	1	5	0	6.104241360e+00	5.157433117e+01	5.596489922e+00
1	1	6	0	8.939901106e+00	5.049440866e+01	5.923964609e+00
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

H	COBR	PR
1.238720882e+01	1.782014847e+00	8.341514772e+02
1.258883066e+01	1.936276412e+00	7.995624691e+02
1.365046157e+01	1.733019024e+00	8.259686553e+02
1.289826629e+01	2.064282981e+00	9.651382588e+02
1.269469113e+01	2.281103626e+00	9.572603914e+02
1.299880918e+01	2.024159698e+00	8.217206520e+02
.	.	.
.	.	.
.	.	.

Fig.6: PDS tool printing the results for the Monte Carlo Method.

Through this set of rupture load values, it is then possible to obtain a normal probability distribution adjustment, determining the mean, the standard deviation, and the coefficient of variation, transforming the resistance function into a random variable. From this, one can then determine the performance function as being

$$g(R, g, q) = R - g - q = 0 \qquad (20)$$

Where R is the slab-resistance function, g is the permanent load, and q is the variable load.

Model error estimation

The random variables chosen are intrinsic to the strength of the materials and the loads imposed on the slabs. However, it is essential to analyze the inherent uncertainties of the resistance and loading model attributed in this work, named eM and eS. The main objective of these variables is to assess uncertainties related to any randomness or numerical simplification present in the model[18]. Thus, eS is assigned a unit mean with a standard deviation of 0.05; that is, the coefficient of variation is also 0.05. The eM can be calculated according to some works in the literature. In this sense, the model error estimation was verified following the guidelines presented in [30].

There are distortions in the experimental and theoretical results due to situations that can be influenced by the computational numerical model, by the variability of the random variables of the system, or even by the variability associated with the experimental activity. Thus, to estimate the model error, it is used the following equation:

$$\left(V_{\frac{e}{m}}\right)^2 = V_m^2 + V_{batch}^2 + V_{test}^2 \qquad (21)$$

Where $V_{e/m}$ is the coefficient of variation of the ratio between the experimental results and the numerical model; V_m is the coefficient of variation of the model error; V_{batch} is the coefficient of variation of the laboratory tests of the system variables represented by the dimensions and strengths and V_{test} the coefficient of variation of the slab experiments.

Through the results obtained in the validation of the model, the value of $V_{e/m} = 0.0886$ was obtained. As for the V_{test}, it is defined that Vtest = 0.02 should be used for elements subjected to bending. Finally, 400 Monte Carlo simulations were performed for one of the slabs used in the model validation to determine the lot variation coefficient. For this simulation, a coefficient of variation of the concrete compressive strength of 0.05 was adopted, and for the steel yield strength, a value of 0.02 was considered. As a result, $V_{batch} = 0.0404$ was obtained. These considerations resulted in a value of $V_m = 0.0763$.

It is then possible to consider these results for the reliability analysis, updating Equation (20) to:

$$g(R, g, q) = R * (eM) - (g - q) * (eS) = 0 \qquad (22)$$

Furthermore, the model error is identified by a normal distribution of the unitary mean value. Using the result of the model error calculation, $V_m = 0.0763$, and the mean $\mu M = 1.00$, an error estimate can be calculated by Equation (23) [31]:

$$eM(z) = \mu M(1 + zVm) \qquad (23)$$

where z is a Gaussian random variable with zero mean and unit standard deviation. Fig. 7 shows the histogram for the error estimate, and in Fig. 8, the normal probability plot verifies that the data converges to a normal distribution curve.

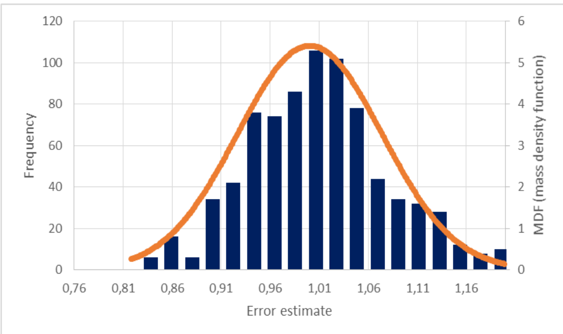

Fig.7: Histogram of model error estimation.

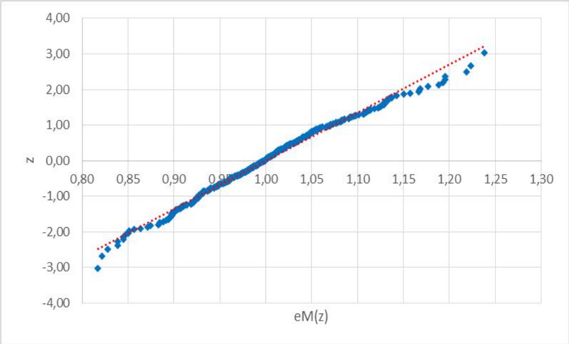

Fig.8: Normal probability graph.

Random variables considered

Some minimum conditions must be considered so that the structures perform their functions satisfactorily. For this, fundamental variables were used to parameterize the element with the limit state function. The variables that most influence the behavior of the structure must be selected. Generally, they are related to geometric properties, materials, and loads. The variabilities of these variables happen in the production, manufacturing control, and loading, among others. Thus, the random variables chosen were the concrete's compressive strength, the steel's yield strength, the slab's thickness, the cover (which measures the variation of the effective depth), and the spacing between bars (which measures the rate of reinforcement). For fc, fy, h, cobr, g and q, were employed the statistical parameters indicated by [32]. For eM and eS, the parameters suggested by [30] were used.

The parameters for the random variables are shown in the Table.

Table 3: Random variables considered.

Variable	µ	C.V.	Distribution
fc	$\frac{f_{ck}}{1-1{,}645V_{fc}}$	0.15	Normal
fy	$\frac{fyk}{1-1{,}645V_{fy}}$	0.05	Normal
esp	esp	0.05	Normal
h	h	0.04	Normal
cobr	cobr	0.125	Normal
g	g_k	0.1	Normal
q	q_k	0.25	Gumbel
eM	1	0.0763	Normal
esp	1	0.05	Normal

Evaluation of the structural reliability of the slabs

Tab. 5 presents the loading parameters for each slab used in its design, where gk is the characteristic permanent loading and qk is the characteristic variable loading. The loading variation was due to the alternation of slab thickness, necessary for all standard checks to be met.

Fig. 9 shows the normal distribution graph of slab L1 for the 400 simulations. The results showed an average rupture load (µCR) of 16.92 kN/m^2, deviation (σ) of 1.51 kN/m^2 with a coefficient of variation (CV) of 8.91% (Tab. 4).

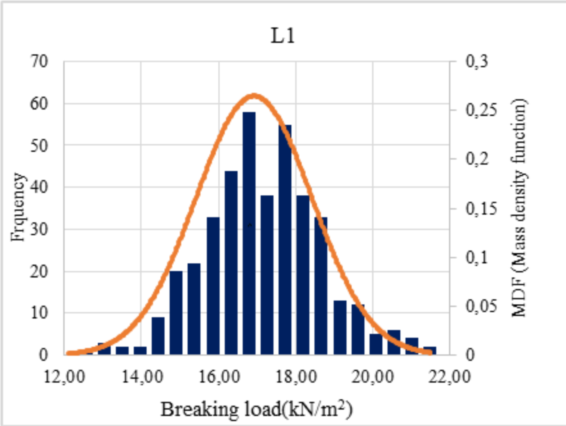

Table 4: Parametric results of the Monte Carlo simulations of the slabs.

LAJE	µCR (kN/m^2)	σ (kN/m^2)	CV
L1	16.92	1.51	0.089052
L2	18.93	1.00	0.052965
L3	19.11	0.82	0.043078
L4	17.20	1.48	0.086152
L5	17.91	1.08	0.060523
L6	18.05	1.39	0.077116
L7	17.28	1.23	0.071332
L8	18.22	0.88	0.048379
L9	18.59	1.44	0.077706

Table 5: Loading distribution parameters according to sizing by NBR 6118 and NBR 6120.

Slab	gk (kN/m2)	qk (kN/m2)	gk+qk (kN/m2)	qk/(gk+qk)
L1	3.5	6	9.5	0.37
L2	4	6	10	0.40
L3	4.75	6	10.75	0.44
L4	3.5	6	9.5	0.37
L5	3.5	6	9.5	0.37
L6	4.25	6	10.25	0.41

L7	3.5	6	9.5	0.37
L8	3.75	6	9.75	0.38
L9	4.25	6	10.25	0.41

Regarding the results (Tab. 6), it can be observed that all the results were above the target reliability index β_0=2.5, indicated as acceptable by the American standard ACI 318 for slabs subjected to bending. This code is the only standard that presents the indicative parameters of β_0 by type of structural element and the internal forces to which it is subjected.

Table 6: Slab reliability results according to design load distribution.

	FORM	
Slab	**β**	**Fp**
L1	2.65	3.9684E-03
L2	2.84	2.1954E-03
L3	2.96	1.5218E-03
L4	2.73	3.1816E-03
L5	2.75	2.9815E-03
L6	2.76	2.8596E-03
L7	2.81	2.4218E-03
L8	3.03	1.2208E-03
L9	2.88	1.9684E-03

Considering qk/(gk+qk)=0.25

In this item, the reliability results will be presented considering the variable loading of the slabs, totaling 25%. It is possible to consider this relationship as the closest to reality since the variable loads of a residential building made of reinforced concrete generally do not exceed 25% of the total, justified by the considerable self-weight of the reinforced concrete. According to that, Araújo[33] describes that in the absence of knowledge of the variation between the two types of loading, a relationship of qk≅0.15gk can be estimated, which results in a proportion of 13% of variable load only.

Therefore, analyzing the results of Tab. 7, none of the slabs indicated a reliability index lower than the target index of ACI 318 (2014).

Table 7: Slab reliability results according to load distribution qk/(qk+gk)=0.25.

	FORM	
Slab	β	Fp
L1	3.50	2.3548E-04
L2	3.83	6.4562E-05
L3	3.99	3.3692E-05
L4	3.63	1.4209E-04
L5	4.18	1.4406E-05
L6	3.60	1.6204E-04
L7	4.40	5.4933E-06
L8	4.24	1.0981E-05
L9	3.76	8.6678E-05

Considering qk/(gk+qk)=0.50

When the results are observed in an analysis submitted to loading divided into 50% variable and 50% permanent (Tab. 8), it is possible to verify a reduction in the reliability indexes. This reduction happens with the increase in the variable loading portion. It is also noted that the minimum reliability is met in all slabs according to the ACI 318 (2014) standard for slabs subjected to bending stresses.

Table 8: Slab reliability results according to load distribution qk/(qk+gk)=0.50.

	FORM	
Slab	β	Pf
L1	2.94	1.6629E-03
L2	3.19	7.0101E-04
L3	3.13	8.6506E-04
L4	3.02	1.2475E-03
L5	3.33	4.3376E-04
L6	2.95	1.5636E-03
L7	3.45	2.7851E-04
L8	3.34	4.2208E-04
L9	3.08	1.0373E-03

Considering qk/(gk+qk)=0.75

When the variable load presents 75% of the total, it results in the lowest values of reliability indices (Tab. 9). Exclusively in this analysis, slab L1 and L6 did not present the minimum results suggested by ACI 318 (2014), but they are close to the target. For all other slabs, β is greater than 2.5.

Table 9: Slab reliability results according to load distribution qk/(qk+gk)=0.75.

FORM		
Slab	β	Pf
L1	2.46	6.9539E-03
L2	2.78	2.7197E-03
L3	2.58	5.0007E-03
L4	2.53	5.7281E-03
L5	2.75	2.9980E-03
L6	2.46	7.0287E-03
L7	2.84	2.2776E-03
L8	2.67	3.7707E-03
L9	2.56	5.2175E-03

Analysis of director cosines

Tab. 10 below represents the director cosines found in the L1 reliability analysis for each load proportion. This result is indispensable for analyzing the influence of each random variable considered in the Ultimate Limit State function.

It is observed that the random variable of slab strength (R) has a more significant influence on the result, reducing with the increase of the variable loading portion (Q). As for the variable Q, it already becomes considerably preponderant from the equal division of loads.

Table 10: Cosine directors generated by FORM method for L1.

L1	**Director Cosine α_{xi}^2**		
	qk/(gk+qk)		
Random variable	0.25	0.5	0.75
R	0.363	0.185	0.121
G	0.117	0.023	0.003
Q	0.219	0.623	0.760
eM	0.023	0.126	0.084
eS	0.069	0.043	0.031

X. CONCLUSION

This work simulated nine different models of reinforced concrete slabs for reliability analysis. Also, the model error was analyzed for its insertion as a random variable to obtain more accurate reliability results. Posteriorly, through the results of probability distributions and using a computational routine developed by Real (2022) for Python language, the structural reliability of 36 models of reinforced concrete slabs was determined through the FORM first-order transformation method. The proportions between variable and permanent loads were varied to verify the influence of the failure probability and structural reliability index between the parameters. Among the 36 slab reliability index results presented, only two were below the one indicated by the American standard ACI 318 (2014). This American code is the only one to suggest a target index related to the type of element and the internal forces to which it is subjected. With this, the satisfactory safety of the reinforced concrete slabs designed following NBR 6118 (2014) stands out.

The director-cosines of FORM for slab L1 were presented to show the significant parameters that influenced the reliability of the slabs. It was verified that the influence of the random variable resistance of the slab is inversely proportional to that of the random parameter of variable loading. The preponderance of the variable load increases in the proportion of the variable loading applied to the slab. It is justified that the random variable loading presents predominance due to its high degree of variability.

Finally, there is a great variation in the reliability indices between the different cases studied here, which, for the most part, are considerably higher than necessary. Thus, the need to calibrate the partial safety factors adopted by the NBR 6118 (2014) is evidenced.

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