

Salient Modeling at offshore Breakwater for oblique wave using least Square weighted Residual Method

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Abstract— This research is the continuation of previous research by the writer using polynomial approach with the purpose of fixing various constraints in the previous method.

The salient equation used in this research is similar to the one used in the previous research, i.e. the principle of static equilibrium coastline where the tangent of the stable coastline is similar to the tangent of the crestline.

In the previous method, the salient equation was approached with a polynomial equation, then polynomial coefficient is obtained by applying the first differential (tangent) of the polynomial in various points with the number of points that is in accordance with the number of polynomial coefficient. There is an obstacle in this method, i.e. the setting of boundary condition points that should be done with trial and error

In this research the tangent equation of the salient is approached with polynomial equation using five points of sample. Salient equation is also approached with polynomial. To obtain polynomial coefficient, the first differential of the salient equation is equalized with tangent equation of the salient and Least Square Weighted Residual Method is applied. Unlike previous research, there is no constraint in this research using this method. Comparison with the result of the research from previous researches shows a conformity of the result of the model with the result of the previous research.

Keywords— Static Equilibrium, Weighted Residual Method.

I. INTRODUCTION

Offshore breakwater is a breakwater constructed parallel with the coast. Even though it is called offshore breakwater, the position of the construction is quite close with the coast, where the construction efficiency is very much determined by the distance between the breakwater and the original coastline. On the coast protected by offshore breakwater, sediment deposit will be formed which is called salient. The success of coastal protection using offshore breakwater is determined by salient Y_s that was formed (Fig.1), where the height of the salient is determined by the length of breakwater L_s and the distance between breakwater and the original coastline or breakwater distance X .

Considering that breakwater is constructed quite close with the coast where the crestline is almost parallel with the coast or the wave direction is almost perpendicular to the coast (Fig.2), therefore in this research a breakwater is developed for a wave that is perpendicular to the coast.

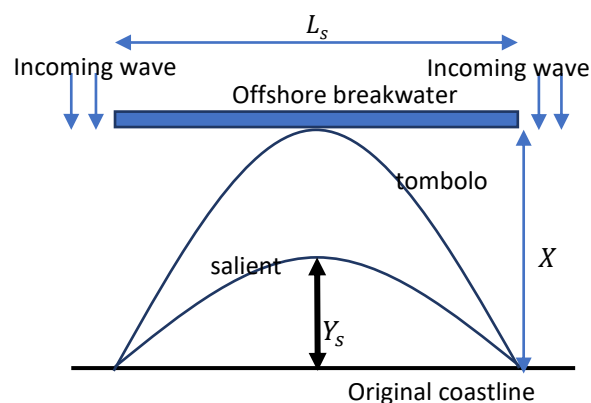


Fig.1. Offshore breakwater and salient

This research is working on an assumption that longshore sediment transport is the primary cause of the changes in coastline. Longshore sediment transport equation of some researcher, such as Komar, P.D., (1998), Shore Protection Manual (SPM), (1984), Van Rijn, Leo C. (2013), Mill-Homens, J., Ranasinghe, R., Van Thiel de Vries, J.S.M. and Stive, M.J.F., (2013), is the sine function of the breaker crest line angle where the value of Longshore sediment transport is zero or no longshore sediment transport if the breaker crestline is parallel to or forming

zero angle against the coastline. In other words, in a stable coastline, the tangent of breaker crestline is similar to the tangent of the coastline (Fig.2). This condition becomes the basis of salient equation in this research.

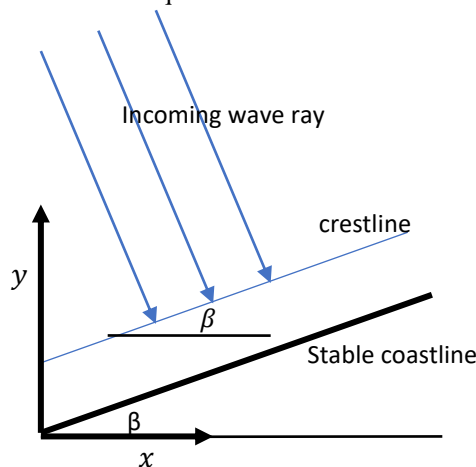


Fig. 2. Crestline of oblique waves

This research aims at obtaining the method of salient height Y_s (Fig.1) for a breaker length L_s , at a breakwater distance X . This research is the continuation of a previous research by the writer where in the previous research polynomial approach was applied where the value of the polynomial coefficient is obtained by applying stable coastline equation on various points (boundary point) on the original coastline. In the previous research, the difficulty was found in determining boundary points. In this research, salient equation is also approached with polynomial, where polynomial coefficient is obtained by using Least Square Weighted Residual Method with Galerkin procedure (Stassa, F.L. (1985)), whereas the tangent of salient is approached with a polynomial equation using five sample points.

Some previous researchers have made relation between (L_s, X) with salient height Y_s but in qualitative form i.e. relation (L_s, X) with the type of salient that was formed. Those researchers are Ahrens, J.P, and Cox, J. (1990), Van Rijn, L.C. (2013), Inman, L.D., and Frautschy, Nir, Y. (1982), . The result of the researches especially from Ahrens, J.P, and Cox, J. (1990), Van Rijn, was used as a comparator of the result of the model where the result of the comparison shows a compatibility of model result with the result of the two researchers.

II. SOME LONGSHORE SEDIMENT TRANSPORT EQUATIONS

As has been mentioned in the previous section that longshore sediment transport equations from some researchers are the sine function of breaker crest line. The

next section will show some longshore sediment transport equations.

1. Komar 1998

$$Q = 0.46 \rho g^{3/2} H_b^{5/2} \sin \theta_{br} \cos \theta_{br} \dots (1)$$

g = gravity acceleration (9.81 m/sec²)

H_b = breaker height (m)

θ_{br} = wave angle at breaker line (angle between wave crest line and coastline)

2. Shore Protection Manual (1984)

$$I = K E C_{g,br} \sin \theta_{br} \cos \theta_{br} \dots (2)$$

I = longshore transport rate (immersed weight)

$$E = \frac{1}{8} \rho g (H_{rms,br})^2 \quad \text{wave energy}$$

H_{rms} = rms wave height at breaker line

$C_{g,br}$ = wave group celerity at breaker line

θ_{br} = wave angle at breaker line (angle between wave crest line and coastline)

$$K = 0.77$$

3. Van Rijn

$$Q_{t,mass} = 128 (H_{s,br})^{2.5} \sin(2\theta_{br}) \dots (3)$$

$Q_{t,mass}$ = longshore transport rate (dry mass kg/s)

$H_{s,br}$ = significant wave height at breaker line (m)

θ_{br} = wave angle at breaker line (angle between wave crest line and coastline)

4. Modified Kamphuis (Mill-Homens et al., 2013)

$$Q_{t,mass} = 0.15 \left(\frac{\rho_s}{\rho_s - \rho} \right) T_p^{0.89} (\tan \beta)^{0.86} d_{50}^{-0.69} H_{s,br}^{2.75} (\sin 2\theta_{br})^{0.5} \dots (4)$$

$Q_{t,mass}$ = longshore transport rate (dry mass kg/s)

ρ_s = sand density (2650 kg/m³)

ρ = sea water density (1030 kg/m³)

T_p = peak wave period (sec.)

$\tan \beta$ = beach slope

d_{50} = median particle size in surf zone (m)

$H_{s,br}$ = significant wave height at breaker line (m)

θ_{br} = wave angle at breaker line (angle between wave crest line and coastline)

Eqs. (1), (2), (3) and (4) show that longshore sediment transport is a sine function of breaker line angel, i.e. $\sin \theta_{br}$ with a value of zero if crest line is parallel to or similar to the coastline. Therefore, the tangent of salient will be similar to the tangent of the diffracted crestline.

III. SOME RESULTS OF THE PREVIOUS STUDIES

There have been many researches on salient formation at offshore breakwater. This section will present some results of previous researches that will be used as comparator on the model development. The results are in the form of qualitative relation between $\frac{L_s}{X}$ and salient without mentioning wave angel.

3.1. Ahrens and Cox (1990)

Ahrens and Cox (1990) used the beach response index classification scheme of Pope and Dean (1986) to develop a predictive relationship for beach response based on ratio of the breakwater segment length to breakwater distance from original shoreline (Table 1). The relationship defining a beach response index I_s is :

$$I_s = e^{(1.72 - \frac{L_s}{X})} \dots\dots(5)$$

Table 1. The value of $\frac{L_s}{X}$ and salient formation, Ahrens and Cox (1990)

I_s	$\frac{L_s}{X}$	Salient formation
1	4,2	Permanent tombolo
2	2,5	Periodic tombolo
3	1,52	Well-developed salient
4	0,81	Subdued salient
5	0,27	No sinuosity

3.2. Leo C. Van Rijn (2013)

The result of Leo C. Van Rijn research (2013) related to this research is on the relation between the value of $\frac{L_s}{X}$ and salient formation (Table 2) , i.e. as follows

Table 2. The Value of $\frac{L_s}{X}$ and salient formation, Leo C. Van Rijn (2013)

$\frac{L_s}{X}$	Salient formation
> 3	Permanent tombolo
$2 < \frac{L_s}{X} < 3$	Permanent or periodic tombolo
$1 < \frac{L_s}{X} < 2$	Well developed salient
$0.5 < \frac{L_s}{X} < 1$	Weak to well developed salient
$0.2 < \frac{L_s}{X} < 0.5$	Incipient to weak salient
$\frac{L_s}{X} < 0.2$	No effect

3.3. Others

Inman and Frautschy (1966)

$$\frac{L_s}{X} \leq 0.17 - 0.33 : \text{no accretion}$$

Nir (1982)

$$\frac{L_s}{X} < 0.5 : \text{no depositional condition}$$

SPM (1984)

$$\frac{L_s}{X} < 1 : \text{tombolo formation prevented}$$

$$\frac{L_s}{X} > 2 : \text{tombolo formation certain}$$

There is a conformity between Ahren and Cox criteria (1990) and Leo C. Van Rijn criteria (2013), whereas Inman and Proudcey (1966), Nir (1982) and SPM (1984) also conform with both criteria.

IV. SALIENT EQUATIONS

The change in the coastline is primarily caused by littoral sediment transportation. Littoral sediment transportation equations are the functions of tangent crestline against coastline and among others are equations from, Komar, P.D., (1998), Shore Protection Manual (SPM), (1984), Van Rijn, Leo C. (2013), Mill-Homens, J.,Ranasinghe, R., Van Thiel de Vries, J.S.M. and Stive , M.J.F., (2013).With this form of littoral sediment transportation, the tangent of the stable coastline is similar to the tangent of the crestline forming it, or for $y(x)$ the stable coastline equation whereas β is crestline angle and hence the stable coastline equation is,

$$\frac{dy}{dx} = \tan\beta \dots\dots(6)$$

Coastline behind breakwater will evolve into a stable coastline in the form of salient or tombolo, where the tangent of the coastline is in line with Eq.(6).

In the previous research (Hutahaeen (2020)), salient $y(x)$ equation was approached with polynomial, i.e. $y(x) = \sum_{i=0}^n c_i x^i$. Then polynomial coefficient c_i was obtained by applying Eq.(6) on a number of n boundary points by calculating beforehand the value of $\tan\beta_i$, with calculation method presented in section (4.1). The difficulty in this method is on the setting of boundary points where different boundary points produced different salient height Y_s .

4.1. Tangent Crestline Equation on Breakwater Lee.

Salient model in this research is developed with an assumption that salient is formed by diffracted wave, therefore it requires tangent from diffracted wave that can reach the coastline. In the calculation of this crestline angle, an assumption was applied that the tangent crestline on a point with x abscissa on original coastline is fixed, even though y ordinate changes due to the changes in bathymetry in the process of salient formation.

The second assumption on the calculation of this crestline angle is that crestline angle is symmetrical, with its center

in the middle of the offshore breakwater where the tangent crestline in the midpoint of the breakwater is zero. Tangent crestline equation is stated with polynomial equation,

$$\tan(\beta(x)) = \sum_{i=0}^{np} c_i x^i \dots\dots(7)$$

Hence (6) becomes,

$$\frac{dy}{dx} = \sum_{i=0}^{np} c_i x^i \dots\dots\dots(8)$$

np is the number of polynomial term where the degree of polynomial is $np - 1$ and in this research $np = 5$ was used. The calculation of polynomial constant c_i was applied with least square method using 5 (five) interpolation points as on Table 3 and Fig.3., with the definition of crestline angle in Fig.4..

Table 3. Interpolation points of crestline angle

No	x	$\tan\beta$
1	0	0
2	$0.25L_s$	$\frac{0.25L_s}{X}$
3	$0.5L_s$	0
4	$0.75L_s$	$-\frac{0.25L_s}{X}$
5	L_s	0

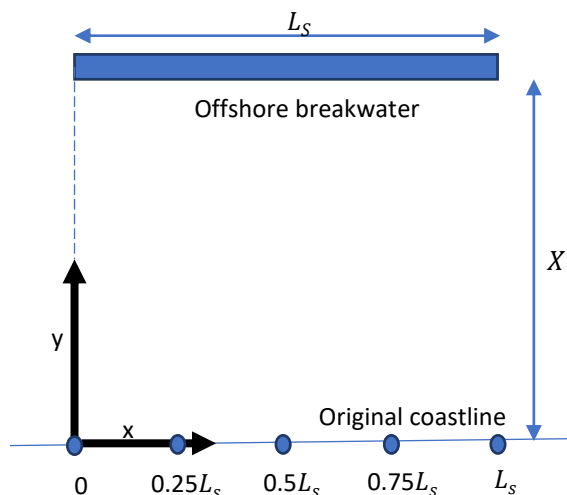


Fig.3. Interpolation points of crestline tangent

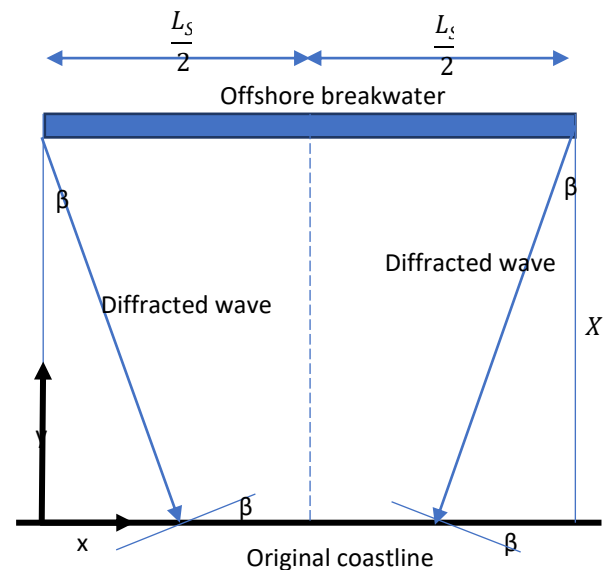


Fig.4. Crestline angle of the diffracted wave

As an example, the result of the interpolation of the tangent value crestline angle for $L_s = 50$ m, with $X = 25$ m and $X = 50$ m is presented in Fig (3). Fig.3 shows that the value of $\frac{dy}{dx} = \tan(\beta(x))$ at $X = 25$ m is bigger than at $X = 50$ m which shows that the salient curve for $X = 25$ m is bigger than the salient curve at $X = 50$ m. This also shows that the closer the position of breakwater to the coastline, the higher the salient height Y_s will be.

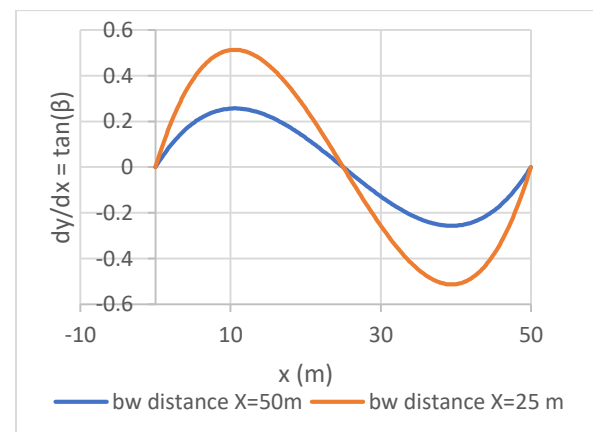


Fig 4. The value of $\frac{dy}{dx} = \tan(\beta(x))$ at $L_s = 50$ m, with breakwater distance $X = 25$ m and $X = 50$ m

V. THE SOLVING OF SALIENT EQUATION WITH WEIGHTED RESIDUAL METHOD

Salient equation $y(x)$ cannot be obtained by integrating the right side of (3) since there are two boundary conditions at the ends of the domain. Hence, (3) will be solved with an approximate method, i.e. Weighted

Residual Method with Galerkin method. Theory on this Weighted Residual Method refers to [5].

5.1. Approximation Equation

In solving differential equation using Weighted Residual Method, the solution is approached with a polynomial, i.e.

$$\bar{y}(x) = \sum_{i=1}^n a_i N_i = [N_i] \{a_i\} \dots (9)$$

$$N_i(x) = x^i(L-x) \dots (10)$$

This approximation equation meets the boundary condition at the ends of the domain, i.e. $y = 0$ at $x = 0$ and at $x = L$, where L is the length of breakwater L_S . n is the number of the terms where in this research $n = 4$.

Substitute approximation equation (9) to salient equation (8), will result in an error of R ,

$$\frac{d\bar{y}}{dx} - \sum_{i=0}^{np} c_i x^i = R \dots (11)$$

Substitute (4) to (5) will result in an error of R ,

$$\left[\frac{dN_i}{dx} \right] \{a_i\} - \sum_{i=0}^{np} c_i x^i = R \dots (12)$$

Equation to obtain the values of polynomial coefficient a_i is obtained by minimizing error R where in this research, the Least Square method is done.

5.2. Minimize Error with Least Square Method

To minimize error, the Least Square Method is done, i.e.

$$I = \int_0^{L_S} R^2 dx$$

$$\frac{\partial I}{\partial a_k} = 0$$

$$\int_0^{L_S} \frac{\partial R}{\partial a_k} R dx = 0 \quad \dots \text{for } k = 1 \text{ to } n$$

$$\int_0^{L_S} \left\{ \frac{dN}{dx} \right\} R dx = 0 \quad \dots (12)$$

Substitute Eq. (11) to Eq. (12)

$$\int_0^{L_S} \left\{ \frac{dN}{dx} \right\} \left(\left[\frac{dN}{dx} \right] \{a_i\} - \sum_{i=0}^{np} c_i x^i \right) dx = 0 \quad \dots (13)$$

or

$$\int_0^{L_S} \left\{ \frac{dN}{dx} \right\} \left[\frac{dN}{dx} \right] dx \{a_i\} = \int_0^{L_S} \left\{ \frac{dN}{dx} \right\} \left(\sum_{i=0}^{np} c_i x^i \right) dx \dots (14)$$

$$[K] = \int_0^{L_S} \left\{ \frac{dN}{dx} \right\} \left[\frac{dN}{dx} \right] dx \dots (15)$$

$$\{F\} = \int_0^{L_S} \left\{ \frac{dN}{dx} \right\} \left(\sum_{i=0}^{np} c_i x^i \right) dx \dots (16)$$

(15) and (16) formed n linear equation system with variabel a_i i.e.

$$[K] \{a\} = \{F\} \quad \dots (17)$$

Equation (17) can be solved with Gauss elimination method or the like, obtain the value of coefficients a_i ,

Then the coefficients are substituted to Eq. (9), obtain salient equation.

VI. RESULT OF THE MODEL

Fig. 5 presents the profile of salient, the result of the model, for breakwater length $L_S = 50$ m, breakwater distance $X = 50$ m and $X = 25$ m. Where $X = 50$ m salient height $Y_S = 4.167$ m is obtained, whereas at $X = 25$ m, salient height $Y_S = 8.333$ m is obtained.

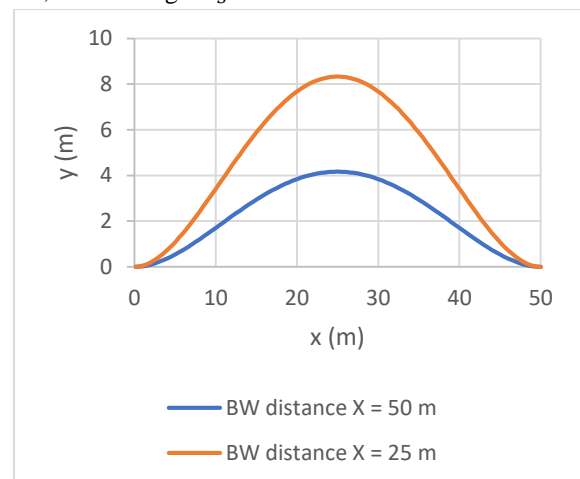


Fig. 5. The profile of salient for breakwater length $L_S = 50$ m, breakwater distance $X = 50$ m and $X = 25$ m.

Table 4 presents the result of the calculation for breakwater length $L_S = 50$ m, whereas breakwater distance X varies. At $\frac{L_S}{X} = 3.5$, where breakwater distance $X = 14,286$ m, $Y_S = 14,583$ m is obtained where actually Y_S cannot exceed X . This shows that tombolo permanent has been formed, as mentioned in the results of research [6] (Table 1.) and [7] (Table 2).

Table 5 presents the result of the calculation for changing length of breakwater, whereas breakwater distance $X = 50$ m. Similar result is obtained namely at $\frac{L_S}{X} = 3.5$ permanent salient is formed.

However, there is a difference between the result on Table 4 and on Table 5. As an example, on Table 4 at $\frac{L_S}{X} = 1.5$, where $L_S = 50$ m and $X = 33.33$, $Y_S = 6.25$ m is obtained, whereas on Table 5., at $\frac{L_S}{X} = 1.5$, where $L_S = 75$ m and $X = 50$, $Y_S = 9.375$ m is obtained. This result shows that in the salient formation, breakwater length plays more role than breakwater distance.

Table 4. The result of the calculation of salient height Y_s , for $L_s = 50$ m.

$\frac{L_s}{X}$	X (m)	L_s (m)	Y_s (m)
0,25	200	50	1,042
0,5	100	50	2,083
0,75	66,667	50	3,125
1	50	50	4,167
1,25	40	50	5,208
1,5	33,333	50	6,25
1,75	28,571	50	7,292
2	25	50	8,333
2,25	22,222	50	9,375
2,5	20	50	10,417
2,75	18,182	50	11,458
3	16,667	50	12,5
3,25	15,385	50	13,542
3,5	14,286	50	14,583

Table 5. The result of the calculation of salient Y_s , for $X = 50$ m.

$\frac{L_s}{X}$	X (m)	L_s (m)	Y_s (m)
0,25	50	12,5	0,26
0,5	50	25	1,042
0,75	50	37,5	2,344
1	50	50	4,167
1,25	50	62,5	6,51
1,5	50	75	9,375
1,75	50	87,5	12,76
2	50	100	16,667
2,25	50	112,5	21,094
2,5	50	125	26,042
2,75	50	137,5	31,51
3	50	150	37,5
3,25	50	162,5	44,01
3,5	50	175	51,042

VII. CONCLUSION

In the salient modeling using Least Square Weighted Residual Method and with salient tangent approach with a polynomial function, salient can be modeled with a result that is suitable with the results of previous research. In addition, there is no obstacle in the setting of boundary point and solution stabilization as contained in the previous author's research.

This research also found out that breakwater length is more decisive than breakwater distance, where the similar comparison value between breakwater length and

breakwater distance will result in higher salient for longer breakwater length.

As for further research, what needed is development of a model for a wave forming an angle with breakwater.

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