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Correlation of Weighting Coefficient at Weighted Total Acceleration with Rayleigh Distribution and with Pierson-Moskowitz Spectrum

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Abstract*— *This paper describes the research on the correlation between weighting coefficient in weighted total acceleration equation with wave height distribution from Rayleigh and with Pierson-Moskowitz spectrum. The research is done using dispersion equation for deep water wave where there is wave amplitude as its variable and a limitation of wave height maximum for a wave period. From that dispersion equation, a maximum wave height equation is formulated for a wave period.

Keywords*— *weighting coefficient, Rayleigh distribution coefficient.

I. INTRODUCTION

Hutahaean (2019) obtained weighted total acceleration equation for a function $F(x,t)$ in the form of $\frac{DF}{dt} = \gamma \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x}$, where γ *is weighting* coefficient with a value around 2.784-3.160 which is obtained analytically using surface water equation and particle velocity equation of the linear wave theory. Using the weighted total acceleration, Hutahaean (2019) formulated dispersion equation for deep water wave where there is wave amplitude as its variable and also maximum wave amplitude for a wave period. From that dispersion equation, wave height maximum equation is formulated for a wave period value resulted in different wave height maximum for different γ value.

There are various waves heights, the most commonly used being the significant wave height H_s or is often also called$H_{1/3}$. Therefore, a hypothesis can be made that there is a value of weighting coefficient that correlates with $H_{1/3}$.

Wiegel (1949 and 1964) and Silvester (1974) produceda relation between wave period and deep water wave height, where with an input of a wave height, its wave period can be calculated. Using wave period from the two equations, wave height maximum is calculated using wave amplitude maximum equation, by changing the value of weighting coefficient until the wave height that is similar to the wave height that becomes the input for Wiegel equation and Silvester equation is obtained. The obtained weighting coefficient is different for the two equations. From the two weighting coefficient values, the characteristics of the closeness with the Rayleigh distribution coefficient are obtained.

Furthermore, studying the Rayleigh distribution coefficientproduced weighting coefficient values that correspondsto Rayleigh coefficient. Silvester equation (1974), formulated using Pierson-Moskowitz spectrum, obtained that wave height at the equation is for a certain weighting coefficient value that is correlated with$H_{1/10}$.

II. DISPERSION EQUATION AT DEEP WATER

Dispersion equation at deep water (Hutahaean, 2019) is a function of wave amplitude A, by ignoring bottom slope is

$$\gamma^2\sigma^2 = gk\left(1 - \left(\frac{kA_0}{2}\right)\right) \quad ....(1)$$

$\gamma$is a weighting coefficient at the weighted total acceleration or total derivative equation, where the result of Hutahaean research (2019) analytically using water surface equation and particle velocity of linear wave theory obtainedγvalue of 2.784-3.160. $\sigma$is angular frequency $\sigma = \frac{2\pi}{T}$, $T$is wave period, $g$is gravity velocity, $k$is wave number and$A_0$is deep water wave amplitude.

Equation (1) has a solution if the value of determinant $D \geq 0$, where

$$D = g^2 - 4\left(\frac{gA_0}{2}\right)(\gamma^2\sigma^2).$$

for$D = 0$

$$A_{0-max} = \frac{g}{2\gamma^2\sigma^2} \quad ....(2)$$

Using (2), wave amplitude A_0at the deep water can be calculated, which is A_0 maximum at the concerned wave period, where with the assumption that at the deep water the wave profile is still sinusoidal, thenH_0 maximum is twiceA_0 maximum. In addition, in (2) there is also a relation that wave height maximum is also determined by the value of γ.

2.1. Comparative Study with Wiegel Equation (1949 and 1964)

Wiegel (1949 and 1964) produced a relation between wave period withH_{0-max}, i.e.

$$T_{Wieg} = 15.6\left(\frac{H_{0-max}}{g}\right)^{0.5} \qquad ....(3)$$

Study at Wiegel equation is done by providing an input of wave height at (3) and is assumed as$H_{0-max}$and $T_{Wieg}$is calculated. Then, using the T_{Wieg}, $H_{0-max,m}$is calculated with (2), byexperimenting the value of γ. Table (1) shows the result of the calculation with $\gamma = 2.483$, where produced $H_{0-max,m} = H_{0-max}$. Therefore (2) provides a result that corresponds to Wiegel equation at the value of $\gamma = 2.483$. Using wave period $T_{Wieg}$and with$A_{0-max} = \frac{H_{0-max}}{2}$, wave length L is calculated with (1) and$\frac{H_{0-max}}{L}$ where constant value of 0.32 is obtained which is a critical wave steepness from Hutahaean (2019), i.e.$\frac{H_b}{L_b} = \frac{1}{\pi}$. $H_b$is breaker height and$L_b$is breaker length. At the wave length L calculation, the value of$T_{Wieg}$is multiplied with coefficient 1.0001, to prevent determinant $D < 0$ from happening.

Table.1: Comparison of (2) with Wiegelequation

H_{0-max} (m)	T_{Wieg} (sec)	$\frac{H_{0-max}}{L}$	$H_{0-max,m}$ (m)
1	4,98	0,32	1
1,5	6,1	0,32	1,5
2	7,04	0,32	2
2,5	7,88	0,32	2,5
3	8,63	0,32	3
3,5	9,32	0,32	3,5
4	9,96	0,32	4
4,5	10,57	0,32	4,5
5	11,14	0,32	5

2.2. Comparative Study with Silvester Equation (1974)

Silvester (1974), formulated a relation between wave period and $H_{1/3}$using Pierson-Moskowitz spectrum, i.e.

$$T_{Sil} = \sqrt{19.68 H_{1/3}}. \qquad ....(4)$$

As with the comparative study with Wiegel equation, the study is done by providing an input of a wave height that is assumed as $H_{1/3}$and the wave period is calculated with (4). Then, usingT_{Sil}, $H_{1/3-m}$is calculated with (2). By experimenting the value ofγ,the value of$\gamma = 2.211$ is produced.The value of i $\frac{H_{1/3}}{L}$is obtained as 0.31, although it is close to $\frac{1}{\pi} = 0.318$it is still somewhat smaller. Thus shows that with wave period (4) the wave is only in critical condition, notbreaking yet. Wave length Lcalculation with (1) is done using wave period 1.0001T_{Sil}.

Table.2: Comparison of (2) with Silvester equation

$H_{1/3}$ (m)	T_{Sil} (sec)	$\frac{H_{1/3}}{L}$	$H_{1/3-m}$ (m)
1	4,44	0,31	1
1,5	5,43	0,31	1,5
2	6,27	0,31	2
2,5	7,01	0,31	2,5
3	7,68	0,31	3
3,5	8,3	0,31	3,5
4	8,87	0,31	4
4,5	9,41	0,31	4,5
5	9,92	0,31	5

2.3. The Correlation between Weighting Coefficient and Rayleigh Distribution

Comparative study between Wiegel equation and Silvester equation produced different γvalue. At (2) it can be seen that the bigger the value of$\gamma$the smaller theH_{0-max}. If wave amplitude calculation is done with (2),

Wiegel, $\gamma = 2.483$: $A_{Wieg} = \frac{g}{2x2.483^2\sigma^2}$

Silvester, $\gamma = 2.211$: $A_{Sil} = \frac{g}{2x2.211^2\sigma^2}$

$$\frac{H_{Sil}}{H_{Wieg}} = \frac{A_{Sil}}{A_{Wieg}} = \frac{2.483^2}{2.211^2} = 1.261$$

The comparative number is close enough with the value of$\frac{H_{1/10}}{H_{1/3}} = 1.271$, where it can be estimated that there is γvalue where$\frac{2.483^2}{\gamma^2} = 1.271$, and other relation. To learn that, coefficient distribution from Rayleigh will be used.

The relation between $H_p$and$H_{1/3}$ for$0 < p \leq 1$is obtained from Rayleigh coefficient (Forristall (1978)),i.e

$$H_p = \alpha_p H_{rms} \qquad ....(5)$$

$$\alpha_p = \ln\sqrt{\frac{1}{p}} + \frac{\sqrt{\pi}}{2p} erfc\left(\sqrt{\ln\frac{1}{p}}\right) \qquad ....(6)$$

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From (4) relation between H_p and $H_{1/3}$ can be made

$$H_p = c_p H_{1/3} \quad \text{...............(7)}$$

$$c_p = \frac{\alpha_p}{\alpha_{1/3}} \quad \text{................(8)}$$

Table.3: Rayleigh distribution coefficient

p	$c_p = \frac{H_p}{H_{1/3}}$
$1/10$	1,271
$1/9$	1,25
$1/8$	1,226
$1/7$	1,198
$1/6$	1,165
$1/5$	1,124
$1/4$	1,072
$1/3$	1
$1/2$	0,887
1	0,626

Table (3) shows that $\frac{H_{Sil}}{H_{Wieg}}$ is quite close with the value of $\frac{H_{1/10}}{H_{1/3}}$. Furthermore, through an experiment, γ values are obtained where $\frac{\gamma^2}{2.483^2} = \frac{H_p}{H_{1/3}}$. The result of the calculation can be seen on Table (4).

Table.4: Correlation between the value of γ and Rayleigh distribution

p	$\frac{H_p}{H_{1/3}}$	$\frac{\gamma^2}{2.483^2}$	γ
$1/10$	1,271	1,272	2,202
$1/9$	1,25	1,25	2,221
$1/8$	1,226	1,225	2,243
$1/7$	1,198	1,198	2,269
$1/6$	1,165	1,164	2,301
$1/5$	1,124	1,124	2,342
$1/4$	1,072	1,072	2,398
$1/3$	1	1	2,483
$1/2$	0,887	0,887	2,636
1	0,626	0,626	3,138

Table (4) shows that if the value of $\gamma = 2.483$ is assumed to correspond to $H_{1/3}$, then $\gamma = 2.202$ corresponds to $H_{1/10}$, $\gamma = 2.342$ corresponds to $H_{1/5}$, etc. This proves that there is a correlation between weighting coefficient value and Rayleigh distribution coefficient and this shows the existence of weighted total acceleration equation with its weighting coefficient in the nature.

Therefore, the study at this section shows that the value of $\gamma = 2.483$, correlates with $H_{1/3}$ so that Wiegel equation (3) is an equation for $H_{1/3}$, whereas the value of $\gamma = 2.202$ is in accordance with Silvester equation (4) and correlates with $H_{1/10}$, therefore Silvester equation is the equation for $H_{1/10}$.

2.4. The Correlation of Weighting Coefficient with Pierson- Moskowitz spectrum

In addition to producing a relation between wave period and wave height, Silvester (1974), also produces a relation between wind velocity $U_{19.5}$ and wave period for FAS (Fully Arisen Sea) condition, where $U_{19.5}$ is wind velocity measured at an elevation of 19.5 m from the surface. The relation is also formulated using Pierson-Moskowitz spectrum. The form of the relation is.

$$T = \frac{2\pi}{g} U_{19.5} \text{ sec.} \quad \text{........(9)}$$

where $g\ (m/sec^2)$ is gravity acceleration and $U_{19.5}\ (m/sec)$ is wind velocity.

Table.5: The calculation of $H_{1/10}$ and $T_{1/10}$ at FAS condition

$U_{19.5}$ (m/sec)	$T_{1/10}$ (sec)	$H_{1/10}$ (m)	$T_{1/10-Sil}$ (sec)
5	3,2	0,53	3,22
10	6,4	2,1	6,43
15	9,61	4,73	9,65
20	12,81	8,41	12,86
25	16,01	13,14	16,08

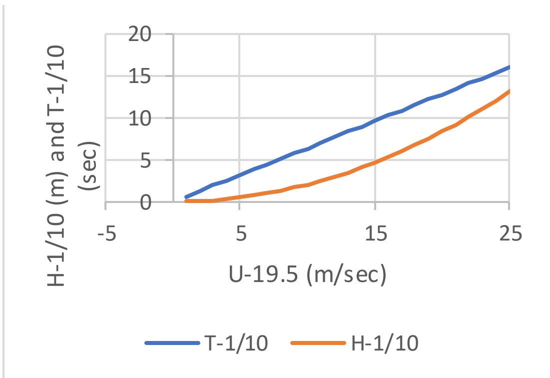

Fig.1: Relation between $U_{19.5}$ with $T_{1/10}$ (wave period for $H_{1/10}$) and $H_{1/10}$

$T_{1/10}$ (wave period for $H_{1/10}$), is calculated with (9), $H_{1/10}$ is calculated with (2) using wave period $T_{1/10}$ and

$\gamma = 2.202$, whereas $T_{1/10-Sil}$ at column 4 is calculated with (4), where the wave height is $H_{1/10}$ at column 3. Wave period in column 2 does not differ much with wave period at column 4 and can be said it is the same. It can be concluded that the calculation of $H_{1/10}$ with (2) using $\gamma = 2.202$ produces a wave height that is in accordance with wave height at the Silvester equation. Therefore, from the result, it can be said that wave height (4) is $H_{1/10}$. It is estimated that this is because (4) and (9) are formulated at the peak of spectrum.

Then to obtain $H_{1/3}$ and $T_{1/3}$ (wave period for $H_{1/3}$) at FAS condition, $H_{1/10}$ at column 2 in Table (5) is divided with 1.271, followed $T_{1/3}$ calculation using Wiegel equation (3). With the $T_{1/3}$, $H_{1/3-(2)}$ is calculated using (2) and $\gamma = 2.483$. The result of the calculation is presented in Table (6), which shows that $H_{1/3} = H_{1/3-(2)}$ which can be interpreted that $\gamma = 2.483$ correlates with $H_{1/3}$ from Pierson-Moskowitz spectrum, whereas wave height in Wiegel equation (3) is an equation for $H_{1/3}$.

Table.6: The Calculation of $H_{1/3}$ and $T_{1/3}$ at FAS condition

$U_{19.5}$ (m/sec)	$H_{1/10}$ (m)	$H_{1/3}$ (m)	$T_{1/3}$ (sec)	$H_{1/3-(2)}$ (m)
5	0,41	0,41	3,2	0,41
10	1,65	1,65	6,41	1,65
15	3,72	3,72	9,61	3,72
20	6,62	6,62	12,81	6,62
25	10,34	10,34	16,01	10,34

The result of the study in this section is that the value of weighting coefficient $\gamma = 2.202$ correlates with $H_{1/10}$ whereas $\gamma = 2.483$ correlates with $H_{1/3}$ at Pierson-Moskowitz spectrum.

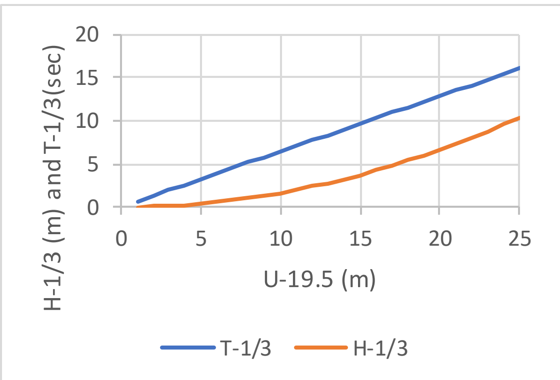

Fig.2: Relation between $U_{19.5}$ and $T_{1/3}$ (wave period for $H_{1/3}$) and $H_{1/3}$

III. CONCLUSION

The result of the study that has been done, obtained that weighting coefficient at the weighted total acceleration equation correlates with Rayleigh distributionand Pierson-Moskowitz spectrum. Considering that Rayleigh distribution and Pierson-Moskowitz spectrum were formulated based on data in the nature, then this proofs the existence of weighting coefficient in the nature.

The calculation of significant wave height can be done using maximum wave amplitude equation using $\gamma = 2.483$. Wave dynamics modeling can be done using weighted total acceleration equation with weightingcoefficient $\gamma = 2.483$.

Analytically, using sinusoidal water surface equation and particle velocity equation of the linear wave theory the value of γ is obtained between 2.784-3.160, whereas in this research the obtained value of γ is smaller, i.e. 2.483 for $H_{1/3}$ and 2.202 for $H_{1/10}$. This can be concluded that there is another sinusoidal water surface equation that will produce weighting coefficient value of 2.483 and 2.202. Therefore, further research that should be done is to conduct a research on the sinusoidal water surface equation, where the equation will have an influence on another wave parameter, i.e. wave length.

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