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Water Wave Modeling Using Complete Solution of Laplace Equation

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***Abstract*— *Analytical solution of Laplace equation using variable separation method, consists of two velocity potentials. However, only one component has been used. This research used both velocity potential equation components. With the potential equation, water wave surface equation and the related wave constants were formulated using kinematic free surface boundary condition and surface momentum equation. The characteristic of water wave surface that was produced was observed, both in deep water and shallow water.*
***Keywords*— Complete Solutionof Laplace Equation, Water wave surface equation.**

I. INTRODUCTION

The completion of Laplace equation using variable separation method in Dean (1991), produces two potential velocities, i.e. cosine and sine components. However, the application only exists in the cosine component in formulating various characters of water wave. This research does not discuss the use of the second component in Dean (1991), rather it studies the characteristic of water wave surface if the two velocity potential components are used simultaneously, i.e. water wave surface equation is formulated using a complete velocity potential.

The formulation begins by formulating the final form of the two velocity potential components. Then, in each velocity potential, water wave surface and wave amplitude equations were formulated. Using wave amplitude equation and surface momentum equation, equations for wave number $k$and wave constant G were formulated.

It is obtained that the two velocity potential components have similar wave number k, wave constant G, but with different water wave surface equations. As the final water wave surface is the superposition or the sum of the two water wave surface equations.

The characteristic of the water wave surface equation consists of maximum wave amplitude in a wave period, wave length, correlation between wave amplitude and wave height and other produced water wave surface profile was studied.

II. TOTAL VELOCITY POTENTIAL EQUATION

The form of Laplace equation solution (Dean(1991)), after periodic boundary condition was performed against time $t$is

$$\varphi(x,z,t) = A\cos kx(Ce^{kz} + De^{-kz})\sin\sigma t + B\sin kx(Ce^{kz} + De^{-kz})\sin\sigma t \quad .....(1)$$

Where$\sigma = \frac{2\pi}{T}$is angular frequency, whereas$T$is wave period.Constant$k$was obtained by performing lateral periodic boundary condition and wave number $k = \frac{2\pi}{L}$ was obtained where $L$is wave length. Therefore,in (1) there is only one value of wave number $k$and one value of wavelength L. Based on the linear characteristic of Laplace equation, then(1) can be written as,

$$\varphi(x,z,t) = \Phi_A(x,z,t) + \Phi_B(x,z,t) \quad ....(2)$$

Where,

$$\varphi_A(x,z,t) = A\cos kx(Ce^{kz} + De^{-kz})\sin\sigma t \quad (3)$$

$$\varphi_B(x,z,t) = B\sin kx(Ce^{kz} + De^{-kz})\sin\sigma t \quad (4)$$

In (2), (3) and (4), the values of constants A, B, C and D should be determined.

Equation (3) was performed at flat bottom (Dean (1991))

$$\varphi_A(x,z,t) = G_A\cosh k(h+z)\cos kx\sin\sigma t \quad (5)$$

was obtained Similar procedure was performed in(4),

$$\varphi_B(x,z,t) = G_B\cosh k(h+z)\sin kx\sin\sigma t \quad (6)$$

was obtained

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As has been mentioned, the two velocity potentials have similar wave number k. There should have been one wave constant, i.e. $G = G_A = G_B$, but to ensure,a proof will be done in the following chapters. In the previous researches Hutahaean (2019 a, b) formulated equations for wave number $k$and wave constant Gusing kinematic free surface boundary condition (KFSBC)and surface momentum equation. So is the case with this research, KFSBC equation and surface momentum equation will be used to formulate equations for wave number k and wave constant G. At the same time, this research is a improvementon the procedure of KFSBC integration against time t, in Hutahaean (2019a,b).

III. THE FORMULATION OF WAVE NUMBER kAND WAVE CONSTANT G_A USING Φ_A.

3.1. Water wave surface equation

The first step in formulating equation for wave number $k$and wave constant $G$is the formulation of water wave surface equation to obtain wave amplitude equation. The formulation was performed using KFSBC. KFSBC equation using weighted total acceleration is (Hutahaean (2019 a,b,c)),

$$\gamma \frac{\partial \eta}{\partial t} = w_\eta - u_\eta \frac{\partial \eta}{\partial x} \quad ....(7)$$

Where$\gamma$is weighting coefficient with the value of 2.87-3.14 (Hutahaean (2019 c)). $\eta = \eta(x,t)$is water wave surfaceelevation against still water level ($z = 0$), $u_\eta$is water particle velocity at horizontal-$x$direction at the water surface ($z = \eta$), whereas$w_\eta$is the water particle velocity at vertical $z$direction at the surface water. Using (5), equations of particles velocity at horizontal and vertical directions were obtained, i.e.

$$u(x,z,t) = -\frac{\partial \Phi_A}{\partial x} = G_A k \cosh k(h+z) \sin kx \, \sin \sigma t \quad ...(8)$$

$$w(x,z,t) = -\frac{\partial \Phi_A}{\partial z} = -G_A k \sinh k(h+z) \cos kx \, \sin \sigma t \quad ...(9)$$

(8) and (9) were performed at $z = \eta$and substituted to (7),

$$\gamma \frac{\partial \eta}{\partial t} = -G_A k \sinh k(h+\eta) \cos kx \sin \sigma t - G_A k \cosh k(h+\eta) \sin kx \sin \sigma t \frac{\partial \eta}{\partial x} \quad ...(10)$$

Water wave surface equation was obtained by integrating (10) against time t. It's visible that (10) is a non-linear function against time $t$which is difficult to complete its integration. However, there are two arguments to make it simple, where the two arguments produce similar conclusion.

The first argument is that the velocity potential equation was obtained using variable separation method, i.e. velocity potential Φthat is regarded to have a form of $\Phi(x,z,t) = X(x)Z(z)T(t)$, where$X(x)$is only a function of x, $Z(z)$is only a function of z and$T(t)$is only a function of time$t$. In this case $Z(z) = \cosh k(h+z)$. In relation with this, ηon the right side of the equation, both in$\sinh k(h+\eta)$or in $\cosh k(h+\eta)$and$\frac{\partial \eta}{\partial x}$are not the function of time t, even though $\eta = \eta(x,t)$. Hence (10) can be written as,

$$\gamma \frac{\partial \eta}{\partial t} = -G_A k \left(\cos kx \sinh k(h+\eta) + \sin kx \cosh k(h+\eta) \frac{\partial \eta}{\partial x} \right) \sin \sigma t \quad .....(11)$$

The second argument is that for a periodical function against time$t$, the element $-G_A k \left(\cos kx \sinh k(h+\eta) + 2 \sin kx \cosh k(h+\eta) \frac{\partial \eta}{\partial x} \right)$should be a constant number against time t, which is strengthened with the formulation of velocity potential as a function of periodical time is just $\sin \sigma t$. From the two arguments, the integration against time in (11) was completed only by integrating the $\sin \sigma t$element, obtained

$$\eta(x,t) = \frac{G_A k}{\gamma \sigma} \left(\cos kx \sinh k(h+\eta) + \sin kx \cosh k(h+\eta) \frac{\partial \eta}{\partial x} \right) \cos \sigma t \quad ....(12)$$

At the characteristics point, where in this research the characteristic point is a point where $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t$, (12) can be written as

$$\eta(x,t) = \frac{G_A k}{\gamma \sigma} \left(\sinh k(h+\eta) + \cosh k(h+\eta) \frac{\partial \eta}{\partial x} \right) \cos kx \cos \sigma t \quad ....(13)$$

The form $\cos kx$was selected because the first term of the elements in the parentheses is more dominant than the second element because of the presence of $\frac{\partial \eta}{\partial x}$element in the second term. It is defined a wave amplitude equation,

$$A = \frac{G_A k}{\gamma \sigma} \left(\sinh k(h+\eta) + \cosh k(h+\eta) \frac{\partial \eta}{\partial x} \right) \quad ......(14)$$

Using (14) the water wave surface equation becomes

$$\eta(x,t) = A \cos kx \cos \sigma t$$

Water wave surface equation was obtained at the velocity potential componentΦ_A, i.e.

$\eta_{0A}(x,t) = Acoskxcos\sigma t$ ....(15)

$$\frac{\partial \eta_{0A}}{\partial x} = -kAsinkxcos\sigma t$$

From (14),

$A_{\eta,1A} = \frac{G_A k}{2\gamma\sigma} sinhk\,(h + \eta_{0A})$ ....(16)

$A_{\eta,2A} = \frac{G_A k}{\gamma\sigma} sinhk\,(h + \eta_{0A})\frac{\partial\eta_{0A}}{\partial x}$ ....(17)

$\eta_A(x,t) = \frac{G_A k}{\gamma\sigma}\left(A_{\eta,1A}coskx + A_{\eta,2A}sinkx\right)cos\sigma t$ .....(18)

So,water wave surface equation consists of 4 (four) equations, i.e. (15), (16), (17) and (18), where wave amplitude in (15) is as input or known number.

3.2. Equation for $k$andG_A.

The next step is formulating equations for wave number $k$and wave constant G_A. The equation to calculate the two parameters can be obtained using (13) and surface momentum equation. (13) is differentiated against horizontal-x axis.

$$\frac{\partial \eta}{\partial x} = -\frac{G_A k^2}{\gamma\sigma}$$

$\left(sinhk\,(h+\eta) + coshk(h+\eta)\frac{\partial\eta}{\partial x}\right) sinkxcos\sigma t$ ....(19)

Bearing in mind that there are two variables that need to be calculated, then two equations are needed. As the second equation is surface momentum equation, where convective velocity is ignored.

$\gamma\frac{\partial u_\eta}{\partial t} = -g\frac{\partial\eta}{\partial x}$ ....(20)

Substitute$\frac{\partial u_\eta}{\partial t}$where$\frac{\partial u_\eta}{\partial t}$was obtained from (8), and substitute $\frac{\partial\eta}{\partial x}$with(19),

$$\gamma\sigma G_A kcoshk(h+\eta)sinkxcos\sigma t = \frac{gG_A k^2}{\gamma\sigma}\left(sinhk\,(h+\eta) + coshk\,(h+\eta)\frac{\partial\eta}{\partial x}\right)sinkxcos\sigma t$$

The equation is divided with $G_A kcoshk\,(h + \eta)\; sinkxcos\sigma t$ for $sinkxcos\sigma t$ which is not the same with zero and remembering that in deep water$tanhk\,(h+\eta) = 1$

$$\gamma^2\sigma^2 = gk\left(1 - \frac{\partial\eta}{\partial x}\right)$$

Substitute$\frac{\partial\eta}{\partial x}$with (19),

$$\gamma^2\sigma^2 = gk\left(1 - \frac{G_A k^2}{\gamma\sigma}\left(sinhk\,(h+\eta) + coshk(h+\eta)\frac{\partial\eta}{\partial x}\right)sinkxcos\sigma t\right)$$

Keeping in mind (14),

$$\gamma^2\sigma^2 = gk(1 - kAsinkxcos\sigma t)$$

The left side of the equation is constant number, therefore the right side should be constant, maximum value of $sinkxcos\sigma t = 1$ is used

$\gamma^2\sigma^2 = gk(1-kA)$......(21)

This equation is an equation to calculate wave number $k$in the deep water. This equation has a maximum wave amplitude value and at the same time is a critical wave steepness for a wave period, i.e. in a large wave amplitude, $(1-kA) = 0$ can occur, or

$A_{max} = \frac{L}{2\pi}$ ....(22)

The calculation of (22) can be done if wavelength $L$is already known. In the case that wavelength is not known, the equation for wave amplitude maximumcan be obtained by bearing in mind that (23) is a quadratic equation for wave number k, with a real root if the determinant is greater than zero. Wave amplitude maksimum was achieved at determinant value$d$equal to zero, $d = g^2 - 4gA\gamma^2\sigma^2 = 0$, so obtained

$A_{max} = \frac{g}{4\gamma^2\sigma^2}$ ....(23)

Byequating $A_{max}$with (22) and (23), critical wavelength in a wave period was obtained, i.e.

$L_{min} = \frac{\pi g}{2\gamma^2\sigma^2}$ ....(24)

(22), (23) and (24) only apply for just one component, in this case is Φ_A.

As has been stated that from the two velocity potentials $\Phi_A$andΦ_B, there is only one value of wave number k, therefore it can be estimated that by using Φ_B the form of wave number equation that is similar with (21) will be obtained.

As an equation for G_A, surface momentum equation (22) and water wave surface equation (15) were used and were performed at characteristic point.

$G_A = \frac{gA}{\gamma\sigma cosk\left(h+\frac{A}{2}\right)}$ .....(25)

IV. THE FORMULATION OF WAVE NUMBER kAND WAVE CONSTANT G_BUSINGΦ_B.

4.1. Water wave surface equation.

Particle velocity equations to horizontal and vertical directions were formulated using Φ_B, in (6).

$$u(x,z,t) = -\frac{\partial\Phi_B}{\partial x}$$

$= -G_B\,kcoshk(h+z)\,coskxsin\sigma t$..(26)

$$w(x,z,t) = -\frac{\partial\Phi_B}{\partial z}$$

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$= -G_B ksinhk\,(h+z)\,sinkxsin\sigma t$ ...(27)

Substitute (26) and (27) that was performed at $z = \eta$ to (7),

$$\gamma \frac{\partial \eta}{\partial t} = -G_B ksinhk(h+\eta)sinkxsin\sigma t$$

$$+G_B kcoshk\,(h+\eta)coskxsin\sigma t \ \frac{\partial \eta}{\partial x} \text{ .....(28)}$$

As has been performed in previous section, the right side of equation (28) can be written as,

$$\gamma \frac{\partial \eta}{\partial t} = -G_B k \left(sinhk\,(h+\eta)sinkx - coshk(h+\eta)coskx \ \frac{\partial \eta}{\partial x} \right) sin\sigma t$$

...(29)

Then, it was integrated against time t.

$$\eta(x,t) = \frac{G_B k}{\gamma\sigma} \left(sinhk\,(h+\eta)sinkx - coshk(h+\eta)coskx \ \frac{\partial \eta}{\partial x} \right) cos\sigma t$$

....(30)

At the characteristic point, (30) can be written as

$$\eta(x,t) = \frac{G_B k}{\gamma\sigma} \left(sinhk\,(h+\eta) - coshk(h+\eta) \frac{\partial \eta}{\partial x} \right) sinkxcos\sigma t$$

......(31)

Selected to use $sinkx$ because the term in the parentheses on the right side is more dominant than the second term where there is $\frac{\partial \eta}{\partial x}$. Defined a wave amplitude equation,

$$A = \frac{G_B k}{\gamma\sigma} \left(sinhk(h+\eta) - coshk\,(h+\eta) \frac{\partial \eta}{\partial x} \right) \text{ ....(32)}$$

Water wave surface equation becomes

$$\eta(x,t) = Asinkxcos\sigma t$$

From (32) water wave surface equation was obtain from velocity potential component Φ_B is

$$\eta_{0B}(x,t) = Asinkxcos\sigma t \text{ ....(33)}$$

$$\frac{\partial \eta_{0B}}{\partial x} = kAcoskxcos\sigma t$$

$$A_{\eta,1B} = \frac{G_B k}{\gamma\sigma} sinhk\,(h+\eta_{0A}) \text{ ....(34)}$$

$$A_{\eta,2B} = \frac{G_B k}{\gamma\sigma} sinhk\,(h+\eta_{0B}) \frac{\partial \eta_{0B}}{\partial x} \text{ ....(35)}$$

$$\eta_B(x,t) = \frac{G_B k}{\gamma\sigma} \left(A_{\eta,1B} sinkx - A_{\eta,2A} coskx \right) cos\sigma t$$

......(36)

Thus, water wave surface equation that was obtained with Φ_B also consists of 4 (four) equations, i.e. (33), (34), (35) and (36), where wave amplitude in (33) is as an input or known number.

4.2. Equation for k and G_B.

(31) was differentiated against horisontal-x axis,

$$\frac{\partial \eta}{\partial x} = \frac{G_B k^2}{\gamma\sigma} \left(sinhk\,(h+\eta) - coshk(h+\eta) \frac{\partial \eta}{\partial x} \right) coskxcos\sigma t$$

......(37)

Next, surface momentum equation was performed where convective velocity was ignored,

$$\gamma \frac{\partial u_\eta}{\partial t} = -g \frac{\partial \eta}{\partial x} \text{ ....(38)}$$

$\frac{\partial u_\eta}{\partial t}$ was obtained from (26) whereas $\frac{\partial \eta}{\partial x}$ from (37) and an equation was obtained ,

$$-\gamma\sigma G_B\, kcoshk\,(h+\eta)coskxcos\sigma t = -g \frac{G_B k^2}{\gamma\sigma} \left(sinhk(h+\eta) - coshk\,(h+\eta) \frac{\partial \eta}{\partial x} \right) coskxcos\sigma t$$

The two terms of the equation were divided with $-G_B kcoshk\,(h+\eta)coskxcos\sigma t$, and keeping in mind that in deep water $tanhk\,(h+\eta) = 1$,

$$\gamma^2\sigma^2 = gk\left(1 - \frac{\partial \eta}{\partial x}\right)$$

Substitute(37)

$$\gamma^2\sigma^2 = gk \left(1 - \frac{G_B k^2}{\gamma\sigma} \left(sinhk\,(h+\eta) - coshk(h+\eta) \frac{\partial \eta}{\partial x} \right) coskxcos\sigma t \right)$$

Bearing in mind (32) and by taking $coskxcos\sigma t = 1$,

$$\gamma^2\sigma^2 = gk(1-kA) \text{ ....(39)}$$

Equation for wave number k, i.e. (39) by using Φ_B is the same as wave number equation formulated using Φ_A, i.e. (21), so it is proven that at potential velocity that is the superposition of two velocity potentials, each component has similar wave number .

Furthermore, the equation for wave constant G_B was obtained using surface momentum equation performed at characteristic point, obtained,

$$G_B = \frac{gA}{\gamma\sigma cosk\left(h+\frac{A}{2}\right)} \text{ .....(40)}$$

Compare to (25), the two velocity potentials have similar wave constant G i.e. $G_A = G_B = G$.

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V. SUMMARY

The description in Chapter III and Chapter IV shows that both velocity potentials have similar wave number equation and wave constant, so that both have similar wave number $k$and wave constant G. Similarly,

1. Wave number equation from (21) and (39)

$$\gamma^2\sigma^2 = gk - gAk^2$$

1. Wave constant Gequation from (25) and (40),

$$G = \frac{gA}{\gamma\sigma coshk\left(h + \frac{A}{2}\right)}$$

2. Water wave surface equation has a slightly different form.

a. Water wave surface equation of Φ_A

$\eta_{0A}(x,t) = Acoskxcos\sigma t$ ....(15)

$$\frac{\partial\eta_{0A}}{\partial x} = -kAsinkxcos\sigma t$$

$A_{\eta,1A} = \frac{Gk}{\gamma\sigma}sinhk(h + \eta_{0A})$ ....(16)

$A_{\eta,2A} = \frac{Gk}{\gamma\sigma}sinhk(h + \eta_{0A})\frac{\partial\eta_{0A}}{\partial x}$ ....(17)

$\eta_A(x,t) = \frac{Gk}{\gamma\sigma}(A_{\eta,1A}coskx + A_{\eta,2A}sinkx)cos\sigma t$ ....(18)

b. Water wave surface equation ofΦ_B

$\eta_{0B}(x,t) = Asinkxcos\sigma t$ ....(33)

$$\frac{\partial\eta_{0B}}{\partial x} = kAcoskxcos\sigma t$$

$A_{\eta,1B} = \frac{Gk}{\gamma\sigma}sinhk(h + \eta_{0B})$ ....(34)

$A_{\eta,2B} = \frac{Gk}{\gamma\sigma}sinhk(h + \eta_{0B})\frac{\partial\eta_{0B}}{\partial x}$ ....(35)

$\eta_B(x,t) = \frac{Gk}{\gamma\sigma}(A_{\eta,1B}sinkx - A_{\eta,2B}coskx)cos\sigma t$ ....(36)

c. Total water wave surface equation is

$$\eta = \eta_A + \eta_B$$

VI. RESULT OF MODEL

6.1. The calculation of deep water depth

Deep water depth was obtained using the criteria $tanh\,k_0\left(h_0 + \frac{A_0}{2}\right) = 1$. For a wave amplitude that is much smaller than deep water depth h_0, $k_0h_0\left(1 + \frac{A_0}{2h_0}\right) = k_0h_0 = constant$ applies. $tanh\,k_0h_0 = 1$can be obtained at $k_0h_0 = \alpha_0\pi$where$tanh\,(\alpha_0\pi) = 1$. Thus, $k_0h_0 = \alpha_0\pi$and $h_0 = \frac{\pi}{k_0}$ ....(41)

$\alpha_0$is a determined constant number, for example in CERC (1984) $\alpha_0 = 1$ was used, so that $\frac{h_0}{L_0} = 0.5$ was obtained.In this research, in addition to the criteria of $tanh(\alpha_0\pi) = 1$, the value $\alpha_0$was also determined based on other reviews. Hutahaean (2019b) obtained that the larger the $\alpha_0$the larger the breaker depth and breaker height will be, so $\alpha_0$can be determined indiscriminately.

For a large wave amplitude, $tanhk\left(h + \frac{A_0}{2}\right) = 1$, where$k_0\left(h_0 + \frac{A_0}{2}\right) = \alpha\pi$, revision on α_0 was done againstα_0,

$\alpha_A = \frac{k_0\left(h_0 + \frac{A_0}{2}\right)}{\pi}$ ....(42)

Therefore the values of wave freuencyσandA_0parameter were absorbed in the value of α_A. For the following calculation,$\alpha = \alpha_A$ was used. The value of $\alpha_0$cannot be used too large, e.g. 2.25, where$tanh\,(2.25\pi) = 1$, but it should take into consideration the characteristic of breaking that was produced, with the best value of $\alpha_0 = 1.6 - 1.9$. Hutahaean (2019b)) obtained that with $\alpha_0 = 1.65$, breaker depth that is in accordance with CERC (1984) was obtained.

Table.1: Wave characteristic in several wave periods

T (sec.)	A_{max} (m)	L_0 (m)	h_0 (m)	$\frac{h_0}{L_0}$
6	0,53	6,9	6,04	0,88
7	0,72	9,39	8,22	0,88
8	0,95	12,26	10,73	0,88
9	1,2	15,52	13,58	0,88
10	1,48	19,16	16,77	0,88
11	1,79	23,19	20,29	0,88
12	2,13	27,6	24,15	0,88
13	2,5	32,39	28,34	0,88
14	2,89	37,56	32,87	0,88
15	3,32	43,12	37,73	0,88

Table (1) shows the result of the calculation of wave characteristic for several wave periods which includes deep water wave amplitude maximumA_{max}, deep water wave length L_0anddeep water depth h_0. The wave amplitude looks small but it will produce a large wave height, where the relation of wave height that is twice wave amplitude cannot be used. The calculation was done using the value of $\alpha_0 = 1.75$ the values of $\gamma = 2.05$, where this value was obtained with the procedure in Hutahaean (2019 c,d), whereas as wave amplitude maximum, (23) was used.

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6.2. Water wave surface profile

The model was performed using wave period8 sec., wave amplitude 0.95 m, $\gamma = 2.05$and$\alpha_0 = 1.75$. The result of the model can be seen in Fig.1.a., Fig.1.b. and Fig.1.c.

Fig.1.a. shows that $\eta_A$andη_B($\eta_A$water wave profile of Φ_A, $\eta_B$water wave profile of Φ_B) have cnoidal profile, and both have similar profile size, i.e. wave crest elevation $\eta_{max} =$ 1.62 m, whereas wave trough elevation$\eta_{min} = -0.66$ m, therefore wave profile is asymmetric where $\eta_{max}$is not the same as $|\eta_{min}|$. Wave height $H = 1.62 + 0.66 = 2.28$ m, Wilson parameter value (1963), $\frac{\eta_{max}}{H} = 0.711$, with this parameter value the wave profile belongs to cnoidal wave profile (Table (2)).

Table.2: Wave type according to Wilson criteria (1963)

Wave Type	$\frac{\eta_{max}}{H}$
Airy waves	< 0.505
Stoke's waves	< 635
Cnoidal waves	$0.635 < \frac{\eta_{max}}{H} < 1$
Solitary waves	$= 1$

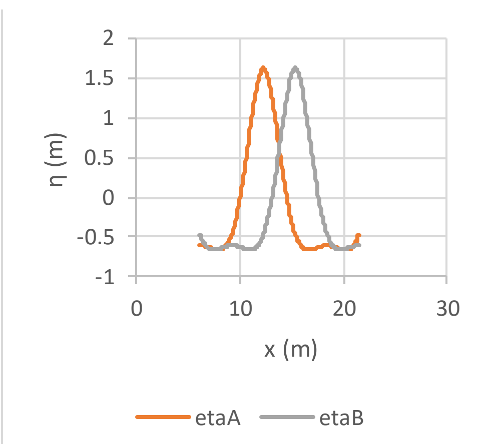

Fig.1.a. Wave profile $\eta_A$and$\eta_B$in the wave period of 8 sec., $A = 0.95$ m

The resultant wave, $\eta = \eta_A + \eta_B$, (Fig.1.b.dan Fig. 1.c.), obtained$\eta_{max} = 1.25$ m, $\eta_{min} = -1.31$ m, can be stated as symmetrical. Wave height $H = 2.561$ m, Wilson parameter$\frac{\eta_{max}}{H} = 0.487$, show that the wave has Airy's wave profile type. The condition is very different from the ones previously known, i.e. Airy's wave type can only be formed in a wave with a very small wave amplitude. One thing that should be noticed is that there is a concavity in in the wave crest. A wave with a sharp wave crest can hardly be seen in a wave in the deep water, it always looks flat. Wave crest in Fig. 1.b can be stated as flat, which is quite in accordance with the one in the nature.

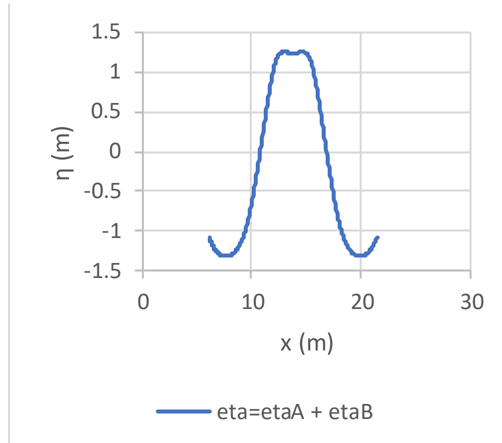

Fig.1.b. Wave profile $\eta = \eta_A + \eta_B$in a wave period of 8 sec., $A = 0.95$ m

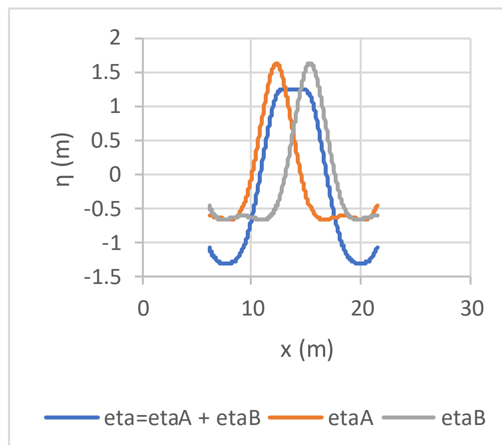

Fig.1.c. Wave profile in a wave period of 8 sec., $A = 0.95$ m

Furthermore, the calculation of water wave surface characteristic was performed in several wave periodswith wave amplitude maximum, i.e. in equation (37), $A_{max} = \frac{g}{4\gamma^2\sigma^2}$. The result of the calculation is presented in Table (3) and Table (4).

Table.3: Water wave surface characteristics at wave amplitude maximum

T (sec.)	η_{min} (m)	η_{max} (m)	H (m)	$\frac{\eta_{max}}{H}$
8	-1,31	1,25	2,561	0,487

9	-1,66	1,58	3,242	0,487
10	-2,05	1,95	4,002	0,487
11	-2,48	2,36	4,843	0,487
12	-2,95	2,81	5,763	0,487
13	-3,47	3,3	6,764	0,487
14	-4,02	3,82	7,844	0,487
15	-4,62	4,39	9,005	0,487

In the maximum wave amplitude, the Wilson parameter value $\frac{\eta_{max}}{H} = 0.487$ for all wave period shows that the wave belongs to to Airy wave, where η_{max} is quite close with $|\eta_{min}|$. Furthermore in Table (4), the value of $\frac{H}{A} = 2.71$ shows that an approach that the value of wave height H is twice the value of wave amplitude A cannot be determined or performed.Wave steepness $\frac{H}{L} = 0.209$, this condition exceeds the criteria of critical wave steepness from Michell (1893) with the value of $\frac{H}{L} = 0.142$

Table.4: The value of $\frac{H}{A}$ and wave steepness $\frac{H}{L}$ in wave amplitude maximum.

T (sec.)	A (m)	L (m)	$\frac{H}{A}$	$\frac{H}{L}$
8	0,945	12,265	2,71	0,209
9	1,196	15,522	2,71	0,209
10	1,477	19,163	2,71	0,209
11	1,787	23,188	2,71	0,209
12	2,127	27,595	2,71	0,209
13	2,496	32,386	2,71	0,209
14	2,894	37,56	2,71	0,209
15	3,323	43,118	2,71	0,209

Note: wave height can be seen in Table (3)

6.5. Water wave surface profile at breaker point

To obtain water wave surface profile at breaker point, the values of G, k and A are needed at breaker point. To obtain the value of the three wave parameters, shoaling and breaking analysis was performed. The shoaling and breaking model used in this research looks similar to the one in Hutahaean (2019 b), the model that was not discussed here. Bearing in mind that the two wave potentials have similar equations for wave amplitude, wave constant and wave number, then the shoaling and breaking model will also be similar to the model in Hutahaean (2019 b) that was formulated using Φ_A.

a. Water wave surface profile

As an example of water wave surface profile at breaker point, a wave with wave period T =8 sec., wave amplitude $A = 0.95$ m and bottom slope $\frac{dh}{dx} = -0.005$ was used. Water wave surface profile at the breaker point is presented in Fig. 2 a. and Fig.2b.

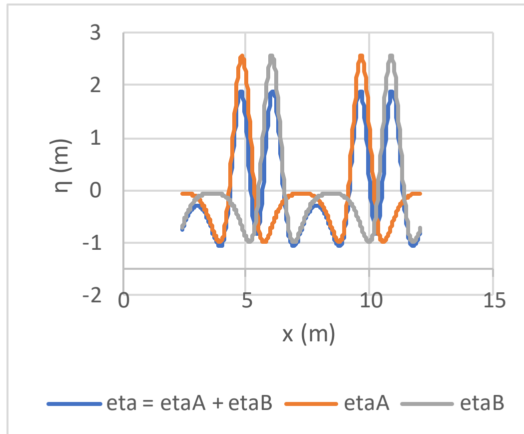

Fig. 2a. Water wave surface profile at breaker point, η, η_A and η_B

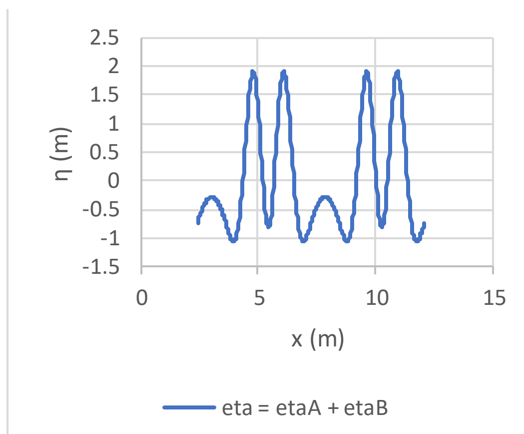

Fig. 2b. Water wave surface profile at breaker point $\eta = \eta_A + \eta_B$.

At the breaker point, wave profile is asymmetrical, where $\eta_{max} = 1.90$ m, $\eta_{min} = -1.059$ m, where wave height $H = 2.958$ m, whereas Wilson parameter $\frac{\eta_{max}}{H} = 0.642$.With this parameter value, the type of the wave is cnoidal wave type. There are two phenomena that should be paid attention to, first, the occurrence of wave setup where in the deep water the wave profile is symmetrical, whereas in the shallow water the wave trough part is lifted. The next phenomenon is the separation of a wave from the two velocity potentials that were used, where there are two wave

crest. The presence of two adjacent waves also found in the coastal water. A more vivid example is tsunami wave on the coast or land, consist of two large main wave crests.
In the profile of the breaking wave, it is also visible that there is a wave trough in fornt of wave crest. This also occurs in tsunami, where prior to the coming of the peak of the tsunami, the coastal water recedes first.

b. Adjustment with breaker height index equation
The result of breaker height model was calibrated against the average value of 5 (five) breaker height indexes. The adjustment was performed by multiplying wave constant $G$with 0.336. Whereas breaker depth $h_b$was adjusted with breaker depth from SPM (1984), by changing the values of α_0, where$\alpha_0 = 1.76$ resulted in a breaker height that fit with breaker depth from SPM (1984). The breaker height index equations used as comparators are breaker height index (BHI) equations from Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992) and Rattana Pitikonand Shibayama (2000), with the comparison result is presented in Table (5).

Table.5: Comparison of breaker height model withBHI

T (sec.)	H_0 (m)	H_b (m) (model)	H_b (m) (BHI)	h_b (m) (model)	h_b (m) (SPM)
8	2,602	2,958	2,955	3,673	3,697
9	3,293	3,741	3,74	4,65	4,678
10	4,065	4,617	4,617	5,741	5,776
11	4,919	5,584	5,586	6,948	6,989
12	5,854	6,644	6,648	8,269	8,317
13	6,87	7,797	7,803	9,705	9,761
14	7,968	9,041	9,049	11,256	11,321
15	9,146	10,378	10,388	12,921	12,996

Note :BHI is the average value of breaker height from 5 (five) breaker height index equations.

As has been stated that the adjustment of breaker height was performed by multiplying wave constant Gat breaker pointwith a coefficient of 0.477.

VII. CONCLUSION

Both components of velocity potential equation as the solution of Laplace equation have similar wave number and wave constant, so that both can be performed as a unity to model water wave mechanics

Water wave surface equation from each velocity potential component has different form, where the total of water wave surface equation is the sum of the two water wave surface equations. However, as has been stated that both have similar wave amplitude value and equation, wave number and wave constant G. Both produced similar wave profile. Therefore, both water wave surface equation are actually identical. Wave separation in the shallow water, also occurs in the nature, shows that the two velocity potentials should have been used. In addition, water wave surface resultant have different wave height with each component of water wave surface. This also strengthens that the two velocity potential components of Laplace equation should have been used all simultaneously.

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