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A Study on Grid-Size and Time-Step Calculation using the Taylor Series in Time Series Water Wave Modeling

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Abstract— *This paper presents the time-step and grid-size calculation methods for solving time series wave differential equations using the Finite Difference Method.*

The methods are formulated using the Taylor series, assuming that at a very small time-step and grid-size, the number of terms in the Taylor series starting with 2nd order with the highest order is much smaller than the order 1.

I. INTRODUCTION

The formulation of basic hydrodynamic equations, both the continuity equation and the momentum equation, is developed using the Taylor series to one derivative (Dean (1991)). It is argued that in which the time-step$\delta t$and grid size $(\delta x, \delta z)$ is very small, calculation on the Taylor series can be done up to one order$O(\delta^1)$.

The solution of the governing equations numerically is by using the discrete method, both the Finite Difference Method (FDM) and the Finite Element Method (FEM) by dividing the domains into grid-points. The grid-point formation requires a small grid-size under the formulation conditions, assuming the time-step$\delta t$and grid size $(\delta x, \delta z)$are very small.

In FDM, the equations of FDM are formulated by truncating the Taylor series, using only the first or second-order (Smith, G.D (1985)). Therefore, the application of the equations isrequired to use a very small time-step and grid-size where the Taylor series can be used in order 1 and order 2 only.

Thus, it takes a small time-step and grid-size following the formulation of governing equations as well as in the formulation of the FDM equation in which FDM is used in the solving governing-equations.

The accuracy of the Taylor series is determined not only by the length of the terms used but also by the grid-size used, where the smaller the grid size, the better the accuracy. At very small time-steps and grid-sizes, the number of terms of order 2 and greater is much smaller than terms of order 1. This, this condition is the basis for the development of the grid-size calculation method in this study.

The grid-size study is carried out using the sinusoidal function, where the water wave equationsare the result of an analytical solution to the Laplace's equation by using the variable separation method is a sinusoidal function in time and space.

II. THE SETTING OF TIME STEPδtAND GRID SIZE δx

The water wave equations are obtained analytically, by solving Laplace's equation with the variable separation method in the form of multiplication between the sinusoidal function and the hyperbolic function (Dean (1991)). The horizontal axis-x and time-t form a sinusoidal function while the vertical axis-z form a hyperbolic function. By substituting the water wave equations into the Taylor series, the equations for the time-step p $\delta t$and grid-size δxandδz were obtained.

The formulation is carried out by two methods. The first method isa separate method where the time-step$\delta t$is calculated first and then inputδt, grid-size $\delta x$and$\delta z$are calculated. The second method is to substitute the relation between$\delta x, \delta z$, and the time step δtin the Taylor series to obtain the equation forδt. Then, δxandδzarecalculated using the relation between$\delta x$and$\delta z$with the obtained δt.

2.1. Separate Calculation

a. δt Calculation

The Taylor series form (Thomas, George B., Finney, Ross L.(1996)) for a function of time-*t* is,

$$f(t+\delta t) = f(t) + \delta t\frac{df}{dt} + \frac{\delta t^2}{2}\frac{d^2f}{dt^2} + \frac{\delta t^3}{6}\frac{d^3f}{dt^3} + \frac{\delta t^4}{24}\frac{d^4f}{dt^4}. \quad \text{.... (1)}$$

To enable Taylor series can be used up to one derivative only, then

$$\left|\frac{\frac{\delta t^2}{2}\frac{d^2f}{dt^2} + \frac{\delta t^3}{6}\frac{d^3f}{dt^3} + \frac{\delta t^4}{24}\frac{d^4f}{dt^4}}{\delta t\frac{df}{dt}}\right| \leq \varepsilon$$

δtin numerator and denominator cancel each other out,

$$\left|\frac{\frac{\delta t d^2f}{2\,dt^2} + \frac{\delta t^2 d^3f}{6\,dt^3} + \frac{\delta t^3 d^4f}{24\,dt^4}}{\frac{df}{dt}}\right| \leq \varepsilon \text{......... (2)}$$

Used

$f(t) = A\cos\sigma t$.... (3)

$\sigma = \frac{2\pi}{T}$:Angular frequency, T: period

Substitution (3) to (2) where the amplitude A on the numerator and denominator cancel each other out is done at the characteristic point and is done at the characteristic point, $\cos\sigma t = \sin\sigma t$, in which the terms $\cos\sigma t$and$\sin\sigma t$in the numerator and denominator also cancel each other out.

$$\left|-\frac{\delta t}{2}\sigma - \frac{\delta t^2}{6}\sigma^2 + \frac{\delta t^3}{24}\sigma^3\right| \leq \varepsilon$$

Since the term in the absolute sign | | is negative, then the positive value of the equation becomes,

$$\frac{\delta t}{2}\sigma + \frac{\delta t^2}{6}\sigma^2 - \frac{\delta t^3}{24}\sigma^3 \leq \varepsilon$$

By taking the equal sign,

$$\frac{\delta t}{2}\sigma + \frac{\delta t^2}{6}\sigma^2 - \frac{\delta t^3}{24}\sigma^3 = \varepsilon \text{....... (4)}$$

Equation (4) can be solved by the Newton-Raphson method (Allen, Myron B.; Isaacson, Eli L.(1998)). The results of the calculation of δtusing equation (4), for several wave periods, are presented in Table (2.1), with an accuracy level of$\varepsilon = 0.001$. Meanwhile, Table (2.2) presents the calculation result of δt for the wave period is 8 sec. with several levels of accuracy.

Table.2.1: The value of δt for several$T$values withε 0.001

T (sec.)	δt (sec.)	$\frac{\delta t}{T}$
6	0,00191	0,00032
7	0,00223	0,00032
8	0,00254	0,00032
9	0,00286	0,00032
10	0,00318	0,00032
11	0,0035	0,00032
12	0,00382	0,00032
13	0,00414	0,00032
14	0,00445	0,00032
15	0,00477	0,00032

Table.2.2: The value of δtat several values ofε*, $T = 8$ sec.*

ε	δt (sec.)	$\frac{\delta t}{T}$
0,1	0,23899	0,02987
0,01	0,0253	0,00316
0,001	0,00254	0,00032
0,0001	0,00025	0,00003

Table (2.1) shows that $\frac{\delta t}{T}$ is constant for all periods of T for a given level of accuracyε. Meanwhile, Table (2.2) shows the change in δt along with the change in the level of accuracyε.

b. δx Calculation

The calculation of δx on the function $f(x,t)$is done using δt as the input obtained by the calculation method as discussed in the previous section. In this section, the Taylor Series is only used up to the 3rd differential or$O(\delta^3)$, considering that theδtvalue obtained is quite accurate, and to facilitate writing. Taylor's series up to the third derivative for a function$f(x,t)$is,

$$f(x+\delta x, t+\delta t) = f(x,t) + s_1 + s_2 + s_3 \text{..... (5)}$$

Where,

$$s_1 = \delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x} \text{...... (6)}$$

$$s_2 = \frac{\delta t^2}{2}\frac{\partial^2 f}{\partial t^2} + \delta t\delta x\frac{\partial^2 f}{\partial t\partial x} + \frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2} \text{..... (7)}$$

$$s_3 = \frac{\delta t^3}{6}\frac{\partial^3 f}{\partial t^3} + \frac{\delta t^2}{2}\delta x\frac{\partial^3 f}{\partial t^2\partial x}$$

$$+\delta t\frac{\delta x^2}{2}\frac{\partial^3 f}{\partial t\partial x^2}+\frac{\delta x^3}{6}\frac{\partial^3 f}{\partial x^3}$$
... (8)

To enable the Taylor series used with only one derivative, then
$\left|\frac{s_2+s_3}{s_1}\right| \leq \varepsilon$...... (9)

The function below is used,
$f(x,t) = A\cos kx\ \cos\sigma t$..... (10)

By using the relation (6) and (7),
$$\frac{s_2}{s_1}=\frac{\frac{\delta t^2}{2}\frac{\partial^2 f}{\partial t^2}+\delta t\delta x\frac{\partial^2 f}{\partial t\partial x}+\frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2}}{\delta t\frac{\partial f}{\partial t}+\delta x\frac{\partial f}{\partial x}}$$
Substitute $f(x,t)$ done at a characteristic point where $\sin\sigma t = \cos\sigma t = \sin kx = \cos kx$,
$\frac{s_2}{s_1}=\frac{\frac{\delta t^2}{2}\sigma^2-\delta t\delta x\sigma k+\frac{\delta x^2}{2}k^2}{\delta t\sigma+\delta xk}$....... (11)
In the same way, it was obtained
$\frac{s_3}{s_1}=\frac{-\frac{\delta t^3}{6}\sigma^3-\frac{\delta t^2}{2}\delta x\sigma^2 k-\delta t\frac{\delta x^2}{2}\sigma k^2-\frac{\delta x^3}{6}k^3}{\delta t\sigma+\delta xk}$..... (12)
Intuitively, it can be estimated that (11) is positive, while (12) is negative. However, considering that (11) is greater than (12), the sum of the two equations will be positive.

Substituting the two equations to (9) and assuming that the term in the absolute sign is positive, the equation is obtained.
$$-\left(\delta t\sigma\varepsilon-\frac{\delta t^2}{2}\sigma^2+\frac{\delta t^3}{6}\sigma^3\right)$$
$$-\left(\delta t\sigma+\frac{\delta t^2}{2}\sigma^2+\varepsilon\right)k\delta x$$
$$+(1+\delta t\sigma)\frac{k^2}{2}\delta x^2-\frac{k^3}{6}\delta x^3=0$$
...(13)

δx can be calculated by (13) with δt as the input, where δt is obtained from the previous procedure.

Table (2.3) presents the calculation results of δx for several T wave periods, at a water depth of 10m. The wavelength is calculated using the dispersion equation from the linear wave theory according to Dean (1991).The level of accuracy used is $\varepsilon = 0.001$.

Table.2.3: δx Calculation

T (sec.)	δt (sec.)	δx (m)	L (m)	$\frac{\delta x}{L}$
6	0,00191	0,04643	48,4062	0,00096
7	0,00223	0,05738	59,8212	0,00096
8	0,00254	0,068	70,8984	0,00096
9	0,00286	0,07839	81,7267	0,00096
10	0,00318	0,0886	92,3739	0,00096
11	0,0035	0,09868	102,887	0,00096
12	0,00382	0,10867	113,299	0,00096
13	0,00414	0,11858	123,633	0,00096
14	0,00445	0,12843	133,905	0,00096
15	0,00477	0,13824	144,128	0,00096

The calculation results show that the value of δx obtained is quite small compared to the wavelength L, which shows that the value is quite relevant to use. Since the value of $\frac{\delta x}{L}$ might be too small, the comparison between the wave phase velocity is reviewed, $C = \frac{L}{T}$, and $\frac{\delta x}{\delta t}$ (Table (2.4)).

Table.2.4: Comparison between $\frac{\delta x}{\delta t}$ and C

T (sec.)	$\frac{\delta x}{\delta t}$ (m/sec.)	C (m/sec)	$\frac{\delta x/\delta t}{C}$
6	24,3257	8,0677	3,01519
7	25,7675	8,54589	3,01519
8	26,7215	8,86229	3,01519
9	27,3802	9,08074	3,01519
10	27,8525	9,23739	3,01519
11	28,2022	9,35337	3,01519
12	28,4682	9,44158	3,01519
13	28,6751	9,51022	3,01519
14	28,8392	9,56465	3,01519
15	28,9716	9,60854	3,01519

Table (2.4) shows that $\frac{\delta x}{\delta t}$ is much greater than C there the ratio $\frac{\delta x/\delta t}{C} = 3.01519$ or $\delta x = 3.01519\ C\ \delta t$. This is following with the Courant criteria, $\delta x = 3.0\ C\ \delta t$ (Courant (2928)).
2.2. δt calculation by substituting δx.

In this section δt and $\delta x$are processed using the same procedure as the previous one where $\delta x$is substituted withδt, in which

$\delta x = \gamma C\delta t = \gamma \frac{\sigma}{k}\delta t$ ...... (14)

$C = \frac{L}{T}$is wave celerity

$L = \frac{2\pi}{k}$Wavelength

γ:Coefficient (when used the results in Table (2.4.),$\gamma = 3.01519$)

Furthermore, using the relation (6) and (7), where$\delta x$is substituted by (14) to obtain:

$\frac{s_2}{s_1} = \left(\frac{1/2-\gamma+\frac{\gamma^2}{2}}{1+\gamma}\right)\sigma\delta t$ ......(15)

By using the relations (6) and (8) and by substitutingδx with (14),

$\frac{s_3}{s_1} = -\left(\frac{1/6+\frac{\gamma}{2}+\frac{\gamma^2}{2}+\frac{\gamma^3}{6}}{1+\gamma}\right)\sigma^2\delta t^2$.... (16)

Substitute (15) and (16) into (9) and taking a value equal to,

$$-\left(\frac{1/6+\frac{\gamma}{2}+\frac{\gamma^2}{2}+\frac{\gamma^3}{6}}{1+\gamma}\right)\sigma^2\delta t^2$$
$$+\left(\frac{1/2-\gamma+\frac{\gamma^2}{2}}{1+\gamma}\right)\sigma\delta t - \varepsilon = 0$$

..... (17)

By inputtingσ, εandγ, δtcan be calculated with (14). Meanwhile, γ can be obtained using the previous calculation results,$\gamma = 3.01519$.

The calculation of δx in the table (2.5) is done in the water depth $h = 10$ m, while the wave number $k$is calculated using the dispersion equation of the linear wave theory (Dean (1991)). It shows that δt and δx in Table (2.5) are more or less the same as the calculation results in Table (2.3). However, the calculation of δt and (17) was preferable considering that there is a clear interaction between δt andδx.

Table.2.5: The results of the calculation of δt and δx with (17), for $\gamma = 3.01519$.

T (sec)	δt (sec)	δx (m)	$\frac{\delta x}{L}$
6	0,00191	0,04619	0,00095
7	0,00223	0,05709	0,00095
8	0,00255	0,06766	0,00095
9	0,00286	0,07799	0,00095
10	0,00318	0,08815	0,00095
11	0,0035	0,09819	0,00095
12	0,00382	0,10812	0,00095
13	0,00414	0,11798	0,00095
14	0,00445	0,12779	0,00095
15	0,00477	0,13754	0,00095

2.3. δz calculation in the function$f(x,z,t)$

This section formulates the calculation method δz for a function $f(x,z,t)$. The calculation method is as discussed in the previous section, where δt and $\delta x$are the inputs obtained from the previous procedure on a sinusoidal function $f(x,t)$.

Taylor series for a function $f(x,z,t)$ is,

$$f(x+\delta x, z+\delta z, t+\delta t) =$$
$$f(x,z,t) + s_1 + s_2 + s_3$$

.... (19)

$s_1 = \delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x} + \delta z\frac{\partial f}{\partial z}$.... (20)

$$s_2 = \frac{\delta t^2}{2}\frac{\partial^2 f}{\partial t^2} + \delta t\delta x\frac{\partial^2 f}{\partial x\partial t} + \frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2}$$
$$+\delta t\delta z\frac{\partial^2 f}{\partial z\partial t} + \delta x\delta z\frac{\partial^2 f}{\partial x\partial z} + \frac{\delta z^2}{2}\frac{\partial^2 f}{\partial z^2}$$

.... (21)

$$s_3 = \frac{\delta t^3}{6}\frac{\partial^3 f}{\partial t^3} + \frac{\delta t^2}{2}\delta x\frac{\partial^3 f}{\partial x\partial t^2}$$
$$+\frac{\delta t^2}{2}\delta z\frac{\partial^3 f}{\partial z\partial t^2} + \delta t\frac{\delta x^2}{2}\frac{\partial^3 f}{\partial x^2\partial t}$$
$$+\delta t\frac{\delta z^2}{2}\frac{\partial^3 f}{\partial z^2\partial t} + \delta t\delta x\delta z\frac{\partial^3 f}{\partial x\partial z\partial t}$$
$$+\frac{\delta x^3}{6}\frac{\partial^3 f}{\partial x^3} + \frac{\delta z^3}{6}\frac{\partial^3 f}{\partial z^3}$$

..... (22)

δzwill be calculated using equation (9).

As a function$f(x,z,t)$, the following form of function is used

$f(x,z,t) = coshk(h+z)coskxcos\sigma t$.... (23)

At the characteristic point, the sinusoidal terms in the numerator and denominator might cancel each other out. The equation is done at$z = \eta$, where$\eta$is water surface elevation and is done in deep water where $tanhk(h+\eta) = 1$or$sinhk(h+\eta) = coshk(h+\eta)$. Thus, the hyperbolic function of the numerator and the denominators might eliminate each other.By using the wave-number conservation law (Hutahaean (2020)), the relation$tank(h+\eta)=1$remains valid in shallow waters.

After removing the sinusoidal and hyperbolic elements, the equations of s_1, s_2 and s_3 are,

$s_1 = -\delta t\sigma - \delta xk + \delta zk$... (24)

$$s_2 = -\frac{\delta t^2}{2}\sigma^2 + \delta t\delta x\sigma k - \frac{\delta x^2}{2}k^2$$
$$-\delta t\delta z\sigma k - \delta x\delta zk^2 + \frac{\delta z^2}{2}k^2$$

.....(25)

$$s_3 = \frac{\delta t^3}{6}\sigma^3 + \frac{\delta t^2}{2}\delta x\sigma^2 k - \frac{\delta t^2}{2}\delta z\sigma^2 k$$
$$+\delta t\frac{\delta x^2}{2}\sigma k^2 - \delta t\frac{\delta z^2}{2}\sigma k^2$$
$$-\delta t\frac{\delta z^2}{2}\sigma k^2 + \delta t\delta x\delta z\sigma k^2$$
$$+\frac{\delta x^3}{6}k^3 + \frac{\delta z^3}{6}k^3$$

......(26)

Substitutings_1, s_2, and$s_3$to (9) assuming that$\frac{s_2+s_3}{s_1}$is positive and using the equal sign, the equation for$\delta z$is in the form of a 3-degree polynomial.

The results of the calculation of δz where δt and $\delta x$are calculated using the procedure in sub-chapter (2.2) are presented in Table (2.6), where the calculation at a water depth of 10m using the accuracy level$\varepsilon = 0.001$.

Table.2.6: δz Calculation

T (sec)	δt (sec)	δx (m)	δz (m)	$\frac{\delta z}{\delta x}$
6	0,00191	0,04619	0,13859	3,00012
7	0,00223	0,05709	0,17127	3,00012
8	0,00255	0,06766	0,20298	3,00012
9	0,00286	0,07799	0,23398	3,00012
10	0,00318	0,08815	0,26447	3,00012
11	0,0035	0,09819	0,29457	3,00012
12	0,00382	0,10812	0,32438	3,00012
13	0,00414	0,11798	0,35396	3,00012
14	0,00445	0,12779	0,38337	3,00012
15	0,00477	0,13754	0,41264	3,00012

Something is interesting enough to note that the value of $\frac{\delta z}{\delta x} = 3.00012$isconstat for allwave periods. Thus, it can be assumed that:

$\delta z = \frac{\gamma^2\sigma}{k}\delta t = \gamma^2 C\delta t$.... (27)

Where $\gamma = 3$ can be used.

Hence, a calculation method might be formulated where δx and δzaresubstituted by δt using the relations in (14) and (27) to (24), (25), and (26).

$s_1 = (-1-\gamma+\gamma^2)\sigma\delta t$... (28)

$s_2 = \left(-\frac{1}{2}+\gamma-\frac{3}{2}\gamma^2-\gamma^3+\frac{1}{2}\gamma^4\right)\sigma^2\delta t^2$.... (29)

$s_3 = \left(\frac{1}{6}+\frac{1}{2}\gamma+\frac{7}{6}\gamma^3-\frac{1}{2}\gamma^4+\frac{1}{6}\gamma^6\right)\sigma^3\delta t^3$.... (30)

Substituting (28), (29), and (30) to (9) and assuming that the term in the absolute sign is positive, the quadratic equation of $\delta t$is obtained as follows,

$a\,\delta t^2 + b\,\delta t + c = 0$...... (31)

$a = \left(\frac{1}{6}+\frac{1}{2}\gamma+\frac{7}{6}\gamma^3-\frac{1}{2}\gamma^4+\frac{1}{6}\gamma^6\right)\sigma^2$..... (32)

$b = \left(-\frac{1}{2}+\gamma-\frac{3}{2}\gamma^2-\gamma^3+\frac{1}{2}\gamma^4\right)\sigma$.... (33)

$c = -(-1-\gamma+\gamma^2)\varepsilon$... (34)

The results of the calculation of δt with (31) for several wave periods, at a water depth of 10m with an accuracy level of$\varepsilon = 0.001$, while$\gamma = 3.00012$are presented in Table (2.7).

Table.2.7: Calculation results of δt, δx and δz with (31)

T (sec)	δt (sec)	δx (m)	δz (m)
6	0,00176	0,04261	0,12782
7	0,00205	0,05265	0,15797
8	0,00235	0,0624	0,18722
9	0,00264	0,07193	0,21581
10	0,00293	0,08131	0,24392
11	0,00323	0,09056	0,27169
12	0,00352	0,09972	0,29918
13	0,00381	0,10882	0,32647
14	0,00411	0,11786	0,35359
15	0,0044	0,12686	0,38059

There is a relatively small difference between the results in Table (2.7) and the results in Table (2.6). It can be said that with (31) simultaneous calculations onδt, δxandδz, the results of calculation with (31), in Table (2.7) will be more reliable than the results of calculations in (2.6).

It is important to note that $\frac{\delta x}{\delta t}$ and $\frac{\delta z}{\delta t}$ correlate with the velocity of the C wave phase, in equations (14) and (27), in interpreting these two parameters.

III. AIRY'S LONG WAVE EQUATION COMPLETION

In this section, Airy's Long Wave Equation or also known as the Shallow Water Equation is resolved. The equationis actually for small amplitude and longwave. To be used for short and high amplitude waves, a modification of the equation might be used. The formulation is not presented in this study considering that this study does not aim to develop a water wave model, but only to test the reliability of the time-step and grid-size produced in the previous sections.

Governing equations consists of two equations, namely

a. Water surface equation

$$\frac{\partial \eta}{\partial t} = -\frac{1}{0.5\pi}\frac{\partial Uh}{\partial x} - 0.025U\frac{\partial \eta}{\partial x} \text{.... (35)}$$

b. Momentum equation

$$\frac{\partial U}{\partial t} = -\frac{1}{9}\left(\frac{3}{2}\frac{\partial UU}{\partial x} + g\frac{\partial \eta}{\partial x}\right) \text{..... (36)}$$

η: water surface elevation

U: depth average particle velocity

h: still water depth

g: gravitation acceleration

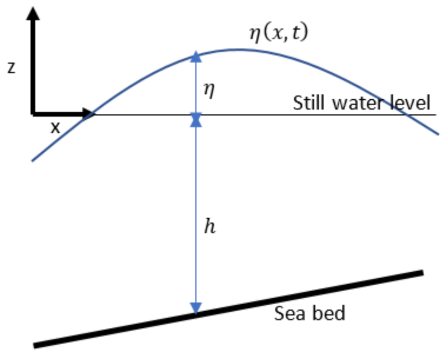

Fig.1: The axes system and variables in SWE

Both equations are solved by modifying MacCormack's predictor-corrector method (Anderson, J.D. Jr.(1994)). The original predictor-connector method is a combination of the forward difference with the backward difference in solving the time differential (Anderson, J.D. Jr.(1994)). In this study, a combination of central and backward differences was used.

The simulation results for waves with a period of 9 sec., at constant still water depth h =10 m, wave amplitude 0.8 m on execution for 10 times the wave period or 80 sec., are presented in Fig. (2). Time-step δt and grid-size δx are generated using the accuracy level of $\varepsilon = 0.001$.

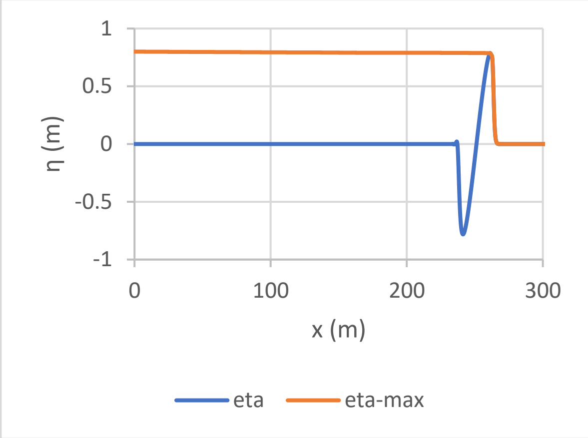

Fig.2: Model results in constant water depth, $h = 10$ m.

Fig. (2) shows that the wave curve is stable at execution for 80 sec. Thus, the wave crest elevation η_{max} was stable. This shows the stability of the model, the numerical method used, as well as the good time-step and grid-size. In case the time-step and grid-size are not correct, for example, the grid-size is too large, and then a reduction in waveamplitude or wave height might occur.

Given the wavelength of the model is shorter than the wavelength of the linear wave theory, the calculation of the grid-size δx used the following dispersion equation to obtain the appropriate grid-size:

$\sigma^2 = gk\ tanh(kh)$....... (37)

However, this study does not propose a new dispersion equation, but only the calculation of the grid-size.

Furthermore, the model is worked on the sloping bottom where the initial still water depth is 10m, while the final still water depth is 1m. The bottom slope is $\frac{9}{300}$ with a channel length of 300m. The wave period is 8 sec, with a wave amplitude of 0.8 m. The model results are presented in Fig (3).

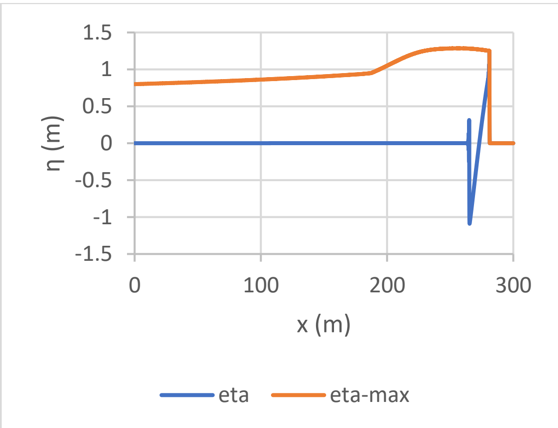

Fig.3: Results of model execution at the sloping bottom.

Fig (3) visualizes the occurrence of shoaling. Meanwhile, in the very shallow waters, there is a large increase in wave crest which possibly breaking the model. The concern, in this case, is the stability of the model in unstable wave conditions, at the time of breaking, which also shows that the time-step and grid-size used are quite good.

IV. CONCLUSION

By using the right time-step and grid-size, the Taylor series can be cut using only one derivative (order 1). Thus, the equations in FDM are exact.

Moreover, the use of the right time-size and grid-size, the solution of the governing equations of the hydrodynamic equations is under the conditions of the formulation, where generally the governing-equations are formulated by cutting the Taylor series in order 1 only.

It is important to note that the results of this study are that the grid-size onthe Taylor series in a sinusoidal function is correlated with the phase velocity instead of the particle velocity or the current velocity. This should be considered in formulating the governing equations of the wave model, especially in the formulation of the momentum equation.

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