

# A Simple Equation for Stable Coastline between Groins and Between T-Head Groins by taking into account diffracted Wave

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**Abstract**— Coastal protection planning using groin requires information on erosion and sedimentation that will occur at the protected coastal segment. An easy and practical yet accurate calculation method is needed, so that the planning cost is not too expensive.

This research developed a method of stable coastline calculation between groin, which includes erosion and sedimentation that occur during the process leading to stable coastline geometry. The governing equation is formulated using the principle of mass conservation where the volume of sedimentation is equal to the volume of erosion, whereas the boundary condition is that stable coastline is oriented perpendicular toward the wave forming it. Therefore, there are two stable coastline orientation, i.e. stable coastline formed by the incoming wave and stable coastline formed by diffracted wave. Using this method, a very simple and easy to use stable coastline equation is obtained.

The equation is formulated for a coastline with groin protector and a coastline with T-Head groin protector. A comparative study was done for the efficiency between the two structures. Stable coastline that was produced is very simple and easy to use. The result of another study stated that for the similar construction volume, T-Head groin provides a better protection.

**Keywords**— stable coastline orientation, diffracted wave.

## I. INTRODUCTION

The coastal segment located between two groins will evolve toward stable coastline geometry, where in this evolution process, erosion and sedimentation occur. The volume of erosion is determined mainly by the distance between groin and also the length of the groin. If the erosion is too large, it can cause damage to the protected coastal construction, then the protection can be stated as a failure. A large distance between groins will result in a large sedimentation at the upstream groin, so that sand bypassing might occur at the end of the groin. Should this happen, the erosion is getting larger. Therefore, the use of groin in the planning of a coastal protection requires a good estimation on the distance between groin and the appropriate length of groin. As a consequence, a model is needed to predict stable coastline geometry with erosion and sedimentation that occur.

The tangent of the stable coastline is perpendicular to the incoming wave ray. In previous researches (Hutahaean (2019a,b)) this characteristic was done on a point as a boundary condition stable coastline. In this research, a development was done, i.e. the boundary condition of the tangent of the stable coastline was done on all points.

The second development is the execution of tangent boundary condition of stable coastline at the shadow zone (Fig 1). In this area, stable coastline is formed by diffracted wave that is perpendicular to the incoming wave ray. This diffracted wave can form sedimentation at the downstream groin. Van Rijn, L.C. (2014) argues that at the downstream groin there is a sedimentation but with different causes.

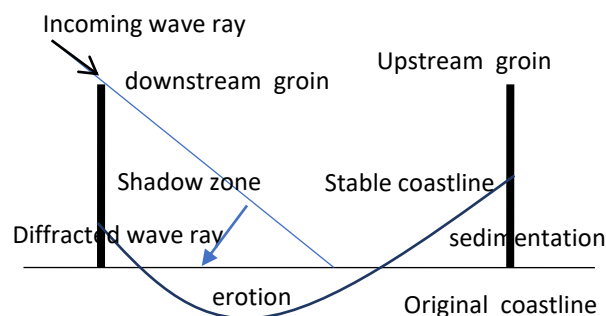


Fig.1. Erosion and sedimentation at the coastal segment located between two groins.

According to Van Rijn, L.C. (2014), as a result of the difference in wave height between the open zone and shadow zone where the wave height in the shadow zone is smaller, there will be a littoral flow toward downstream groin, so that an elliptic littoral flow is formed (Fig 2). This flow carries a littoral drift that is deposited at the downstream groin.

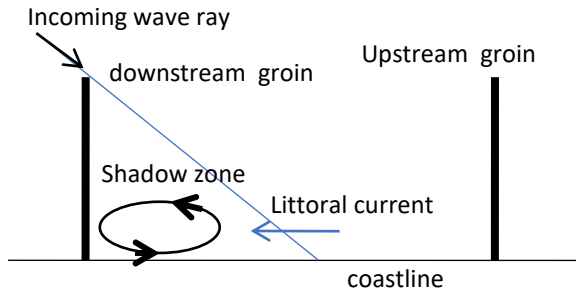


Fig 2. Elliptic littoral current at the downstream groin

## II. STUDY ON THE CHARACTERISTIC OF STABLE COASTLINE

This section will study the characteristic of stable coastline based on longshore sediment transport equation and geometry stable coastline, where the characteristic will become the boundary condition at the formulation of stable coastline equation.

### 2.1. Review on longshore sediment transport formula.

Shoreline change is mainly caused by longshore sediment transport. The equation of longshore sediment transport always uses breaking wave angles as its variable. The followings are the three longshore sediment transport formula.

a. Kamphuis' Longshore sediment transport formula, Kamphuis, J.W. (1991)

$$Q_{ls} = (C_K K_R H_b^2 T^{1.5} m_b^{0.75} D_{50}^{-0.35}) \sin^{0.6}(2\alpha_b) \dots (1)$$

$Q_{ls}$  longshore sediment transport rate,  $b$  = subscript denoting breaking condition; a complete information can be seen at, Kamphuis, J.W. (1991). The concern of this equation is the element  $\sin^{0.6}(2\alpha_b)$ , where  $\alpha_b$  = angle of breaking waves to local shoreline. In this case, if  $\alpha_b = 0$ , the tangent of crestline is parallel or equal to the tangent of the coastline, then  $Q_{ls} = 0$ .

b. Longshore sediment transport of SPM (1984)

$$Q_{ls} = \left( \frac{K_C}{16 \left( \frac{\rho_s}{\rho} - 1 \right) (1-p)} \sqrt{\frac{g H_b^{3/2}}{\gamma 2.386}} \right) \sin(2\alpha_b) \dots (2)$$

Similar to equation (1), the concern is the element  $\sin(2\alpha_b)$  where if  $\alpha_b = 0$  then  $Q_{ls} = 0$ . Complete

information on equation (2) can be seen at Shore Protection Manual (SPM), (1984).

c. Longshore sediment transport formula of Hanson, H., and Kraus, N.C. (1989)

This longshore sediment transport equation from Hanson and Kraus is used at the widely used shoreline change model, i.e. GENESIS. The form of the equation is as follows.

$$Q_{ls} = (H^2 C_g)_b \left( a_1 \sin 2\alpha_b + a_2 \cos \alpha_b \frac{\partial H}{\partial x} \right) \dots (3)$$

In the case of  $\frac{\partial H}{\partial x} = 0$  or is very small, then equation (3) becomes,

$$Q_{ls} = \left( (H^2 C_g)_b a_1 \right) \sin 2\alpha_b \dots (4)$$

In this equation (4)  $\alpha_b = 0$ , then  $Q_{ls} = 0$ . Complete information on equation (3), can be seen at Hanson, H., and Kraus, N.C. (1989).

From the three longshore sediment transport equations, it can be stated that at the stable coastline, the tangent of the coastline is parallel or equal to the tangent of the crestwave that forms the coastline. In an open area where the coast is formed by incoming wave, the tangent of the coastline is parallel with the crestline of the incoming wave, whereas at the shadow zone, stable coastline is parallel with the crestline diffracted wave.

### 2.2. Review on the form of stable coastline.

It has been known that in the nature there is geometrical form of the stable coastline in static equilibrium condition, and there are plenty of researches that have been done on the form of that stable coastline. There are several terminologies for the form of the stable coastline, Silverster, R. (1960) called it zeta bays, half-heart bay Silverster, R., Tsuchiya, Y., and Shibano, Y. (1980), crenulate shaped bays Silverster R, Hsu, J.R.C. (1993), Hsu, J.R.C., and Silverster R., Member et al (1989). The form of the stable coastline, Silverster R, Hsu, J.R.C. (1993), Hsu, J.R.C., and Silverster R., Member et al (1989) studying stable coastline between two headlands are as follows

Stable coastline consists of two parts (Fig.3), i.e. coastline directly facing the incoming wave ( $\overline{BC}$  line) and coastline facing the diffracted wave,  $\overline{AB}$  line. At the segment of the coastline facing the incoming wave, the tangent of the stable coastline is equal to the tangent of the crestline of the incoming wave, whereas at the shadow zone facing the diffracted wave, the tangent of the coastline is parallel with the tangent of the crestline of the diffracted wave.

From the review of the longshore sediment transport equation and the geometry of stable coastline, it can be

concluded that stable coastline has a tangent that is parallel with the crestline forming it.

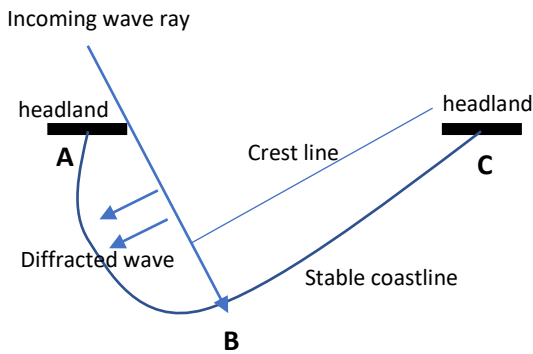


Fig.3. Stable coastline between headlands, Hsu, J.R.C., and Silverster R., Member et.al (1989)

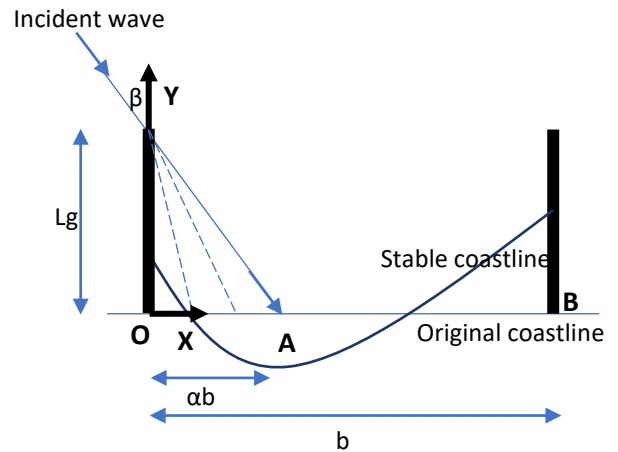


Fig.4. Stable coastline between two groins

For the coastline directly facing the incoming wave, the tangent stable coastline is equal to the tangent of the crestline of the incoming wave, whereas the coastline formed by the diffracted wave will have a tangent that is equal to the tangent of diffracted wave crestline. This condition will be used as boundary condition at the formulation of stable coastline equation.

### III. GOVERNING EQUATION AND BOUNDARY CONDITIONS

#### 3.1. Governing Equation

As is the case in Hutahaean (2019a,b), the stable coastline equation is approximated with quadratic polynomial equation,

$$y(x) = c_0 + c_1x + c_2x^2 \dots\dots(5)$$

$y(x)$  is coastline stable ordinate,  $x$  is an abscissa, where the  $x$  axis coincides with the original coastline (Fig.4).  $c_0$ ,  $c_1$  and  $c_2$  is polynomial coefficient, of which the value should be determined.

With an assumption that there is no sediment that goes in and out of the calculation area, then the volume of sedimentation should be equal to the volume of erosion. By ignoring the porosity of the sand, then the mass conservation equation can be stated as follows.

$$\int_0^b y(x) dx = 0 \dots\dots(6)$$

where  $b$  is the distance between groin (Fig.4). By completing the integration, the governing equation of the stable coastline is obtained, i.e.

$$c_0 + \frac{1}{2}c_1b + \frac{1}{3}c_2b^2 = 0 \dots\dots(7)$$

#### 3.2. Boundary Conditions.

The calculation area consists of two parts, i.e. area directly facing incident wave, i.e. the  $\overline{AB}$  line and shadow zone area directly facing the diffracted wave, i.e. along the  $\overline{OA}$  line where the two areas have different stable coastline characteristic. Therefore, based on the wave, there are two boundary conditions.

##### a. Boundary condition in line $\overline{AB}$

In general, the characteristic of this stable coastline is perpendicular to the incident wave. However, considering that stable coastline is a curve as stated in (5), then the characteristic is not appropriate to be applied only on just one point as in Hutahaean (2019a,b) where the characteristic is represented on a point, i.e.  $\frac{dy}{dx} = \tan\beta$  or  $\frac{dy}{dx} - \tan\beta = 0$ ,  $\beta$  is the incoming wave angle (Fig.4). In this research, the boundary condition of the tangent in the stable coastline is done in all point, i.e. at the part of coast facing the existing incoming wave applies,

$$\int_{x_A}^{x_B} \left( \frac{dy}{dx} - \tan\beta \right) dx = 0 \dots\dots(8)$$

where  $x_A = b - L_g \tan\beta$  and  $x_B = b$ .

##### b. Boundary Condition in line $\overline{OA}$

Boundary condition in this area is equal to the one in area  $\overline{AB}$ , i.e. that stable coastline is perpendicular to the incoming wave; in this area the incoming wave is the diffracted wave that is perpendicular to the original wave i.e.  $(\beta + \frac{\pi}{2})$ , (Fig.3). Whereas the form of boundary condition equation in this area is,

$$\int_0^{x_A} \left( \frac{dy}{dx} - c_d \tan \left( \beta + \frac{\pi}{2} \right) \right) dx = 0 \dots\dots(9)$$

Where,  $c_d$  is a diffraction coefficient. Bearing in mind that the values of diffraction coefficient varies along the  $\overline{OA}$  line then the area is divided into a number of line segments (Fig.4), hence (9) becomes,

$$\sum_{i=1}^n \int_{x_i}^{x_{i+1}} \left( \frac{dy}{dx} - c_{di} \tan \left( \beta + \frac{\pi}{2} \right) \right) dx = 0 \quad \dots(10)$$

$c_{di}$  is a diffraction coefficient at the center of a segment  $i$ , whereas  $n$  is the number of segment. Therefore, there are three linear simultaneous equations, i.e. Eq.(7),(8), and (10), with three unknowns i.e.  $c_0$ ,  $c_1$  and  $c_2$ . The simultaneous linear equation system can be completed with Gauss elimination system or similar type of method.

Equation (10) can be simplified by using (9), where the average diffraction coefficient is used as the diffraction equation, i.e.

$$c_d = \frac{1}{n} \sum_{i=1}^n c_{di} \quad \dots\dots(11)$$

Where  $c_{di}$  is the diffraction coefficient at segment  $i$ . Using this method will result in polynomial coefficient at (5) i.e.

$$c_2 = \frac{1}{b} \left( \tan(\beta) - c_d \tan \left( \beta + \frac{\pi}{2} \right) \right) \quad \dots\dots(12)$$

$$c_1 = -(1 + \alpha) b c_2 + \tan(\beta) \quad \dots\dots(13)$$

$$c_0 = -\frac{c_1}{2} b - \frac{c_2}{3} b^2 \quad \dots\dots(14)$$

Where,

$$\alpha = \frac{L_g \tan \beta}{b} \quad \dots\dots(15)$$

3.3. T-Head Groin.

The governing equation and boundary conditions for T-head groin are equal to the governing equation and boundary conditions at the groin, only the point A shifted as far as half the length of the head, i.e.  $\frac{t}{2}$  (Fig. 5), where  $x_A = \frac{t}{2} + L_{tr} \tan \beta$ ,  $L_{tr}$  is the length of trunk (Fig.5). The form of the equation coefficient is equal to (12),(13) and (14), where for T-head groin

$$\alpha = \frac{L_{tr} \tan \beta + \frac{t}{2}}{b} \quad \dots\dots(16)$$

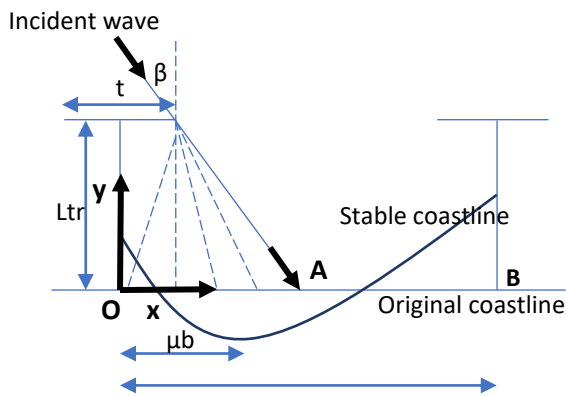


Fig.5. Stable coastline between T-Head groin.

3.4. Diffraction Coefficients

According to Kamphuis, J.W. (1992) diffraction coefficient is, for a point A located at the coastline, where the line  $\overline{BA}$  formed an angle  $\delta$  against the incident wave (Fig.6), then the diffraction coefficient is  $K_d = 0.7 - 0.0077\delta$ , this equation is for a positive value of  $\delta$ , where a positive  $\delta$  is spinning clockwise. Whereas a negative  $\delta$  spins counter clockwise.

$$K_d = 0.7 - 0.0077\delta \quad \text{for } 0^\circ \leq \delta \leq 90^\circ$$

$$K_d = 0.7 - 0.37 \sin \delta \quad \text{for } 0^\circ \leq \delta \leq -40^\circ$$

$$K_d = 0.83 - 0.17 \sin \delta \quad \text{for } -40^\circ \leq \delta \leq -90^\circ$$

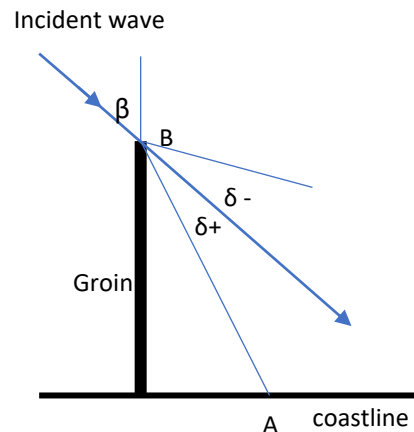


Fig.6. Wave diffraction at a groin

IV. THE RESULT OF THE MODEL

All calculations in this section is done using polynomial coefficients (12), (13) and (14).

4.1. Erosion and Sedimentation at The Coastal Segment B between Groins.

This section studies the erosion and sedimentation at the coastal segment between groins in the process of stable coastline formation. The study was done with several scenarios, i.e. varied wave angel, varied groin length and the distance between groins.

4.1.1. The Study on the Influence of Wave Angel  $\beta$

In this section, the model is done in the distance between groin  $b = 100$  m, the length of groin  $L_g = 40$  m with wave incident angel  $\beta = 15^\circ$  and  $\beta = 30^\circ$ .

Fig.7. shows that the smaller the wave incident angel, the bigger the erosion-sedimentation will occur. Table 1 presents stable coastline condition for several incident wave angel, at a distance between groin  $b = 100$  m, and length of groin  $L_g = 40$  m.

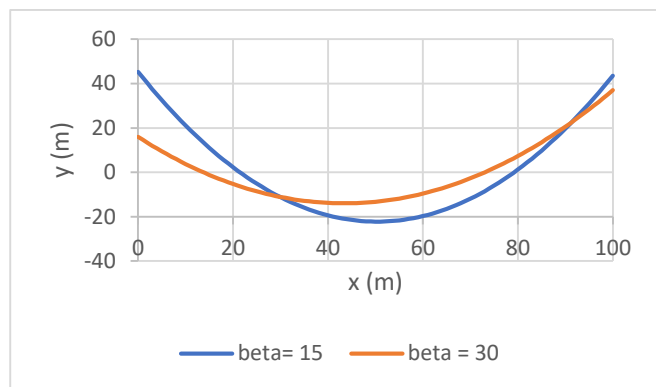


Fig.7. Stable coastline for  $\beta = 15^\circ$  and  $\beta = 30^\circ$ .

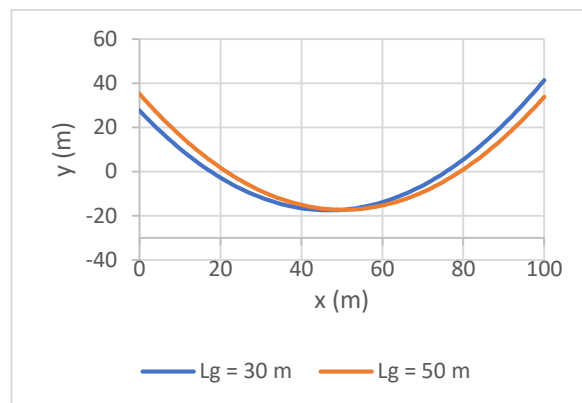


Fig.8. The comparison of stable coastline with different length of groin.

Table (1) shows that the bigger the  $\beta$  the smaller the sedimentation at the downstream groin ( $y_{ds}$ ) and upstream groin ( $y_{us}$ ) as well. Whereas the volume of erosion  $y_{min}$  was initially getting smaller with the increase of  $\beta$  but then it was increasing. Sedimentation at the downstream groin decreases with the increase in wave angel  $\beta$ . Sedimentation at the upstream groin was initially getting smaller with the increase  $\beta$  but then it increases. The position of  $y_{min}$  i.e.  $x_{min}$  is getting closer to the downstream groin with the increase of  $\beta$ . It can be concluded that the dangerous incident wave angel at the coastal segment between groins is a small incident wave angel, where at the coastal water, the angel is getting smaller.

Table 1 Erosion and sedimentation in several values of  $\beta$ .

$\beta$ ( $^\circ$ )	$y_{ds}$ (m)	$y_{min}$ (m)	$y_{us}$ (m)	$x_{min}$ (m)
10	70,5	-32,8	60,43	51,28
15	45,3	-22,21	43,53	50,33
20	31,51	-17,34	37,68	48,51
25	22,46	-14,95	36,24	46,09
30	15,98	-13,94	37,01	43,39

4.1.2. Study on The Influence of The Length of Groin  $L_g$ .

This section will study the influence of the length of groin at the erosion-sedimentation at the coastal segment between groins, with wave angel  $\beta = 20^\circ$  and the distance between groins  $b = 100$  m.

Fig.8 presented erosion and sedimentation that occurred for lengths of groin  $L_g = 30$  m and  $L_g = 50$  m. There is indeed the influence of the length of groin, i.e. the longer the length of groin, the smaller the erosion will be, but with a very small reduction. For a clearer description see Table 2.

The calculation in Table 2 was done with  $\beta = 20^\circ$  and  $b = 100$  m for several values of  $L_g$ , which shows that the longer the length of groin, the sedimentation at the downstream groin ( $y_{ds}$ ) gets bigger, the biggest erosion ( $y_{min}$ ) gets smaller but with a relatively small reduction, with the position ( $x_{min}$ ) that gets closer to upstream groin, whereas the sedimentation at the upstream groin is decreasing. From this phenomenon, a conclusion can be made that the addition of the length of groin is not effective in reducing erosion. It is only needed to prevent sand bypassing at the end of the groin from happening.

Table 2 The influence of the length of groin  $L_g$  on sedimentation and erosion.

$L_g$ (m)	$y_{ds}$ (m)	$y_{min}$ (m)	$y_{us}$ (m)	$x_{min}$ (m)
25	25,84	-17,67	43,35	45,78
30	27,73	-17,52	41,46	46,69
35	29,62	-17,42	39,57	47,6
40	31,51	-17,34	37,68	48,51
45	33,39	-17,3	35,79	49,42
50	35,28	-17,3	33,91	50,33

4.2. Erosion and Sedimentation at the Coastal Segment between T-Head Groins.

This section will study erosion and sedimentation at the coastal segment between T-Head groins at the stable coastline formation. In general, the phenomenon exists in the groin also exists at the T-head groin, i.e. among others at the variation of angel  $\beta$ , the length groin and distance between groins. What will be learned at this T-head groin is the influence of the length of head T and the comparison between groin and T-head groin. The picture of T-head groin can be seen in Fig. 9.

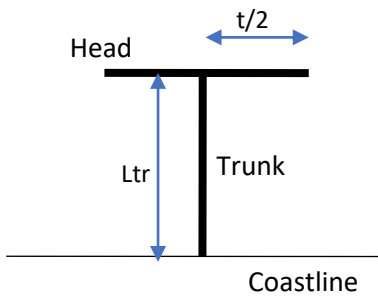


Fig. 9. T-head groin

4.2.1. Study on The Influence of The Length of Head  $\frac{t}{2}$

The model was done at the distance between groin  $b = 100$  m, the length of trunk  $L_{tr} = 25$  m, the length of half the head  $\frac{t}{2}$  varies, with wave incident angel =  $20^0$ .

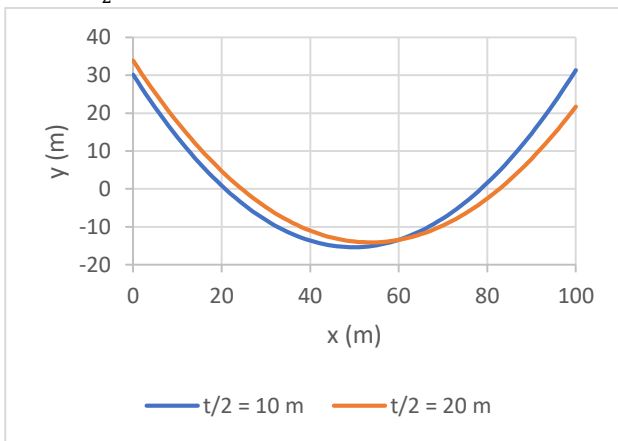


Fig.10. Study on the influence of the length of T-head

As can be seen in Fig.10 that the length of trunk reduces erosion and sedimentation at the upstream groin but increases sedimentation at the downstream groin. The complete result can be seen in Table 3.

Table 3 Erosion and Sedimentation at several lengths of head  $\frac{t}{2}$ .

$\frac{t}{2}$ (m)	$y_{ds}$ (m)	$y_{min}$ (m)	$y_{us}$ (m)	$x_{min}$ (m)
10	30,172	-15,377	31,33	49,686
12	30,911	-15,041	29,236	50,464
14	31,639	-14,746	27,237	51,246
16	32,363	-14,493	25,323	52,034
18	33,092	-14,281	23,49	52,829
20	33,83	-14,109	21,727	53,631

4.2.2. Comparison Between Groin and T-head Groin  
The comparison of construction efficiency was done at the similar construction length, i.e. if the length of groin  $L_g$

and the length of total T-head groin  $L_{total} = L_{tr} + \frac{t}{2}$ , then the comparison was done at  $L_g = L_{total}$ . Fig.11 presents the comparison between erosion and sedimentation between the protection and groin with  $L_g = 40$  m, where the protection with T-head groin,  $L_{tr} = 30$  m,  $\frac{t}{2} = 10$  m. Wave angel  $\beta = 20^0$ , the length of coastal segment  $b = 100$  m. The result of the calculation is presented in Fig.11, where it shows that erosion and sedimentation at T-Head groin is smaller than the erosion and sedimentation at groin.

Furthermore, erosion-sedimentation at groin are compared with erosion-sedimentation at the T-head groin with various length of  $L_g$ , the length trunk is fixed at  $L_{tr} = 30$  m, the length of head  $\frac{t}{2}$  changes where  $L_g = L_{tr} + \frac{t}{2}$ . The result of the calculation is presented in Table 4 which shows that for similar length of construction, T-head groin is more effective than groin.

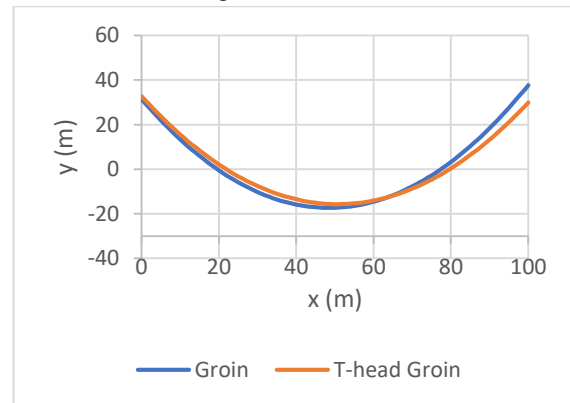


Fig. 11. The comparison of erosion-sedimentation at groin and T-Head Groin

Table 4 Comparison between Groin and T-Head Groin

$L_g$ (m)	Groin			T-head groin		
	$y_{ds}$ (m)	$y_{min}$ (m)	$y_{us}$ (m)	$y_{ds}$ (m)	$y_{min}$ (m)	$y_{us}$ (m)
35	29,62	17,42	39,57	30,45	-16,49	35,4
40	31,51	17,34	37,68	32,82	-15,68	29,87
45	33,39	-17,3	35,79	34,97	-15,1	24,85
50	35,28	-17,3	33,91	37,04	-14,74	20,27

V. CONCLUSION

Stable coastline equation that was obtained in this research is very simple and taking into account diffracted wave. Qualitatively, the model provides a clear description on erosion and sedimentation at coastal segment between

groins, but quantitatively it needs to be studied using physical model data or measurement data.

Study on the comparison between groin and T-head groin obtains that T-head groin provides a better protection than groin.

In this research, sand porosity has not been taken into account where the volume of sand will expand if the sand is inundated with water. The next research that will be done should take into account sand porosity. The inclusion of sand porosity factor can be done in the mass conservation equation.

Another development that can be done is developing numerical model using either finite difference method or finite element method. By using this method, diversity factor can be easily included, such as diffraction coefficient.

### REFERENCES

- [1] Hutahaean, S. (2018a). Stable Coastline Between Two Groins Equation. International Journal of Advanced Engineering Research and Science (IJAERS), [vol-5, Issue-8, Aug-2018]. ISSN:2349-6496P/ 2456-1908(O). <https://dx.doi.org/10.22161/ijaers.5.8.28>
- [2] Hutahaean, S. (2018b). Comparative Study between Groin and T-Head Groin. International Journal of Advanced Engineering Research and Science (IJAERS), [vol-5, Issue-11, Nov-2018]. ISSN:2349-6496P/ 2456-1908(O). <https://dx.doi.org/10.22161/ijaers.5.11.1>
- [3] Van Rijn, L.C. (2014). A simple general expression for longshore transport of sand and gravel. Coastal Engineering Vol.90, 23-29.
- [4] Kamphuis, J.W. (1991). Alongshore Sediment Transport Rate. J. of Waterway, Port, Coastal, and Ocean Eng., ASCE, Vol. 117, pp 624-660.
- [5] Shore Protection Manual (SPM), (1984), U.S. Government Printing Office, Washington D.C.
- [6] Hanson, H., and Kraus, N.C. (1989), GENESIS: Generalized Model for Simulating Shoreline Change, CERC Rep 89-19, Coast.Engnr. Res.Ctr., U.S. Army Engr. Waterway Experiment Station, Vicksburg, Miss., 185 pp.
- [7] Silverster, R. (1960). Stabilization of sedimentary coastlines. [J], Nature 467-469.
- [8] Silverster, R., Tsuchiya, Y., and Shibano, Y. (1980). Zeta Bays, pocket beaches and headland control. Proceedings, 17th. International Conference on Coastal Engineering. American Society of Civil Engineers, Sidney, Australia.
- [9] Silverster R, Hsu, J.R.C. (1993). Coastal Stabilization, Innovative Concepts. Prentice-Hall Inc., Englewood Cliffs, NJ.
- [10] Hsu, J.R.C., and Silverster R., Member et.al (1989) Static Equilibrium bays : New relationships. Journal of Waterways, Port, Coastal and Ocean Engineering, 285-289.
- [11] Kamphuis, J.W. (1992). Short course on design in reliability of coastal structure. Chapter 9, Proc. 23<sup>rd</sup> ICCE 1992, Venice Italy.