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# Quadratic Bounded Knapsack Problem Solving with Particle Swarm Optimization and Golden Eagle Optimization

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***Abstract*—** *Optimization problems are the most interesting problems to discuss in mathematics. Optimization is used to modeling problems in various field to achieve the effectiveness and efficiency of the desired target. One of the optimization problems that are often encountered in everyday life is the selection and packaging of items with limited media or knapsack to get maximum profit. This problem is well-known as knapsack problem. There are various types of knapsack problems, one of them is quadratic bounded knapsack problem. In this paper, the authors proposed a old and new algorithm, which is Particle Swarm Optimization (PSO) and Golden Eagle Optimization (GEO). Furthermore, the implementation of the proposed algorithm, PSO is compared to the GEO. Based on the results of this study, PSO algorithm performs better and produces the best solution than the GEO algorithm on all data used. The advantage obtained by the PSO algorithm is better and in accordance with the knapsack capacity. In addition, although the convergent iteration of the PSO takes longer time than GEO with the same number of iterations, GEO is able to find better solutions faster and able to escape from the local optimum. However, the computation time required by the PSO algorithm is faster than the GEO algorithm.*

## I. INTRODUCTION

Mathematics is one part of science that has an important role in the world of technology and companies. The rapidity of development, along with technological advances, increases the competition between industries, so companies are required to maximize performance in various fields. One of those fields is optimization problems that are often encountered in everyday life. Companies often experience some difficulties related to packaging of goods with limited media, or known as knapsack, to transport all goods even though the number of storage media is more than one.

The knapsack problem is about how to choose goods from many choices where each item has its own weight and advantages, taking into account the capacity of the storage media, so that from the selection of these goods the maximum profit is obtained. The knapsack problem consists of several problems, including binary knapsack, bounded knapsack, and unbounded knapsack. The division is based on the pattern of storage of goods with various values and weights. The Binary knapsack problem, or knapsack 0-1, is a knapsack problem where the items that are inserted into the storage media must be included all (1) or not at all (0). The bounded knapsack problem is a knapsack problem where each item is available as *n* units and the number of items inserted into the storage media is limited. It can be included in part or in full. The unbounded knapsack problem is a knapsack problem where each item is available in more than one unit and the number of items inserted into the storage media is unlimited [5].

Metaheuristic algorithms that have been used in research on optimization problems are as follows. The Particle Swarm Optimization (PSO) algorithm was first introduced, and their research based on the behavior of a flock of birds or fish in nature. PSO algorithms have been widely applied to almost every area of optimization, computational intelligence and scheduling design applications [3]. Another research is a metaheursitic algorithm approach to solving non-linear equation systems containig complex roots. From the results of the research, the PSO algorithm is considered to have the best accuracy results compared to the Firely Algorithm and the Cuckoo Search algorithm because the value of its function is geting closer to zero [4]

Another metaheursitc algorithm that has been used is the Golden Eagle Optimization (GEO). The GEO algorithm was first introduced in his nature-inspired research to solve global optimization problems. In this study, the GEO algorithm was tested for its performance and efficiency using 33 problems from different classes. Furthermore, the performance results are compared with six other weel-known metaheuristic algorithms throgh different statistical measures. It is proven that GEO can find global optimal and avoid local optima effectively, thas is through intense movement by utilizing the best solution found during iteration [6].

Based on the basic problems that exist in the knapsack, there are several variations of the knapsack problem, which are multi-objective knapsack, multiple constraint knapsack, multiple knapsack, and quadratic knapsack. The multi-objective knapsack problem is a knapsack problem that has more than one objective function to maximize profits. The multiple constraint knapsack problem is a knapsack problem that has more than one constraint to maximize its profits. The multiple knapsack problem has more than one storage medium in which all items must be packed to maximize profits. The last, the quadratic knapsack problem, is a knapsack problem that aims to maximize the objective function in quadratic form for binary and linear capacity constraints [2].

Optimization problems, including knapsack problems, can be solved using several methods or algorithms. One of the algorithms that is often used is the metaheuristic algorithm. Many studies use this algorithm because it is an efficient way to produce a solution. Metaheuristic algorithms are algorithms created to solve optimization problems through approaches that are inspired by nature, such as biology, physics, or animal behavior [1].

Based on the description above, the writer is interested in researching a new problem, the quadratic bounded knapsack with multiple constraints. This problem arises when the objective function is obtained in the form of a quadratic with more than one constraint function and the minimum and maximum limits are known. These problems are adapted to everyday life; for example, the price of goods can change at any time. Research will be carried out using data in the form of simulation data. The data created will be adjusted based on the circumstances real and in accordance with the research problem, namely quadratic bounded knapsack with multiple constraints. In this study, the use of simulation data is intended to be able to represent data types that are more varied and universal.

Furthermore, the interesting thing that will be discussed in this research is how the application of PSO and GEO algorithms in solving quadratic problems bounded knapsack. Researchers would compare the results of the solutions given by the two algorithms to the problem. The purpose of this research is to analyze the application and review the comparison of the PSO and GEO algorithms for solving quadratic bounded knapsack problems.

## II. PROBLEM AND ALGORITHM

### 3.1 Quadratic Bounded Knapsack

The Quadratic bounded knapsack problem with multiple constraints is a variation problem based on the parameters where there is a quantity of goods available of each type and there is more than one constraint. The purpose of the quadratic problem bounded knapsack with multiple constraints is to select a subset of units that have a weight that overall does not exceed the given knapsack capacity (C) so that it can be determined the amount of each type of good by obtaining the total profit maximum and meeting all constraints. The obstacles to this problem are: storage media capacity coverage in the form of weight and space, as well as cost or capital provided. An example of this problem is that it is assumed that each type of good has a minimum or maximum quantity availability limit that must be bought. The limitation has the aim of ensuring the minimum number of items to get maximum profit and do not exceed load capacity or cost.

Some explanations regarding the quadratic bounded knapsack with multiple constraints Among other things, each type of good has a number of goods available $(m_j)$. Advantages of goods are calculated or obtained by multiplying the number of selected types of goods $(y_j)$ by the unit profit $(p_{jj})$. There is an additional profit for each pair of item types $i$ and $j$ $i < j$ If the number of selected goods types and types of goods are both greater than zero (0), and there are three constraints that must be met, namely weight, volume, and capital.

Based on the description above, the quadratic bounded problem knapsack with multiple constraints can be defined as follows:

Purpose function:

$$\text{Maximize } Z = \sum_{j=1}^{n} y_j p_{jj} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} t_i t_j p_{ij} \quad (1)$$

Constraint:

$$\sum_{j=1}^{n} y_j w_j \leq C \quad (2)$$

$$\sum_{j=1}^{n} y_j v_j \leq S \quad (3)$$

$$\sum_{j=1}^{n} y_j b_j \leq M \quad (4)$$

$$y_j \in \{0,1,\ldots,m_j\}, j = 1,2,\ldots,n \quad (5)$$

$$t_i \,\&\, t_j = \begin{cases} 0, & \text{if } y_j = 0 \\ 1, & \text{if other} \end{cases} \quad (6)$$

The optimum value of the objective function or total profit $(Z)$, the number of types of goods $(n)$, profit or profit of goods type $i$ and $j$ $(p_{ij})$. The decision variable is the number of goods type j which is inserted into storage media means if 1 if selected or 0 if not selected $(y_{i,}y_j)$the decision variable is the number of items of type $i$,$j$ that is entered into storage media means if 1 if selected and gets additional profit $(t_{i,}t_j)$, weight or the weight of the type of goods $j$ $(w_j)$, volume of goods type $j$ with negligible dimensions of goods $(v_j)$, purchase price of goods type $j$ $(b_j)$, the amount of availability of goods type $j$ $(m_j)$, weight capacity of storage media kg unit ($C$), storage media space capacity unit $cm^3$ ($S$), and modal ($M$).

### 3.2 Particle Swarm Optimization (PSO)

PSO algorithm there are stochastically generated particles in the search space. Each particle is a candidate solution for the problem whic is represented by position, velocity and has a memory to help it remember the previous best position. The PSO algorithm consists of $N$ particles. Each particle swarm has a type of topology that is used to identify several other particles to affect each individual so that it can describe the relationship between particles. Topologies that are often used include global best $(gbest)$ and local best $(lbest)$. Global best is the best posiition of the entire population used for fast search, while local best is the best position of each particle used for slow search [3].

In summary, the steps of the PSO algorithm are presented in the Flowchart in Figure 1 below.

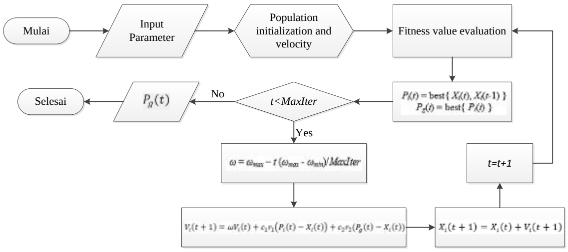


*Fig. 1: Scheme of PSO algorithm*

### 3.3 Golden Eagle Optimization (GEO)

The golden eagle is a bird of prey belonging to the Accipitridae family. Golden eagles are professional hunters who can catch prey of all sizes from insects to medium-sized mammals. This bird can fly at a speed of 190 $km/h$, with excellent eyesight and very strong claws [6].

In summary, the procedure of the Golden Eagle Optimization algorithm is presented in Algorithm 1:

Algorithm 1: Procedure GEO

Initialization population

Fitness value evaluation

Initialization of $p_a$ and $p_c$

**for** every iteration

update $p_a$ and $p_c$

**for** every golden eagle

randomly select prey from population memory

calculate exploitation vector $\vec{A}$

**if** exploitation vector length is not equal to zero

calculate exploration vector $\vec{C}$

calculate exploration vector $\Delta x$

update position

evaluate the fitness value of the new position

**if** fitness is better than the $i$-th eagle memory

update the memory of the $i$-eagle with its newest position

**end**

**end**

**end**

**end**

## III. METHODOLOGY

The researcher used an experimental type of research. This study used simulation data consisting of data on a number of goods, vehicle data, and capital. For the type of product, the goods data used are the product name, minimum and maximum limits, weight, volume, purchase price, selling price, and profit.

The simulation data generation used was then generated using software. A random data generation program was used to generate simulation data, including the minimum limit for the number of goods ($lb_j$), the maximum limit for the number of goods ($ub_j$), weight, volume, purchase price, and selling price of goods. The size of the data used was 100 types of goods. This types of goods consisting of several data, which were the number of goods, weight capacity, volume capacity, and capital. The data generation program was written in a script with several rules to adjust the value of each type of data and identify the data according to the quadratic bounded knapsack problem.

The problem in this research will be solved using several steps. The steps used to solve the problem can be seen in Figure 2 below.

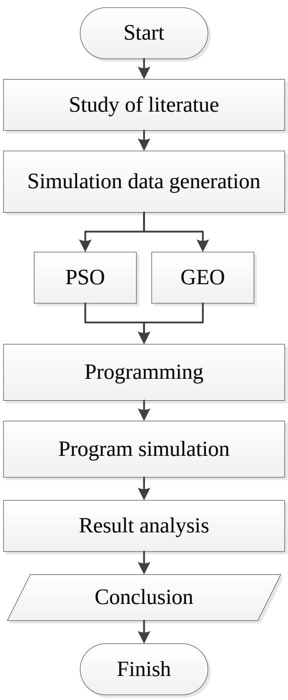


*Fig. 2: Scheme of research method*

The problem that became the object of this research would be processed using a metaheuristic algorithm that included the Pasrticle Swarm Optimization (PSO) and Golden Eagle Optimization (GEO). The form of the solution for each metaheuristic algorithm that must be carried out was as follows.

1. Entering research data included the amount of availability of goods ($m_j$), weight of goods ($w_j$), volume of goods ($v_j$), purchase price of goods ($b_j$) and profit matrix ($p$), as well as determining the limit of constraints, which include knapsack weight capacity ($C$), capacity knapsack space ($S$) and modal ($M$).
2. Determining the parameter values, which included: population size ($N_{pop}$), number of iterations ($I_{max}$), control coefficient to set the local and globak best position influence ($c_1, c_2$), inertial weights govern the effect of particle velocity iterations before ($\omega_{max}, \omega_{min}$).
3. As a candidate solution, a randomly generated initial value vector ($x_i; 1 \le i \le N_{pop}$) was generated at the interval [0, 1]. $x_i = [x_{i1}, x_{i2}, \dots, x_{iD}]$ where D is the number of items
4. Changed the value ($x$) to the vector form of the decision variable ($y$) as the number of selected items. For the bounded knapsack problem, the conversion of vector to vector can be done using the following equation (7).
$$y_{i,j} = round(x_{i,j} * m_j), \quad j = 1,2,\dots,D \qquad (7)$$
5. Check the constraints of each candidate solution. The solutions of each knapsack must satisfy the following constraints:
$$\sum_{j=1}^{n} w_j y_j \le C, \qquad (8)$$
$$\sum_{j=1}^{n} v_j y_j \le S, \qquad (9)$$
$$\sum_{j=1}^{n} b_j y_j \le M, \qquad (10)$$
$$y_j \in \{0,1,\dots,m_j\}, j = 1,2,\dots,n \qquad (11)$$
If from these checks it is found that there are candidate solutions that do not meet the constraints, then the candidate solutions must be penalized using the following equation (12)
$$x_{ik} = \left| x_{ik} - \frac{1}{m_k} \right| \qquad (12)$$
Where $i$ was the index of candidate solutions that did not meet the constraints, and $k$ was the index of the type of good that must be reduced (chosen randomly). The penalty step must be repeated until the candidate solution satisfies the constraint.
6. Calculated total profit (objective function). The profit value of each solution was calculated based on the total profit.
$$Z = \sum_{j=1}^{n} y_j p_{jj} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} t_i t_j p_{ij} \qquad (13)$$
where $t_i, t_j$ was zero (0) if no item $i, j$ was selected, and one (1) if any item $i, j$ was selected.

7. Apply PSO and GEO algorithm. Solution representation and evaluation steps are carried out as the PSO and GEO algorithms described above.

After the application of the algorithm was completed, it was continued with the creation and simulation of the program. The experiment was run ten times because the metaheuristic algorithm contains random or stochastic values that allow the algorithm solution to vary. The parameter test of the PSO and GEO algorithms consists of six parameters, namely population size ($Npop$), control coefficient to set the local and globak best position influence ($c_1, c_2$), inertial weights govern the effect of particle velocity iterations before ($\omega_{max}, \omega_{min}$), and maximum iteration ($Imax$). Next, the program was tested to complete the entire research data. The next step was to analyze the results and draw conclusions.

## IV. RESULT AND DISCUSSION

In this section, we will describe the results, the application of the quadratic bounded knapsack problem using simulation data, and the discussion. In the discussion section, it will be explained the influence of parameters, the comparison of PSO and GEO algorithms, and the best results from these algorithms. The solution was carried out using the help of MATLAB software, which was run on a laptop with an Intel (R) Core (TM) i7-4510U @ 2.00GHz CPU, 4GB RAM, and a 64-bit OS. The results of the research on the PSO and GEO algorithms that has been carried out are as follows.

### 3.1 Tested Parameters

The program for implementing the PSO and GEO algorithms that have been developed was tested on the data that has been collected. In this study, experiments were carried out according to the data taking as many as 100 kinds of goods with the provisions of the weight capacity is 8100 kg, volume capacity is 12100000 $cm^3$, and capital of Rp 88.700.000,00. Each parameter value was tested with a population of 100 and a maximum iteration of 1000.

In the parameter test ($c_1$ and $c_2$), the value ($\omega_{max}$ and $\omega_{min}$) used was 0.9 and 0.1. the simulation program was run ten times for each parameter value used. The best result obained from the parameter test ($c_1$ and $c_2$) was both 1. In the parameter test ($\omega_{max}$ and $\omega_{min}$), the value ($c_1$ and $c_2$) used was 1 and 1. the simulation program was run ten times for each parameter value used. The best result obained from the parameter test ($\omega_{max}$ and $\omega_{min}$) was 0.9 and 0.5.

### 3.2 Final Simulation

After the parameter test was completed, a final simulation was carried out to test the PSO and GEO algorithms in solving 100 types of items quadratic bounded knapsack problem. The value of the parameters used in this final simulation was based on the results of the parameter test, namely the value that was able to produce or improve a better solution. The parameter values include: $N = 100; I_{max} = 1000; c_1 = 1; c_2 = 1; \omega_{max} = 0.9$ and $\omega_{min} = 0.5$. The final simulation results obtained from the best parameter test for each algorithm are presented in Table 1.

*Table.1: The best final profit simulations for PSO and GEO*

| No | PSO | | GEO | |
|---|---|---|---|---|
| | Best profit | Average | Best Profit | Average |
| 1 | 18997000 | 18801200 | 17640000 | 1777800 |
| 2 | 19051000 | | 17881000 | |
| 3 | 18009000 | | 17763000 | |
| 4 | 18916000 | | 17859000 | |
| 5 | 18964000 | | 17630000 | |
| 6 | 18646000 | | 17780000 | |
| 7 | 18644000 | | 17806000 | |
| 8 | 18898000 | | 17894000 | |
| 9 | 18921000 | | 17810000 | |
| 10 | 18966000 | | 17717000 | |

Furthermore, from the final simulation results, the average iteration of non-improved solutions was obtained, and the average computation time (execution time) of the program was run ten times. The results of the convergent iteration averages and computational times for the PSO algorithm and the GEO algorithm are presented in Table 2.

*Table.1: The simulation of the final convergent iteration and the computing time of the PSO and GEO*

| No | PSO | | GEO | |
|---|---|---|---|---|
| | Convergent Iteration | Computing Time | Convergent Iteration | Computing Time |
| 1 | 971 | 56,1118 | 987 | 194,3299 |
| 2 | 815 | 49,4042 | 977 | 154,8863 |
| 3 | 986 | 24,0281 | 989 | 112,1724 |
| 4 | 995 | 25,7097 | 975 | 121,0958 |
| 5 | 867 | 27,6347 | 990 | 118,9619 |
| 6 | 538 | 24,9073 | 982 | 112,7621 |
| 7 | 1000 | 34,2521 | 923 | 120,5419 |
| 8 | 999 | 25,3172 | 986 | 115,1079 |
| 9 | 983 | 26,4481 | 987 | 122,2256 |
| 10 | 973 | 25,3768 | 929 | 117,1695 |

Based on the results of the final simulation (Table 1) that has been carried out, the best profit was calculated from 100 types of goods that has been determined using the best parameters that have been tested. It can be seen that the existing PSO algoritm has provided a better solution than the new algorithm, whis is GEO. The difference in profits between the PSO and GEO algorithms, it can be seen that the difference in the average in the average profit is RP 1.110.400,00. The comparison of the advantages obtained by the PSO and GEO algorithms can be seen in Figure 2.

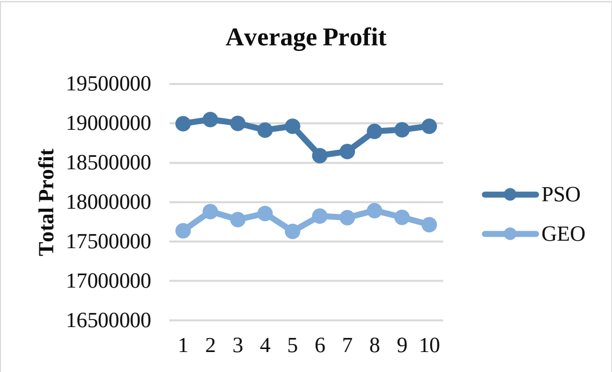


*Fig. 3: Graph of the best profits of PSO and GEO*

Furthermore, based on the final simulation with the combination of values used (see Table 1), it can be seen that the GEO algorithm is superior in its speed of finding a better solution, meaning that it quickly meets the convergence limit compared to the PSO algorithm. A convergent iteration is an iteration that indicates the algorithm is not able to find a better solution until the iteration reaches the maximum limit specified. The graph of the convergent iteration average can be seen in Figure 3.

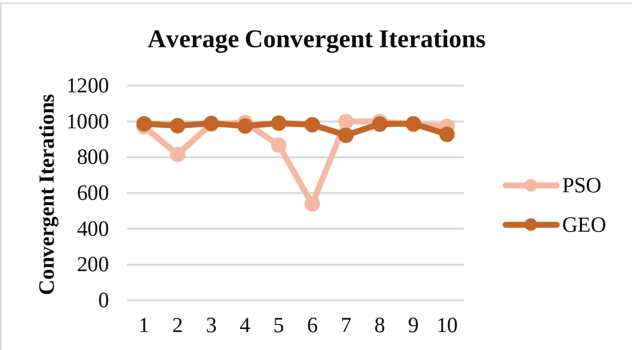


*Fig. 4: Convergent iterations of PSO and GEO*

Based on the convergent iteration and computational time of the algorithm presented in Table 2, it can be seen that the GEO algorithm is faster to find the convergence limit than the PSO algorithm. The computation time of the GEO algorithm is relatively larger than that of the PSO algorithm along with the larger data size. The graph of the average computational time of the PSO and GEO algorithms can be seen in Figure 4.

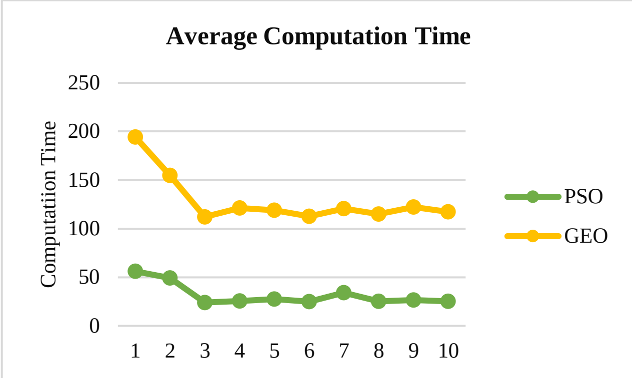


*Fig. 5: PSO and GEO computation times*

Based on the results of the research that has been conducted, it can be said that the old algorithm is PSO algorihtm can compete with the newly discovered algorithm, which is GEO algorithm. Mathematically, the PSO and GEO algorithms can be said to be effective and efficient even though there are shortcomings for each algorithm.

## V. CONCLUSION

Based on the results and discussion, it can be cocluded that PSO algorithm gives the best profit of Rp 19.051.000,00 and GEO algorithm gives the best profit Rp 17.894.000,00 for every 100 types of items taken. From these two advantage, it can be concluded that the PSO algorithm is superior to the GEO algorithm for the quadratic bounded knapsack problem.

## REFERENCES

[1] Boussaid, I., J. Lepagont dan P. Siarry. 2013. A survery on Optimization Metaheuristics. *Information Sciences* 237: 82-117.J. Clerk Maxwell (1892), A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, pp.68–73.

[2] Kellerer, H., D. Pisinger dan U. Pferschy. 2004. *Knapsack Problem*. Berlin: Springer.

[3] Kennedy, J. dan R. Eberhart. 1995. Particle Swarm Optimization. *Proceedings of ICNN'95: International Conference on Neural Networks*.

[4] Sari, M. P., A. Kamsyakawuni, A. Riski dan K. A. Santoso. 2020. Metaheuristic algorithm approach to solve non-linear equations system with complex roots. *Soedirman's International Conference on Mathematics and Applied Sciences* (SICoMAS).

[5] Supriadi, D. 2016. Perbandingan penyelesaian knapsack problem secara matematika, kriteria greedy dan algoritma greedy. *Indonesian Journal on Computer and Information*

*Technologi*, 1(2) : 91, https://doi.org/10.31294/ijcit.v1i2.1835

[6] Yazdi, M. T., A. M. Balani, M. D. Nayeri dan A. Azar. 2021. Golden Eagle Optimizer: A nature-inspired metaheuristic algorithm. *Computers and Industrial Engineering* 152: 107050.