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A review of the fractal geometry in structural elements

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***Keywords*— Fractal geometry, Hausdorff fractal dimension, structure elements.**

***Abstract*— Fractal geometry is a secret language nature follows to grow, to face unknown challenges, and to bloom and blossom with optimal energy. The fractal property of self-similarity, fractional dimensionality, optimality, and innovative fractal patterns, attracted the author(s) to pose the question, what could be the direct relation between fractal geometry and the structures?*

To inquire about the relation between the two, the work of Benoit Mandelbrot is referred to develop the understanding of fractal geometry and its relationship with nature. Simultaneously, research review is framed by referencing published articles, which explicitly discusses the fractal geometry and their application in structural forms. In addition to the above, a brief study about contemporary works and computational tools are discussed, which has enhanced the productivity, efficiency, and optimality of structures, architects, and engineers.

This interdisciplinary research presents a brief overview of fractal geometry and some of its applications in structural forms. Concluding as The mathematics is a key language between nature and engineering. Fractal geometry gives us an optimal solution to the problem with aesthetics and architectural valued structures. Computational tools like machine learning, digital robotic fabrication, high-end modelling software's and coding, help to imitate, imagine and fabricate nature-inspired structures in an ontological, optimal, and sustainable way.

I. INTRODUCTION

Nature grows progressively in metric space, by repeating, copying, and evolving infinite geometric patterns. This growth is non-linear in metric space which results in the form of fractional dimension. This observation of French mathematician Benoit Mandelbrot gave a new view of the real geometry of nature. Mandelbrot explains in his book, “The fractal geometry of nature” that all-natural forms have fractal dimensions and the form is generated by following the fractal properties [4]. This research raises questions about: The fractal property of self-similarity and self-structuring creates structural forms. In this regard, can we contemplate the direct relation between fractal geometry and structures? How fractal geometry is applied by architects and engineers in their practice? How efficient and sustainable are the structures inspired by fractal geometry? The fundamental objectives of this research are (1) to research fractal geometry exhibits in nature and its properties. (2) To research existing structures designed by architects and engineers inspired by the fractals. In addition to the above, a brief study of contemporary works and computational tools are discussed. Which has enhanced productivity, efficiency, and optimality.

II. FRACTAL GEOMETRY IN NATURE AND ITS CLASSIFICATION.

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." _ Benoit Mandelbrot

Benoit Mandelbrot in 1982 in his book, "The fractal geometry of nature" [4] described the word, "Fractal" comes from the Latin word frangere means, "to break, fragment". The geometrical shapes composed of fragments that may be similar, identical, repetitive, or random are called fractals [1]. In nature, everything is formed from fragments and disperse into fragments. For example, the smallest flower of cauliflower is self-similar to the whole flower, the branching pattern of a tree, the more we zoom, a self-similar pattern is observed. The fractals are self-similar and create structural form by self geometrical repetition Mandelbrot, in his paper in 1989, Fractal geometry: what is it, and what does it do? Defines fractal geometry as a link between Euclidean geometry and nature's mathematical chaos [5]. Figure -1 shows the photographs of some natural elements having fractal geometry

2.1 Classification of fractals

Benoit Mandelbrot in his books and research papers in 1982, 1989, Also Vrdoljak et. al in his paper, "Principle of fractal geometry in architecture and civil engineering" in 2019[4][5][27] described that fractals can be classified based on the degree of self-similarity and type of formation [30].

2.1.1 Degree of self-similarity

1. Exactly self-similar fractals - Contains exact scale similar copies of the whole fractal. (Strongest self-similar fractals) also called geometric fractals.
2. Quasi self-similar fractals - Contains few scaled copies of whole fractals and few copies not related to whole fractals. Also called algebraic fractals
3. Statically self-similar fractals- Do not contain copies of themselves but some fractal properties remain the same. (lowest degree of self-similarity)

2.1.2 Type of formation

1. Iterative fractals - Such fractals are formed after translation, rotation, copy, replacing elements with copies. Such fractals are self-similar.
2. Recursive fractals - Such fractals are defined from recursive mathematical formulas. Which identifies the given point in space (Complex space) falls under a domain or not such fractals are quasi self-similar fractals.
3. Random fractals - Such fractals contain partial properties of iterative fractals and recursive fractals hence it is very natural fractals. Nature's creations like clouds, snowflakes, etc. are the best example of random fractals. As Benoit Mandelbrot in his book "fractal geometry of nature" said, "the best fractals are those that exhibit the maximum of invariance."

III. FRACTAL DIMENSION

To justify the fractal geometry and patterns mathematicians developed the concept of fractal dimension (roughness). Benoit Mandelbrot in 1982 in his book, "Fractal geometry of nature" defines fractal as "A fractal is by definition a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension" [4] [5]

In 1918 the great mathematician Felix Hausdorff., introduced the Hausdorff dimension. It is a measure of roughness. Hausdorff dimension for Euclidian's geometry, say point, line, square, cube is zero, one, two, three respectively, such shapes with Hausdorff dimension as an integer also known as the topological dimension. But the Hausdorff dimension of rough shapes is a fraction that is calculated by the ratio of the logarithm of the number of self-similar copies (M) obtained after (N) number of iterations.

i.e.

$D = \log(M)/\log(N)$

Observation from the above pattern denotes that a single line has divided into three parts but the middle part is removed and iterated progressively in a similar pattern. Two similar patterns after each iteration are obtained (Figure -2). As per definition, the Hausdorff dimension after three iterations will be 1.584 (calculated by using equation 1). In this way, the Hausdorff dimension of fractal geometry is calculated. As we can see above geometry is not one dimensional or two but it is in fractional dimension.

IV. APPLICATION OF FRACTAL GEOMETRY IN STRUCTURAL ELEMENTS

Consciously or unconsciously architects and engineers are using the concept of fractal geometry. Either in contemporary modern design innovation or architectural ornamentation of ancient Hindu temples, Buddhist temples, or roman churches [18][28]. The work of Benoit

Mandelbrot on fractal geometry and its mathematics changed the perception of the scientific and technological world. The use of fractal geometry in image processing, virtual reality, artificial intelligence, antenna, etc. are revolutionary ideas. Which has changed the computational, medical, technological world. The impact of that has also been seen in the architecture and civil engineering world.

The fractal property of self-similarity and self-organization can easily be observed in the branching pattern of trees. Trees are organisms that stand by themselves, so their shape has an inherent structural rationality' [20]. They are non-static structural forms, a seed takes the form of a tree after a long time. The challenge to the upcoming form is unknown. It uses its natural intelligence to obtain the best form at minimum use of energy. Trees are fractal-like structures following the rule of self-similarity and random fractals.

The paper, "The mechanical self-optimization of trees" by C. Mattheck & I. Tesari[6], explains the optimized growth of trees and relation between forces, stresses with the form and their fiber organization in correlation with the five theorems, minimization of the lever arm, Axioms of uniform stresses, minimization of critical shear stresses, Adaptation of the strength of wood to mechanical stresses, Growth stresses counteract critical loads[7]. The tree is a natural vertical member, designed by the intuition of nature to withstand the dynamic self-weight and lateral loads. Tree as a structural form, always been a keen inspiration for architects and engineers. The term dendriform is used for the forms and shape which are imitations of tree or plants. 'Dendron' is a Greek word for 'tree'. The branching-like structure is also known as the 'dendritic structure' (Schulz and Hilgenfeldt, 1994). the term 'dendritic structure' uses this natural entity for describing a mesh-free ramified system or branching structure(KullandHerbig,1994)[8]

4.1 Capital

Md Rian et. al. in his paper in 2014 "Tree-inspired dendriform and fractal-like branching structures in architecture:"[17] explained - The true wooden dendriform can be seen in Chinese Dougong Brackets, 'Dou' means wooden block or piece and 'gong' means wooden bracket. The Typical Construction Of dugong is an interlocking assemblage of some 'gongs'. The 'gongs' are interlocked, to form the structural cantilever capital which takes the load of the roof and transmits it into columns.[17] Refer figure 3.a. Xianjie Menga et. al. in 2019 their paper "Experimental study on the seismic mechanism of a full-scale traditional Chinese timber structure"[29], they studied the behavior of dugong in dynamic loading condition, in which they modeled the full-scale timber structure which has this dendriform in it. They generated experimental data on 15 sets of shake table models, compared the horizontal and vertical displacement with the acceleration, and concluded that such structure can resist large earthquakes[29] refer to figure 3.b & 3.c. This system of interlocking was also practiced in India, roman, Egypt, and Greece by using stone as a material.

4.2 Column

Tang et. al. in 2011 in his paper "Developing evolutionary structural optimization techniques for civil engineering applications." And Fernández-Ruiz et. al. in 2014 in his paper "Patterns of force: length ratios for the design of compression structures with inner ribs."[24][10] concluded that in the 19th century, poetic architect Antoni Gaudi used some tree-inspired structures in his designs like in Sagrada Familia, in Barcelona refer figure 4.a. He developed a unique technique of hanging chain models to develop stable structural forms. Gaudi studied the member's loads by suspending the cables under gravity. He produced a group of the arch that was only subjected to compressive axial forces, hence free from bending [10][24]Inspired by the mechanical and structural characteristics of nature. Ahmeti et. al, in 2007 in his paper "Efficiency of lightweight structural forms: The case of treelike structures-A comparative structural analysis." And in 2016 in his paper L. Aldinger, "Frei Otto: Heritage and Prospect," [1][16]concludes that, During the 20th century, Frei Otto, a very experimental German architect, has introduced the term lightweight structure in his practice and research [16]. His design philosophy is focused on the relationship between architecture and nature, and their performance. Otto scrutinized the new concepts of form-finding by experimenting with lightweight tents, soap films, suspended constructions, dome and grid shells, and branching structures [1]. He is also fascinated by the tree's fractal-like geometry and started using them in his practice, at Stuttgart airport, Stuttgart Germany refers to figure 4.b. Another architect, structural engineer, educator at Harvard University, Allen and Zalewski in his book "Form and force" [2] exemplified the used graphic static for finding the optimized form for steel-made dendriform structures by achieving maximum force equilibrium in designing a long-span market roof. [2]

4.3 Beam and trusses

Benoit Mandelbrot in his book nature's geometry [4] mentioned that even before Koch, Peano and Sierpinski. The tower that was built by French engineer Gustave Eiffel in Paris deliberately incorporated the idea of a fractal curve, full of branch points. The A's and tower are not solid beams but every member is a colossal truss, with every sub-member as a truss. Which makes the structure stiff and lightweight [4]

Roderick lake in 1993 in his paper "Materials with structural hierarchy", which was published in Nature. Gives us insight into bone structure hierarchy and its implication in materials. Also Meenakshi Sundaram et.al. in 2009 in his paper "Gustave Eiffel and his optimal structures," justify more clearly structure hierarchy and its role in optimization of structure which as follows.[23][21] Fractal patterns are even observed at the microscopic level by the scientist and practiced by engineers like Gustave Eiffel (Consciously or unconsciously) understanding this by relating the structure of bone and Eiffel tower design. Cortical or compact bone and trabecular or cancellous bone are the outer and inner parts of our bone respectively refer to figure 5. Haversian canals are layered rings carrying blood vessels that are surrounded by lamellae. Lamellae are made of collagen fibers, which are in turn made of fibrils. These five layers inside one another, if we denote structural hierarchy level by n, our compact bones are hierarchy level 5. Such structure imparts special structural property. A similar structural hierarchy is observed in Gustave Eiffel works like Eiffel tower, Garabit Viaduct Bridge, Maria Pia Bridge.[21] [23]

P. Weidman in 2004 in his paper "Model equations for the Eiffel Tower profile: Historical perspective and new results," And C. Roland in 2004 in his paper "Proposal for an iron tower: 300 meters in height," discusses the topology and behavior of the tower under wind condition. The core of their research is [22][27][7]- To withstand heavy wind load and self-weight by the tower itself, proper geometry selection is needed. The four legs of the tower are supported at the bottom but only bottom support is not sufficient enough to resist the wind load. So four structural belts are provided at different heights of 91,129, 228, and 309 meters from the ground. Also to resist the wind load the exterior profile of the tower is considered as nonlinear and at a determined scale of the curve of the bending due to wind[22]. Eiffel and co. are very familiar in construction with truss systems(trails/cross beam) and piers, where horizontal forces are taken by viaduct but in the case of the Eiffel, tower piers have to counter the thrust of wind[7]. But in the case of the Eiffel tower, they have to give away the cross beams. Which has been explained by M Meenakshi Sundaram and G K Ananthasuresh in their paper "Gustave Eiffel and his optimal structure" [21]

4.4 Slab

This section mainly reviews the work of Pier Lungi Neirve. The research work of T. Iori et. al. in 1960 "Pier Luigi Nervi's Works for the 1960 Rome Olympics,". In 2018 D. Thomas, "The Masters and Their Structures," in *Masters of the Structural Aesthetic* were majorly referred and explored, The Victoria Amazonica leaves (figure -6.a) appear to be very delicate but due to the fractal branching of the ribs and the veins, it gets enough structural strength. Its delicacy, fractal pattern, and strength attracted architects and engineers to understand and develop the architectural structural form based on its geometry. Victoria Amazonica has radial and circular veins, the intersection of two makes the ribs like a pattern which gives it great structural strength [3]. Above mentioned rib pattern is made up of airy tissues which make it light and have a high bouncy which enables the leaf to float above the surface of the water[27]. Such pattern also observed in equiangular spiral, growth spiral, logarithmic spiral, can be constructed from equally spaced rays by starting at a point along one ray, and drawing the perpendicular to a neighboring ray. As the number of rays approaches infinity, the sequence of segments approaches the smooth logarithmic spiral [9]. The fractal property of self-similarity and self-organization is observed in the equiangular spiral, sunflower, and many natural elements.[26]

Above mentioned geometric pattern is seen frequently in the work of a great structural engineer, architect, constructor Pier Luigi Nervi (figure 6b). He confluences the geometry and construction technique so intelligently which gives captivating aesthetical structural elements without any embellishment.

Using such a pattern along with the concept of prefabrication and Ferro cement gave a very optimal solution for large span roofs, half dome, vaults, and shell structures.[26] [25] [8]. Which can be seen in Palazzetto dello sports Arena in Hanover, New Hampshire, Thompson Arena in Hanover, New Hampshire, and many more.

4.5 Contemporary work and computational tool

Fractal geometry has been of keen interest for architects and engineers for all time. But imitating them in practice is far easier in the contemporary world due to technological advancement. The computational tools like rhino, grasshopper, python, robotic fabrication, Machine learning, etc. made the process of modeling, designing, analysis, and fabrication very quick, easy, and efficient. The fractal branching of trees inspired the structure of a modern chapel in Nagasaki, Japan refers figure 7.a. Designed by architect Yu Momoeda, the building uses a branching timber column system that begins with four pillars each splitting into eight branches. These branches are connected by white steel rods and in turn support the next level of eight smaller pillars, which branch to support the top section of 16 branching pillars[25]. Another example is the Sierpinski pyramid 17.25m (56ft 7in) tall[15] refer to figure 7.b. Which has been constructed by using a 3D printing machine. Made for the International Science Festival in Gothenburg, Sweden. Last but not least, Yijiang Huang, Caelan R. Garrett, Caitlin T. Mueller used the automated sequence and motion planning for robotic spatial extrusion of 3D trusses [14]. Figure 7.c

Software like rhino, grasshopper, python in their research for modeling, form-finding, stress distribution, and structural behavior to analyze their design concepts. Also, the fabrication techniques like robotic fabrication used by Professor Catlin Muller in her research lab, 'Digital structure' at MIT in various projects like making a crystal truss system, Islamic shells. Explored the various possible computational techniques which bridge the structure and architecture [12].

V. FIGURES

Fig.1: Source : Zdimalova, Maria & Škrabul'áková, Erika. (2019). Magic with Fractals.[30]

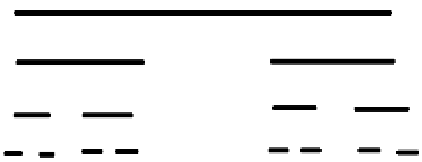

Fig.2: Self similar division on line

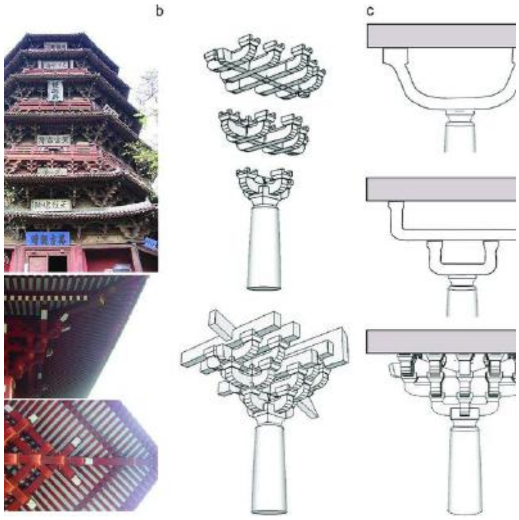

Fig.3.a Chinese Dougong Brackets

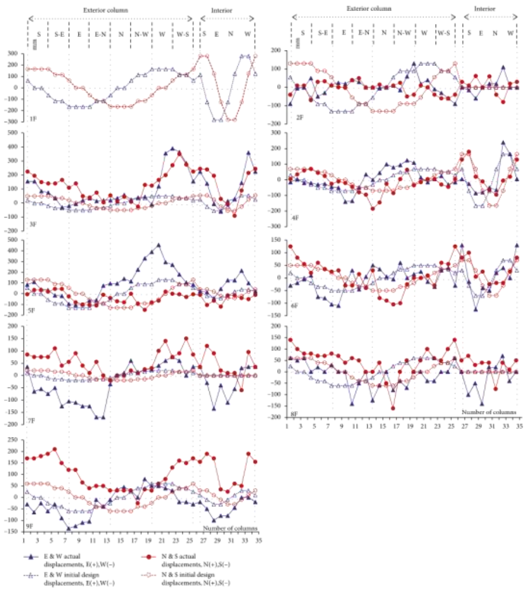

Fig: 3.b graphs showing horizontal deflection under dynamic loads

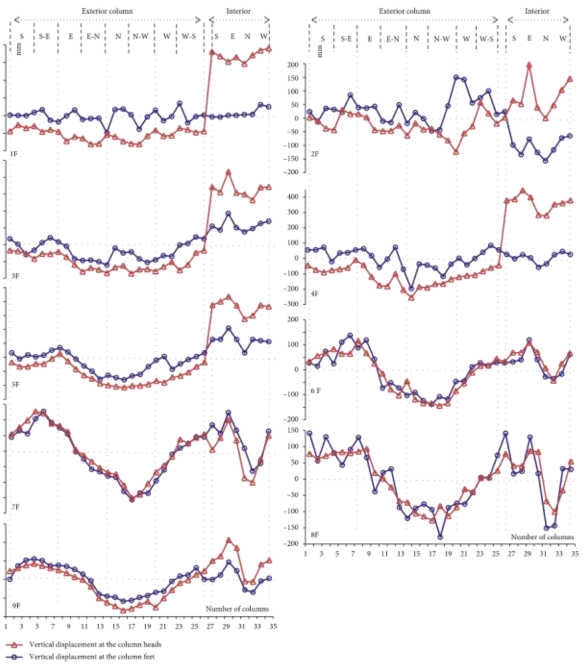

Fig 3.c graphs showing vertical deflection under dynamic loads

Source:

Fig 3.a Md Rian, Iasef. (2014). Tree-inspired dendriforms and fractal-like branching structures in architecture: A brief historical overview. Frontiers of Architectural Research. 3. 10.1016/j.foar.2014.03.006.[17]

Fig 3.b,c Xianjie Menga , Tieying Lia,⁎ , Qingshan Yang,cXianjie Menga , Tieying Lia,⁎ , Qingshan Yang,c in their paper "Experimental study on the seismic mechanism of a full-scale traditional Chinese timber structure"[29]

Fig 4.a Sagrada familia

Fig 4.b Stuttgart airport

Source:

Fig 4.a
https://www.flickr.com/photos/7455207@N05/5491325900/in/photostream/

Fig 4.b
https://in.pinterest.com/pin/962645107568888828/

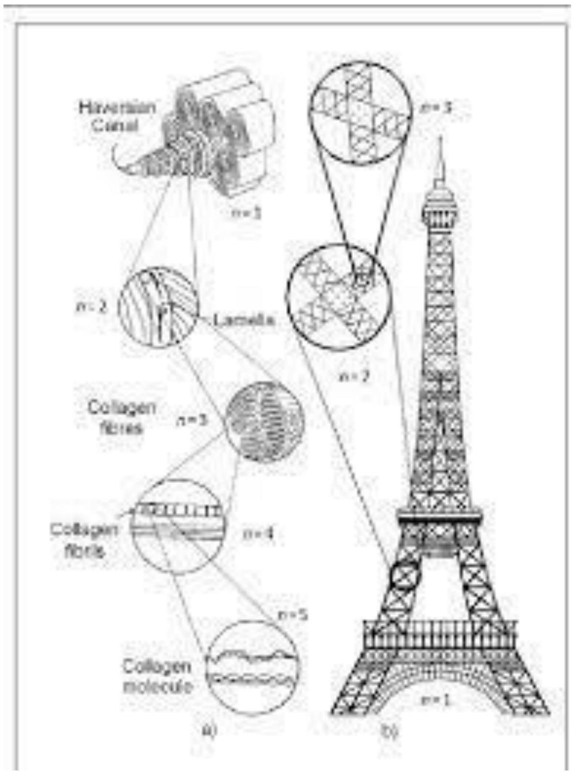

Fig 5 Bone internal structure & Eiffel tower

Source:

Meenakshi Sundaram and G K Ananthasuresh in their paper "Gustave eiffel and his optimal structure"[21]

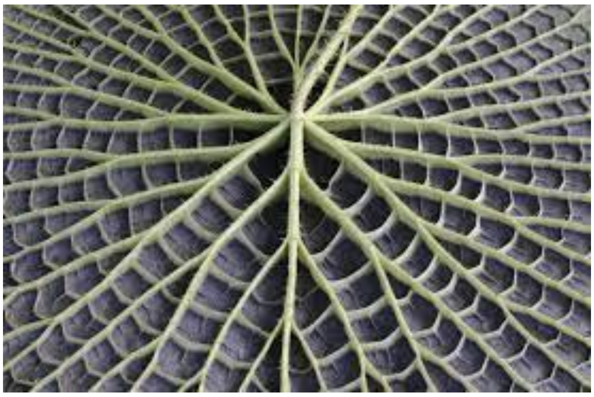

Fig 6.a Victoria Amazonica leaves

Fig 6.b Pier Luigi Nervi roof

Source:

Fig 6.a
https://in.pinterest.com/pin/318277898639049456/

Fig 6.b
https://www.pinterest.co.uk/pin/544794886167830366/

Fig 7.a chapel in Nagasaki, Japan

Source:
https://www.designboom.com/architecture/yu-momoeda-architecture-office-agri-chapel-japan-01-03-2018/

Fig 7.b 3D print fractal pyramid

Source:
https://www.pinterest.co.kr/pin/286541595019046905/

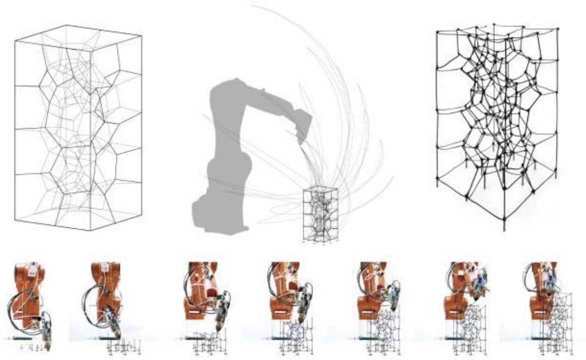

Fig 7.c Robotic fabrication

Source:
https://web.mit.edu/yijiangh/www/publications/

VI. CONCLUSION

This paper briefs about one of the greatest secrets of nature's design: irregularity, self-similarity, repetition, optimality, fractal dimensionality. The degree of their self-similarity and their mode of formation is the basis of their classification. There are infinite types of fractals present in nature. A research review is established to identify the direct relation between fractal geometry and the structural elements. Architects and engineers are using this concept of self similarity and fractal geometry from the ancient to contemporary time consciously or unconsciously. Which give beautiful structural forms with great efficiency and optimality. Fractal geometry supports creativity and builds a connection between human and nature. The idea for new structural forms helps architects and engineers in defining new senses of structures. Many research has used this concept in form finding and optimization problems. Furthermore, computational tools and advancement in technology will act as catalyst and supportive agent to explore the new structural forms, which are efficient, lightweight weight, optimal, and economical along with aesthetical beauty.

REFERENCES

[1] Ahmeti, Flamur. "Efficiency of lightweight structural forms: The case of treelike structures-A comparative structural analysis." (2007).

[2] Allen zalwiski et. al. "Form-and-forces designing efficient, expressives structure - A graphical approach" book _ publisher - wiley

[3] A. L. Polesel, "Fractal Branching in the Victoria Amazonica," *WeWantToLearn.net*, Jan. 23, 2020. https://wewanttolearn.wordpress.com/2020/01/23/fractal-branching-in-the-victoria-amazonica/ (accessed Jan. 24, 2021).

[4] Benoit Mandelbrot - The Fractal Geometry of Nature. Book _ publisher – W.H. Freeman and company

[5] B. B. Mandelbrot, A. Blumen, M. Fleischmann, D. J. Tildesley, and R. C. Ball, "Fractal geometry: what is it, and what does it do?," *Proc. R. Soc. Lond. Math. Phys. Sci.*, vol. 423, no. 1864, pp. 3–16, May 1989, DOI: 10.1098/rspa.1989.0038.

[6] C. Mattheck & I. Tesari,"The mechanical self optimization of trees", WIT Transactions on Ecology and the Environment, WIT press, 2004, vol 70,10.2495/DN040201

[7] C. Roland and P. Weidman, "Proposal for an iron tower: 300 metres in height," *Archit. Res. Q.*, vol. 8, pp. 215–245, Dec. 2004, DOI: 10.1017/S1359135504000260.

[8] D. Thomas, "The Masters and Their Structures," in *Masters of the Structural Aesthetic*,

[9] E. W. Weisstein, "Logarithmic Spiral." https://mathworld.wolfram.com/LogarithmicSpiral.html (accessed Jan. 25, 2021).

[10] Fernández-Ruiz, M. A., et al. "Patterns of force: length ratios for the design of compression structures with inner ribs." *Engineering Structures* 148 (2017): 878-889.

[11] G. C. Mattheck, *Trees: The Mechanical Design*. Springer Science & Business Media, 2012.

[12] http://digitalstructures.mit.edu/

[13] https://archive.curbed.com/2018/1/4/16848894/japan-chapel-architecture-fractal-yu-momoeda

[14] https://arxiv.org/abs/1810.00998

[15] https://www.pinterest.com.mx/pin/286541595019046905/?amp_client_id=CLIENT_ID(_)&mweb_unauth_id=%7B%7Bdefault.session%7D%7D&_url=https%3A%2F%2Fwww.pinterest.com.mx%2Famp%2Fpin%2F286541595019046905%2F

[16] I. L. Aldinger, "Frei Otto: Heritage and Prospect," *Int. J. Space Struct.*, vol. 31, no. 1, pp. 3–8, Mar. 2016, DOI: 10.1177/0266351116649079.

[17] Md Rian and M. Sassone, "Tree-inspired dendriforms and fractal-like branching structures in architecture: A brief historical overview," *Front. Archit. Res.*, vol. 3, no. 3, pp. 298–323, Sep. 2014, DOI: 10.1016/j.foar.2014.03.006.

[18] Iasef Md Rian, Jin-Ho Park, Hyung Uk Ahn, Dongkuk Chang, Fractal geometry as the synthesis of [19] Hindu cosmology in Kandariya Mahadev temple, Khajuraho, Building and Environment, Volume 42, Issue 12, 2007, Pages 4093-4107, ISSN 0360-1323,

[19] L. B. Leopold, "Trees and streams: The efficiency of branching patterns," *J. Theor. Biol.*, vol. 31, no. 2, pp. 339–354, May 1971, DOI: 10.1016/0022-5193(71)90192-5.

[20] M. Meenakshi Sundaram and G. K. Ananthasuresh, "Gustave Eiffel and his optimal structures," *Resonance*, vol. 14, no. 9, pp. 849–865, Sep. 2009, DOI: 10.1007/s12045-009-0081-x.

[21] P. Weidman and I. Pinelis, "Model equations for the Eiffel Tower profile: Historical perspective and new results," *Comptes Rendus Mec.*, vol. 332, pp. 571–584, Jul. 2004, DOI: 10.1016/j.crme.2004.02.021.

[22] Roderick "Materials with structural hierarchy | Nature." https://www.nature.com/articles/361511a0 (accessed Jan. 22, 2021). Singapore: Springer Singapore, 2018, pp. 47–107.

[23] Tang, Jiwu. "Developing evolutionary structural optimization techniques for civil engineering applications." *A thesis submitted in fulfilment of the requirement for the degree of Doctor of Philosophy* (2011).

[24] T. Iori and S. Poretti, "Pier Luigi Nervi's Works for the 1960 Rome Olympics," p. 9, 1960.

[25] T. Leslie, "Form as Diagram of Forces: The Equiangular Spiral in the Work of Pier Luigi Nervi," *J. Archit. Educ. 1984-*, vol. 57, no. 2, pp. 45–54, 2003.

[26] Unav, "Climate change-oriented design: Living on the water. A new approach to architectural design," 2020, DOI: 10.24425/JWLD.2020.135036.

[27] Vrdoljak anton et. al. 2019, "Principle of fractal geometry and its application in architecture and civil engineering

[28] V. Bharne and K. Krusche, *Rediscovering the Hindu Temple: The Sacred Architecture and Urbanism of India*. Cambridge Scholars Publishing, 2014.

[29] X. Meng, T. Li, and Q. s Yang, "Experimental study on the seismic mechanism of a full-scale traditional Chinese timber structure," *Eng. Struct.*, vol. 180, pp. 484–493, Feb. 2019, DOI: 10.1016/j.engstruct.2018.11.055.

[30] Zdimalova, Maria & Škrabul'áková, Erika. (2019). Magic with Fractals.