International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-9, Issue-1; Jan, 2022
Journal Home Page Available: https://ijaers.com/
Article DOI: https://dx.doi.org/10.22161/ijaers.91.3

Scalar Dark Matter Formation in Electron - Axion Like Collider

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Received: 19 Nov 2021,

Received in revised form: 20 Dec 2021,

Accepted: 28 Dec 2021,

Available online: 11 Jan 2022

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***Keywords*— Axion – Axion like, Dark Matter, Standard Model, New Physics**

***Abstract*— The aim paper is devoted to study the interaction of axion like (ALPs) with fermions when expanding the standard model. The process of collider between axion-electron, which produce scalar dark matter and release electron has been studied. The scattering amplitude and cross section of this collision in the center of mass system (CMS) have been calculated by using Mandelstam variables and then plotted its dependence on scattering angle and energy.*

I. INTRODUCTION

In recent years, the search for particles outside the standard model with small mass and weak interaction with the standard model has attracted a lot of attention from the scientific community, especially those who interested in interactions of high-energy elementary particles. The motive of this interest is to answer the question of whether there is new physics; Are the new particles energetic or light? Theoretical studies also want to show that the light particles that interact weakly with the particles in the standard model are spontaneously generated by the extension of the standard model, as well as the dark matter (DM) formation process.

Axion like particles (ALPs) are the scalar (or pseudo – scalar) particles that appear in many different physical models, they can act as a Goldstone Boson of the U(1)PQ group (Peccei Quinn) [1,2] and can also appears as a component of the Chiral super-field in the super-symmetry theory (SUSY) [2].

ALPs with mass around MeV are associated with a wide range of phenomena for cosmology and astrophysics [3], such as affecting the Bigbang Nucleosynthesis (BBN), CMB, and stellar evolution. ALPs are also involved in the formation of cold dark matter and can be used to account for a large number of astronomical singularities such as the extreme cold efficiency of a class of stars, the transparency to the strangeness of the universe with its super-high-energy gamma ray [4] or the hiding of monochromatic X-ray around the energy of 3.5KeV [5]. ALPs also play a key role in breaking electroweak symmetry and in solving hierarchy problems through relaxation mechanisms. ALPs also give us the exciting ability to connect the standard model to potential dark matter particles [6]

The aim of this paper is to consider the interaction of scalar particles as ALPs with fermions when expanding the standard model, it occurs in most of the strong CP problems, in the model with break super-symmetry. The ALPs are pseudo-Goldstone bosons, they are light and very weakly bound together. The process of axion-electron interaction through positron exchange will study. It produces scalar dark matter and release electrons. We will calculate the scattering amplitude and scattering cross section of the collision in the center of mass system (CMS) with Mandelstam variables and investigate the graphing number.

The paper is constructed as follow: In Sec.2, we find an expression of the scattering amplitude based on the Feynman diagram at the s - channel and the t - channel with Mandelstam variables used. In Sec. 3, the scattering cross section is deduced and numerically calculated, and then plotted its dependence on scattering angle and energy. In Section 4, we give some discussion and conclusion about the obtained results.

II. ALPS – ELECTRON SCATTERING AMPLITUDE

In this section, we will derive an expression for the scattering amplitude of the process $a(k_1)e^-(p_1)\xrightarrow{e}\phi(k_2)e^-(p_2)$, which ALPS collider with electron to create e$^-\phi$ through positron exchange.

The propagator of a electron has the form

$$D(q)=\frac{1}{q^2-m_e^2+i\varepsilon} \quad (1)$$

q^2 is the square of momentum transfer.

The Lagrangian of axion – electron interaction is [7,8]

$$L_{ae^-e^+}=-g_e\bar{e}i\gamma_5 ea. \quad (2)$$

where $g_e\approx\frac{m_a m_e}{m_\pi f_\pi}=4,07.10^{-11}m_a$ is the axion – electron coupling [7,9]

From (1) and (2), we use Feynman diagram law to compute the scattering amplitude for above process. In this process, ALPs – electron interaction is carried out simultaneously in both channels, s- channel and t – channel. So the total scattering matrix is

$$M=M_s+M_t \quad (3)$$

Where M_s and M_t are the scattering matrices of s - channel and t – channel respectively

2.1. s- Channel scattering amplitude

In this channel (Fig.1), an electron with momentum p_1 absorbs an axion with momentum k_1 to produce a positron, which then radiates out scalar dark matter with momentum k_2 and produces an electron with momentum p_2.

Based on the rule of calculating amplitude according to Feynman diagram, we have

$$\begin{aligned} iM_s&=\bar{u}(p_2)\left(-\frac{im_e g_e}{m_a}\gamma_5\right)u(p_1)\left(\frac{1}{q_s^2-m_e^2}\right)\\ &=-\frac{im_e g_e}{m_a}.\frac{1}{q_s^2-m_e^2}\bar{u}(p_2)\gamma_5 u(p_1)\end{aligned} \quad (4)$$

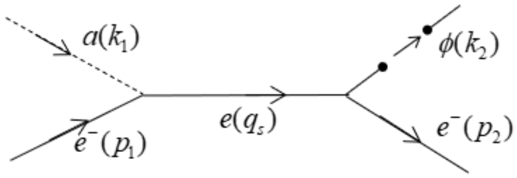

Fig. 1: s – Channel

Note that the sum is taken according to the spin states of the particle

$$u(p).\bar{u}(p)=\left(p_\mu\gamma^\mu\pm m\right)_{mn}=\left(\hat{p}\pm m\right)_{mn}$$

Here (+) for particles and (-) for anti-particles. Thus,

$$\left|M_s\right|^2=\frac{m_e^2 g_e^2}{m_a^2(q_s^2-m_e^2)^2}Tr\left[(\hat{p}_2+m_e)\gamma_5^2(\hat{p}_1-m_e)\right] \quad (5)$$

We already know the properties of gamma matrices

$$\begin{aligned}&\bar{\gamma}_5=-\gamma_5,\\ &(\gamma_0)^+=\gamma_0,(\gamma_k)^+=-\gamma_k(k=1,2,3),\\ &\bar{\gamma}_\mu=\gamma_0\gamma_\mu\gamma_0=\gamma_\mu,\overline{\gamma_\mu\gamma_\nu}=\gamma_\nu\gamma_\mu,\\ &\gamma_5^2=1\end{aligned} \quad (6)$$

and γ_5 is anti-commutative with the other gamma matrices, then

$$(\hat{p}_1+m_e)\bar{\gamma}_5=\gamma_5(\hat{p}_1-m_e) \quad (7)$$

Putting eqs. (6), (7) into eq.(5) we get

$$\left|M_s\right|^2=\frac{m_e^2 g_e^2}{m_a^2(q_s^2-m_e^2)^2}Tr\left[\hat{p}_2\hat{p}_1-\hat{p}_2 m_e+m_e\hat{p}_1-m_e^2\right] \quad (8)$$

Since the product of an odd number of Dirac matrices equals zero, and

$$\begin{aligned}&Tr\left[\hat{p}_1 m_e\right]=Tr\left[\hat{p}_2 m_e\right]=0;\\ &Tr\left[\hat{p}_2\hat{p}_1-m_e^2\right]=Tr\left[p_2^\mu p_1^\nu\gamma_\mu\gamma_\nu-m_e^2\right]=p_2^\mu p_1^\nu Tr\left[\gamma_\mu\gamma_\nu-m_e^2\right]\\ &=p_2^\mu p_1^\nu\left(4g_{\mu\nu}-4m_e^2\right)=4\left(p_2 p_1-m_e^2\right)\end{aligned} \quad (9)$$

Substitute (9) into (8), we obtain:

$$\left|M_s\right|^2=\frac{m_e^2 g_e^2}{m_a^2(4E_1E_2)^2}4(p_2 p_1-m_e^2) \quad (10)$$

Considering the center of mass reference system, using the law of conservation of momentum:

$$\vec{p}_1 + \vec{k}_1 = 0 \Rightarrow \vec{p}_1 = -\vec{k}_1 \equiv \vec{p}; \quad \vec{p}_2 + \vec{k}_2 = 0 \Rightarrow \vec{p}_2 = -\vec{k}_2 \equiv \vec{k} \tag{11}$$

Here Mandelstam variables are used

$$p_1 = (E_1, \vec{p}); k_1 = (E_2, -\vec{p}); \quad p_2 = (E_3, \vec{k}); k_2 = (E_4, -\vec{k}); \tag{12}$$

p_1, p_2, k_1, k_2 are the 4 – component vectors; $\vec{p}, \vec{k}$ are the 3 – component momentums of incident and scattered particles.

We have $\vec{p}^2 = E^2 - m^2$ then

$$\vec{p}^2 = E_1^2 - m_e^2 = E_2^2 - m_a^2; \ \vec{k}^2 = E_3^2 - m_e^2 = E_4^2 - m_\phi^2 \tag{13}$$

$$s = q^2 = (p_1 + k_1)^2 = m_e^2 + m_a^2 + 2\left(E_1E_2 + \sqrt{E_1^2 - m_e^2}\sqrt{E_2^2 - m_a^2}\right) \tag{14}$$

$$s = m_e^2 + m_\phi^2 + 2\left(E_3E_4 + \sqrt{E_3^2 - m_e^2}\sqrt{E_4^2 - m_\phi^2}\right) \tag{15}$$

Since the law of conservation of energy:

$$E_1 + E_2 = E_3 + E_4 \tag{16}$$

Ignore the mass of the particles besides the energy terms

$$|\vec{p}| = E_1 = E_2 = E; \ |\vec{k}| = E_3 = E_4 = E \tag{17}$$

$$q_s^2 - m_e^2 \approx 4E_1E_2 \tag{18}$$

So we have

$$\begin{aligned} p_2 p_1 &= E_1E_3 - \vec{p}.\vec{k} = E_1E_3 - |\vec{p}|.|\vec{k}|.\cos\theta \\ &= E_1E_3 - \sqrt{E_1^2 - m_e^2}.\sqrt{E_3^2 - m_e^2}.\cos\theta \\ &\approx E_1E_3 - E_1E_3.\cos\theta = E_1E_3(1-\cos\theta) \end{aligned} \tag{19}$$

Then substitute (19) into (10), we derive

$$\begin{aligned} |M_s|^2 &= \frac{m_e^2 g_e^2}{m_a^2 (4E_1E_2)^2} 4[E_1E_3(1-\cos\theta) - m_e^2] \\ &\approx \frac{m_e^2 g_e^2}{4m_a^2 (E_1E_2)^2} E_1E_3(1-\cos\theta) \end{aligned} \tag{20}$$

From (13), (16), (17) and (18) while ignoring the electron mass m_e

$$\sqrt{E_4^2 - m_\phi^2} + E_4 = 2E \Rightarrow E_4 = E\left(1 + \frac{m_\phi^2}{4E^2}\right) = E\left(1 + \frac{m_\phi^2}{4s}\right) \tag{21}$$

$$E_3 = 2E - E_4 = E - \frac{m_\phi^2}{4E} = E\left(1 - \frac{m_\phi^2}{4s}\right) \tag{22}$$

$$s = q_s^2 - m_e^2 \approx q_s^2 = 4E_1E_2 = 4E^2 \tag{23}$$

So the final expression for the s -channel scattering amplitude is

$$\begin{aligned} |M_s|^2 &= \frac{m_e^2 g_e^2}{4m_a^2 E^4} E^2 \left(1 - \frac{m_\phi^2}{s}\right)(1-\cos\theta) \\ &= \frac{m_e^2 g_e^2}{4m_a^2 E^2}\left(1 - \frac{m_\phi^2}{4E^2}\right)(1-\cos\theta) \end{aligned} \tag{24}$$

2.2. t- Channel scattering amplitude

In this channel, an electron radiates out scalar dark matter and produces positron, which immediately combines with axion to produce electron.

Doing the same as the s - channel transform, we have

$$\begin{aligned} |M_t|^2 &= \frac{m_e^2 g_e^2}{m_a^2 (q_t^2 - m_e^2)^2} 4E_1E_3(1-\cos\theta) \\ &= \frac{m_e^2 g_e^2}{m_a^2 (q_t^2 - m_e^2)^2} 4E^2\left(1 - \frac{m_\phi^2}{4E^2}\right)(1-\cos\theta) \end{aligned} \tag{25}$$

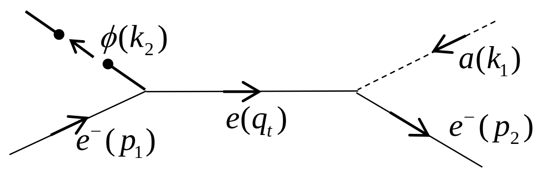

Fig. 2: t – Channel

Further more

$$\begin{aligned} q_t^2 &= (p_1 - k_2)^2 = p_1^2 + k_2^2 - 2p_1k_2 \\ &= m_e^2 + m_\phi^2 - 2\left(E_1E_4 + \sqrt{E_1^2 - m_e^2}\sqrt{E_3^2 - m_e^2}\cos\theta\right) \end{aligned} \tag{26}$$

Ignore the mass of the particles besides the energy term E_1, E_3 to derive

$$q_t^2 - m_e^2 = 2E^2\left(\frac{m_\phi^2}{4E^2} - 1\right)(1+\cos\theta) \tag{27}$$

Thus, the expression of the t – channel scattering amplitude is

$$\begin{aligned} |M_t|^2 &= \frac{m_e^2 g_e^2\left(1 - \frac{m_\phi^2}{4E^2}\right)(1-\cos\theta)}{m_a^2 E^2\left(\frac{m_\phi^2}{4E^2} - 1\right)^2 (1+\cos\theta)^2} \\ &= \frac{m_e^2 g_e^2 (1-\cos\theta)}{m_a^2 E^2\left(1 - \frac{m_\phi^2}{4E^2}\right)(1+\cos\theta)^2} \end{aligned} \tag{28}$$

2.3. Total scattering amplitude

To continue calculating the total scattering amplitude according to (3), we have to calculate two more terms $M_s.M_t^*, M_t.M_s^*$.

Firstly,

$$M_s.M_t{}^* = \frac{m_e^2 g_e^2}{m_a^2(q_s^2 - m_e^2)(q_t^2 - m_e^2)} Tr\left[(\hat{p}_2 + m_e)\gamma_5(\hat{p}_1 + m_e)\overline{\gamma}_5\right] = -\frac{m_e^2 g_e^2 (1-\cos\theta)}{8m_a^2.E^2(1+\cos\theta)} \quad (29)$$

and similarly $M_s{}^*.M_t = -\frac{m_e^2 g_e^2(1-\cos\theta)}{8m_a^2.E^2(1+\cos\theta)}$ (30)

Then, the expression of scattering amplitude is

$$|M|^2 = \frac{m_e^2 g_e^2}{4m_a^2 E^2}\left(1-\frac{m_\phi^2}{4E^2}\right)(1-\cos\theta) + \frac{m_e^2 g_e^2(1-\cos\theta)}{m_a^2 E^2\left(1-\frac{m_\phi^2}{4E^2}\right)(1+\cos\theta)^2} - \frac{m_e^2 g_e^2(1-\cos\theta)}{2m_a^2.E^2(1+\cos\theta)}$$
$$= \frac{m_e^2 g_e^2}{m_a^2 s}(1-\cos\theta)\times\left[\left(1-\frac{m_\phi^2}{s}\right)+\frac{4}{\left(1-\frac{m_\phi^2}{s}\right)(1+\cos\theta)^2}-\frac{2}{(1+\cos\theta)}\right] \quad (31)$$

III. CROSS SECTION

In this section we will calculate the differential scattering cross section and the total scattering cross section of the above collision. These are important physical parameters that can be compared with experimental measurements

By definition, the differential cross-section is equal to

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{k}|}{|\vec{p}|}|M|^2 \quad (32)$$

Substitute the expression of the total scattering amplitude in (31) into (32), we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\cdot\frac{E_3}{E_1}\cdot\frac{m_e^2 g_e^2}{m_a^2 s}(1-\cos\theta)\times\left[\left(1-\frac{m_\phi^2}{s}\right)+\frac{4}{\left(1-\frac{m_\phi^2}{s}\right)(1+\cos\theta)^2}-\frac{2}{(1+\cos\theta)}\right] \quad (33)$$
$$= \frac{1}{64\pi^2 s^2}\cdot\frac{m_e^2 g_e^2}{m_a^2}(1-\cos\theta)\left[\left(1-\frac{m_\phi^2}{s}\right)-\frac{2}{(1+\cos\theta)}\right]^2$$

Integrating this expression with attention $d\Omega = 2\pi d(\cos\theta)$, we get the total scattering cross section

$$\sigma = \frac{1}{32\pi s^2}\cdot\frac{m_e^2 g_e^2}{m_a^2}\int(1-\cos\theta)\left[\left(1-\frac{m_\phi^2}{s}\right)-\frac{2}{(1+\cos\theta)}\right]^2 d(\cos\theta) \quad (34)$$

Set $A = 1-\frac{m_\phi^2}{s}; x = 1+\cos\theta$ then

$$\sigma = \frac{1}{32\pi s^2}\cdot\frac{m_e^2 g_e^2}{m_a^2}\left[-\frac{1}{2}A^2x^2 + 2(A^2+2A)x - 4(2A-1)\ln x - \frac{8}{x}\right] \quad (35)$$

We choose the input parameters as follows [8]:

$$m_e = 0{,}511 MeV = 5{,}11.10^{-4} GeV; g_e = 4{,}07.10^{-11} m_a \quad (36)$$

We then plot the dependence of the differential cross section in terms of cosθ when choosing s = 14 TeV with different m_ϕ.

According to the Fig. 3, it is clear that the dark matter effect in this scattering process can only be clearly observed at small scattering angles (*θ ≈ 0)*, while at large angles, the effect is negligible

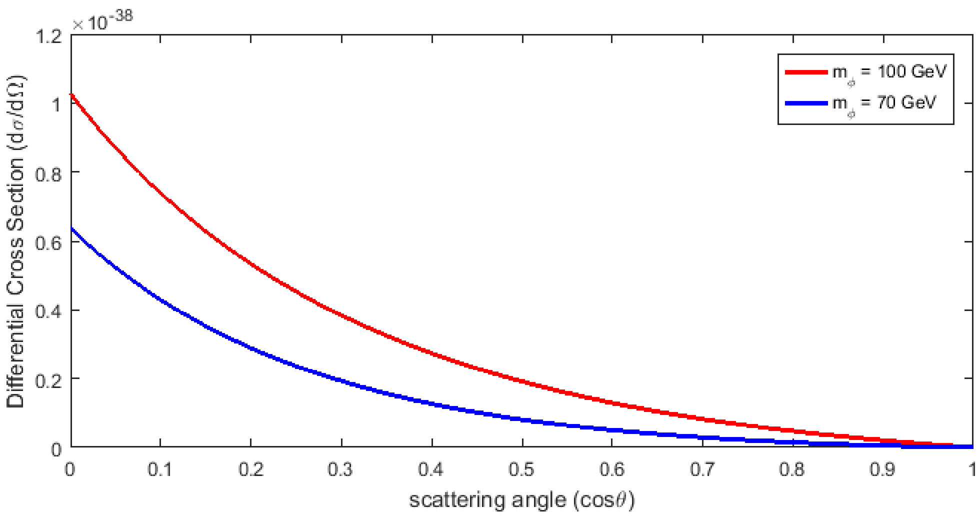

Fig. 3: Graph of the dependence of the differential cross section on the scattering angle

We plot the dependence of the total scattering cross section on the collision energy, with the scattering angle θ = 0. We see, the total scattering cross section in this case is inversely proportional to the collision energy, the larger energy collision, the smaller the total scattering cross-section.

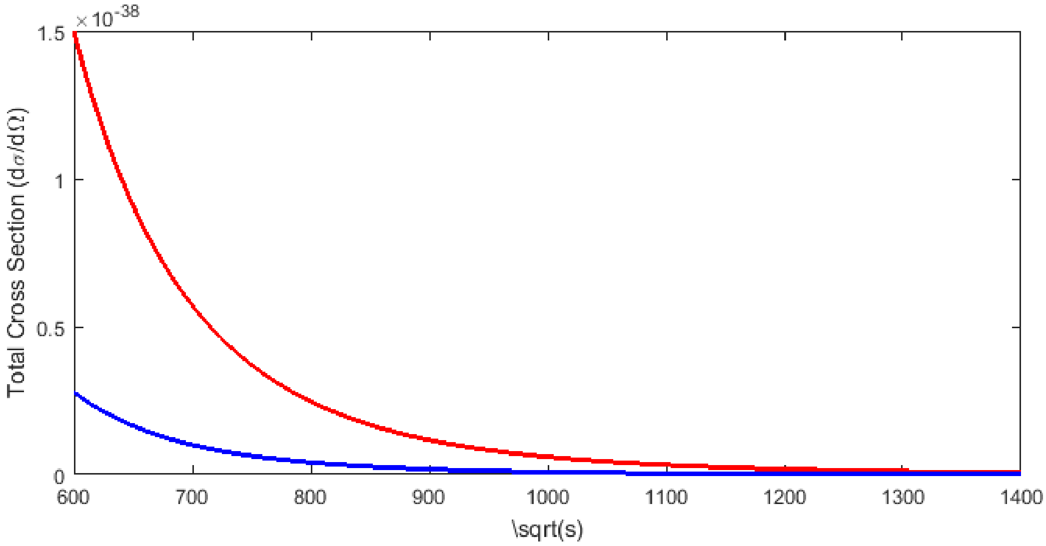

Fig. 4: Graph of the dependence of the total cross section on the collision energy

And finally we plot the dependence of the total scattering cross-section on the m_ϕ represented as follows

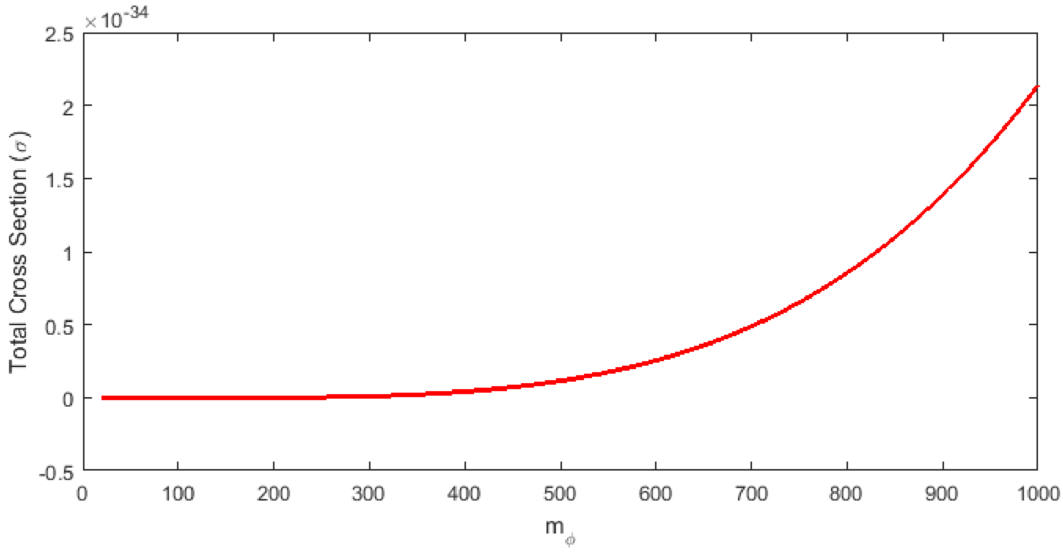

Fig. 5: Graph of the dependence of the total cross section on the mass of dark matter

The total scattering cross section in this process is directly proportional to the mass, as the mass increases; the total scattering cross section also increases. This is the same as in the case of $\gamma\gamma \rightarrow \gamma\gamma$ scattering [12,13] where radions are involved.

IV. CONCLUSION

In this work we have discussed theorical situation concerning ALPs particles that interac with Standard Model particles via couplings to electron. It should be emphasized that the effects of the radion have been found to be quite strong [11,12,13]. A scenario of particular interest is ALPs coupled electron to a light scalar DM particle. In this case, DM may pair – annihilate into photons DM can couple-annihilate into photons and therefore it is very difficult for us to observe DM-generating effects experimentally. Our results are attractive because of possible connection to radion and dark matter. We hope that future experiments will confirm the existence of radion. Works along these lines are in progress.

ACKNOWLEDGEMENTS

The author (Xuan N.N) would like to thank to Prof. N. S. Han for supporting and making useful remarks to improve this paper. This work is also supported by Le Qui Don University.

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