International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-9, Issue-4; Apr, 2022
Journal Home Page Available: https://ijaers.com/
Article DOI: https://dx.doi.org/10.22161/ijaers.94.40

Analysis nonlinear vibrations of the three - phase composite shallow cylindrical shell

Huyen TTH, Luot QT

Department of Mechanics, Le Quy Don Technical University, Hanoi City 100000, Vietnam
Email: huonghuyen.hvktqs@gmail.com

Received: 21 Mar 2022,

Received in revised form: 20 Apr 2022,

Accepted: 25 Apr 2022,

Available online: 30 Apr 2022

***Keywords*— Nonlinear vibrations; Three-phase composites.**

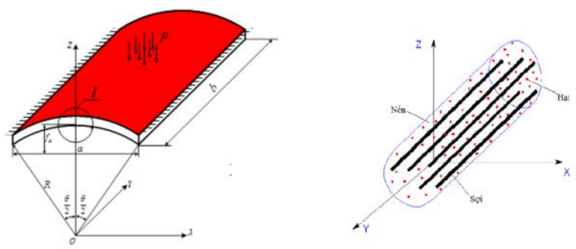
***Abstract*—** *This paper presents the results the study nonlinear vibrations of the three-phase composite shallow cylindrical shell. The differential equation describing the nonlinear vibrations of the shell is solved by the Newmark direct integration method combined with the Newton Raphson iterative method. Numerical simulation by finite element method is used to calculate the vibration of the structure. The results of the digital survey allow to make quantitative comments, technical recommendations, help the designers and users to orient effective applications in technical fields.*

I. INTRODUCTION

Three-phase composite is a material consisting of a matrix phase, a fiber phase and a particle phase, which has been studied by Vanin G.A and Duc N.D. (1996a, 1996b)[7,8]. In the 1997 publications, Vanin G.A. and Duc N.D determined the elastic modulus for three-phase composite materials 3Dm (Vanin G.A. and Duc N.D., 1997) and 4Dm (Duc N.D., 1997a). An overview of three-phase composite materials was also found in the study of Minh D.K. (2011). Recently, Duc N.D. et al. (2011) also studied the nonlinear stability of three-phase polymer composite panels under thermal and mechanical load conditions (Duc N.D. et al., 2013; 2014). In this report, the authors studied the nonlinear dynamic response of the multilayer three-phase composite shallow cylindrical shell. The formulas are based on classical shell theory, taking into account geometric nonlinearities.

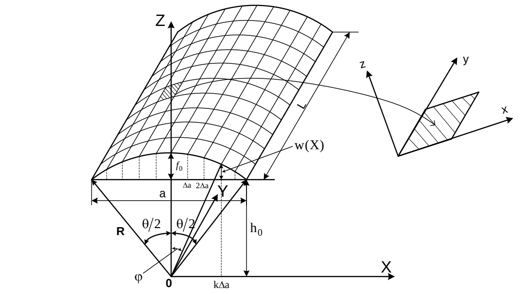
II. MATHEMATICAL MODELS AND ASSUMPTIONS

The cylindrical shell is considered as the shell with the ratio $\frac{f_0}{l_{min}} \leq \frac{1}{5}$, where f_0 - the curvature of the shell, l_{min} = min(a, L), a, L- the equal projection dimensions of the shell (Fig.2). According to the finite element method, the cylindrical shell can be discretized by flat elements, whereby the shell is a finite combination of 9 node-pointed flat elements, called the flat shell element, where each “flat shell element” can be seen as a combination of two types of elements: a 9 node flat element, each with 2 degrees of freedom (u_i, v_i) and a 9 node flat shell element subjected to combined bending - torsion, each node has 4 degrees of freedom (w_i, θ_{xi}, θ_{yi}, θ_{zi}), as shown in Fig.2. In the report, the authors use the thick-shell theory, which satisfies the Reissner-Mindlin theory.

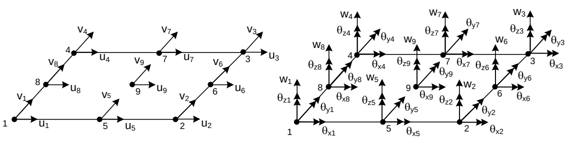


a, Shell model, b, Three-phase polymer composites model

Fig. 1(a,b): The model of the problem



a, Finite element model of the shell



b, Flat element in tension (compression)and flat shell element in combined bending - torsion

Fig. 2 (a,b): Finite Element Model

III. DETERMINATION OF ELASTIC COEFFICIENTS FOR THREE-PHASE COMPOSITE

For three-phase composite materials (polymer matrix, fiber and particle). According to [2], the elastic coefficient of three-phase composites Vanin G.A. determined in 2 steps:

The first step: Considering a two-phase composite consisting of the initial matrix phase and particles, such a composite is considered to be homogeneous, isotropic, and has two elastic coefficients. The determination of the elastic coefficients for composites filled with spherical particles is determined, taking into account the interaction between the particles and the matrix. The elastic coefficients of the grain-reinforced composite are now called hypothetical composites.

Second step: determine the elastic coefficients of the composite between the assumed foundation and the reinforcing fibers.

Assuming the components of the composite (matrix, fibers, particles) are all homogeneous, isotropic, then we will denote: E_n, G_n, ν_n, ψ_n; E_s, G_s, ν_s, ψ_s; E_h, G_h, ν_h, ψ_h are denoted by the modulus of elasticity, modulus of elasticity of shear, modulus of volume deformation, Poisson's coefficient, and composition ratio (by volume) of the matrix and particles, respectively. From here on, the quantities related to the matrix will have the *n*-index; relative to the particle is the *h*-index. According to [3], the elastic modulus of the assumed composite as follows:

$$\bar{E} = \frac{9\bar{K}\bar{G}}{3\bar{K}+\bar{G}};\ \bar{\nu} = \frac{3\bar{K}-2\bar{G}}{6\bar{K}-2\bar{G}} \qquad (1)$$

Where:

$$\bar{G} = G_n \frac{1-\psi_h(7-5\nu_n)H}{1+\psi_h(8-10\nu_n)H};\ \bar{K} = K_n \frac{1+4\psi_h G_n L(3K_n)^{-1}}{1-4\psi_h G_n L(3K_n)^{-1}} \qquad (2)$$

with

$$L = \frac{K_h - K_n}{K_h + (4G_n/3)};\ H = \frac{(G_n/G_h)-1}{8-10\nu_n+(7-5\nu_n)(G_n/G_h)};$$

$$G_i = \frac{E_i}{2(1+v_i)};\ \ i=n,s,h.$$

The modulus of elasticity for the three-phase homogeneous fiber-reinforced composite is calculated according to Vanin's formula [1]:

$$E_{11} = \psi_s E_s + (1-\psi_s)\bar{E} + \frac{8\bar{G}\psi_s(1-\psi_s)(\psi_s-\bar{\nu})}{2-\psi_s+\bar{\chi}\psi_s+(1-\psi_s)(\chi_s-1)(\bar{G}/G_s)}$$

$$E_{22} = \left[\frac{\nu_{21}^2}{E_{11}} + \frac{1}{8\bar{G}}\left(\begin{array}{l}\dfrac{2(1-\psi_s)(\bar{x}-1)+(x_s-1)(\bar{x}-1+2\psi_s)(\bar{G}/G_s)}{2-\psi_s+\bar{x}\psi_s+(1-\psi_s)(x_s-1)(\bar{G}/G_s)} \\ +2\dfrac{\bar{x}(1-\psi_s)+(1+\psi_s\bar{x})(\bar{G}/G_s)}{\bar{x}+\psi_s+(1-\psi_s)(\bar{G}/G_s)}\end{array}\right)\right]^{-1}$$

$$G_{12} = \bar{G}\frac{1+\psi_s+(1-\psi_s)(\bar{G}/G_s)}{1-\psi_s+(1+\psi_s)(\bar{G}/G_s)};$$

$$G_{23} = \bar{G}\frac{\bar{x}+\psi_s+(1-\psi_s)(\bar{G}/G_s)}{(1-\psi_s)\bar{x}+(1+\bar{x}\psi_s)(\bar{G}/G_s)};\ x_s = 3-4\nu_s;$$

$$\nu_{23} = \left[\frac{-\nu_{21}^2 E_{22}}{E_{11}} + \frac{E_{22}}{8\bar{G}}\left(\begin{array}{l}2\dfrac{(1-\psi_s)\bar{x}+(1+\psi_s\bar{x})(\bar{G}/G_s)}{\psi_s+\bar{x}+(1-\psi_s)(\bar{G}/G_s)} \\ -\dfrac{2(\bar{x}-1)(1-\psi_s)+(x_s-1)(\bar{x}-1+2\psi_s)(\bar{G}/G_s)}{2+\bar{x}\psi_s-\psi_s+(1-\psi_s)(x_s-1)(\bar{G}/G_s)}\end{array}\right)\right]$$

$$\nu_{21} = \bar{\nu} - \frac{(\bar{x}+1)(\bar{\nu}-\nu_s)\psi_s}{2-\psi_s+\bar{x}\psi_s+(1-\psi_s)(x_s-1)(\bar{G}/G_s)};$$

$$\nu_{12} = \frac{E_{11}}{E_{22}}\nu_{21};\ \bar{x} = 3-4\bar{\nu};$$

IV. DOMINANT EQUATION

4.1. The relationship between strain and displacement

When taking into account the deformation of the mean surface of the element, the strain vector components are related to the displacement field according to the expression:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \gamma_{xy}^o\end{Bmatrix} + z\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix} + \begin{Bmatrix}\varepsilon_x^N\\ \varepsilon_y^N\\ \gamma_{xy}^N\end{Bmatrix}, \qquad (3)$$

Where:

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & -\frac{1}{R_x} \\ 0 & \frac{\partial}{\partial y} & -\frac{1}{R_y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix}$$ is the linear strain vector,

R_x, R_y are the radius of curvature in the x and y directions, respectively

$$\{\kappa\} = \{\kappa_x \ \kappa_y \ \kappa_{xy}\}^T = \left\{\frac{\partial\theta_y}{\partial x} \ \frac{\partial\theta_x}{\partial y} \ \frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right\}^T$$ is the bending and torsion curvature vectors,

$$\begin{Bmatrix} \varepsilon_x^N \\ \varepsilon_y^N \\ \gamma_{xy}^N \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{\partial w_0}{\partial x} & 0 \\ 0 & \frac{\partial w_0}{\partial y} \\ \frac{\partial w_0}{\partial y} & \frac{\partial w_0}{\partial x} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} w_0$$ is the nonlinear strain vector.

4.2. Relationship between stress and strain

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} \\ Q'_{12} & Q'_{12} & Q'_{26} \\ Q'_{16} & Q'_{26} & Q'_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k \quad (4)$$

Where:

$$Q'_{11} = Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta$$
$$Q'_{12} = Q_{12}(\cos^4\theta + \sin^4\theta) + (Q_{11} + Q_{12} - 4Q_{66})\sin^2\theta\cos^2\theta$$
$$Q'_{16} = (Q_{12} - Q_{22} + 2Q_{66})\cos\theta\sin^3\theta) + (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta$$
$$Q'_{22} = Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta$$
$$Q'_{26} = (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta) + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta$$
$$Q'_{66} = Q_{66}(\cos^4\theta + \sin^4\theta) + (Q_{11} + Q_{22} - 2(Q_{12} + Q_{66}))\sin^2\theta\cos^2\theta$$
$$Q_{11} = \frac{E_{11}}{1-(E_{22}/E_{11})v_{12}^2}; Q_{22} = \frac{E_{22}}{1-(E_{22}/E_{11})v_{12}^2} = \frac{E_{22}}{E_{11}}Q_{11};$$
$$Q_{12} = v_{12}Q_{22} \quad Q_{66} = G_{12}$$

4.3. Internal force components

The surface force vector $\{N\} = \{N_x \ N_y \ N_{xy}\}^T$, bending moment, torsion moment, $\{M\} = \{M_x \ M_y \ M_{xy}\}^T$ in the shell element with n composite layers are determined as follows:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_0\} + \{\varepsilon_N\} \\ \{\kappa\} \end{Bmatrix} \quad (5)$$

Where:

$$[A] = \left(\sum_{k=1}^{n}[Q_{ij}']_k (z_{k+1} - z_k)\right),$$
$$[B] = \left(\frac{1}{2}\sum_{k=1}^{n}[Q_{ij}']_k (z_{k+1}^2 - z_k^2)\right),$$
$$[D] = \left(\frac{1}{3}\sum_{k=1}^{n}[Q_{ij}']_k (z_{k+1}^3 - z_k^3)\right),$$

4.4. Differential equation of vibration of a three phase composite shell

After building the finite element model of the structure is built, the structural analysis is performed. This work includes:

- Build element equations (element stiffness matrix, element load vector);
- Connect elements to create the overall stiffness matrix;
- Set up the general equation;
- Solve the general equation;
- Calculate the necessary results from the solutions of the general equation.

The nonlinear differential equation which describes the vibration of the shell:

$$[M_g]\{\ddot{q}\} + [C_g]\{\dot{q}\} + [K_g]\{q\} = \{F\} \quad (6)$$

Where: $[M_g]$- Matrix of the overall mass of the shell; $[K_g]$ - the the overall stiffness matrix of the shell; $[C_g]$- The overall resistance matrix of the shell, calculated by the formula: $[C_g] = (\alpha[M_g] + \beta[K_g])$.

Equation (6) is a nonlinear differential equation that is solved by the Newmark method combined with the Newton-Raphson iterative method.

V. NUMERICAL RESULTS AND DISCUSSION

5.1. The starting problem:

Structural parameters: Three - phase composite pillar panel, rectangular projection size, total thickness h = 0,0025m, radius of curvature R = 1,0m, length L = 0,30 m, opening angle θ = 30^0. The composite shell consists of 5 layers, the composite layers are made of Graphite/Epoxy T300/976, each layer has a thickness of h_1 = 0,0005m; the ratio of grain and matrix is 0,3. Considering the case of composite layers arranged symmetrically [-α/ α /0/ α /- α], with α = 45^0,

Material characteristics of Graphite/Epoxy T300/976 are as follows: Graphite-Epoxy T300/976 : E_{11} = 150.10^9 N/m², E_{22} = E_{33} = 9.10^9 N/m², G_{12} = G_{13} = $7,1.10^9$ N/m²,

G_{23} = 2,5.10^9 N/m^2, $\nu_{12} = \nu_{23} = \nu_{32}$ = 0.3, ρ_{GE} = 1600 kg/m^3.

Load parameters: a short-term load in the form of a shock wave distributed on the upper surface of the shell, the load is as follows:

$$p(t) = p_{max} F(t),$$
$$F(t) = \begin{cases} 1 - \dfrac{t}{\tau_\theta}: & 0 \le t \le \tau_\theta \\ 0: & t > \tau_\theta \end{cases} . \qquad (7)$$

In which: p_{max} = 1.10^5 N/m^2, τ_θ= 0,025s. The shell was clamped along two straight edges: u = 0, v = 0, w = 0, θ_x = 0, θ_y = 0, θ_z = 0 at x = 0 and x = a. The center point on the upper surface of the shell (point A (Fig.3)) was considered. The finite element model of the problem is shown in Fig.4.

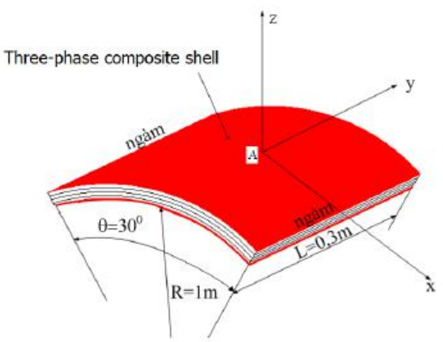


Fig. 3: Real model of the problem

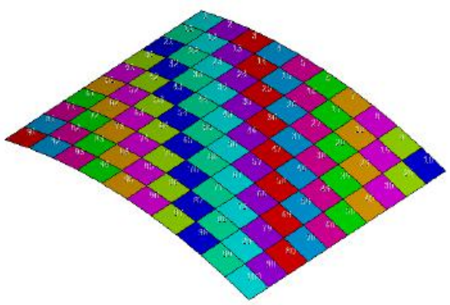


Fig. 4: Finite element model of the problem

5.2. The influence of some factors on the nonlinear vibration of the shellI

5.2.1. Effect of nonlinear properties

To examine the effect of nonlinearity, the linear problem was compared with the solved nonlinear problem. Fig. 6 and Fig.6, Fig.7, Fig.8 and table 1 show the displacement and stress variations at point A in two cases. The time response of displacement, stress at the calculation point of the linear problem is different from the nonlinear problem both in amplitude and cycle. In particular, the response values of the nonlinear problem are much larger than that of the linear problem, which shows that the calculation by the nonlinear method is more stable and safer. According to the authors, this is the advantage of solving nonlinear problems for this particular case.

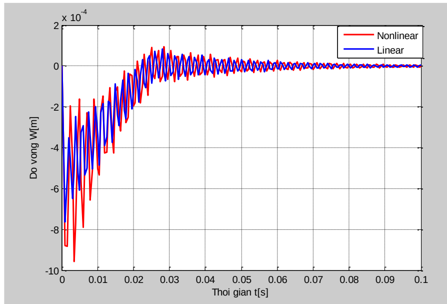


Fig. 6: Time history response of vertical displacement W at point A

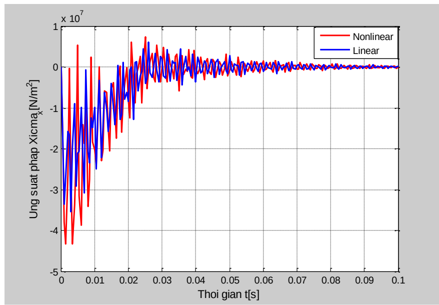


Fig. 7:. Time history response of stress σx at point A

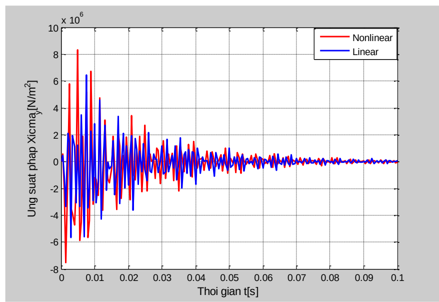


Fig. 8: Time history response of stress σ_y at point A

Table 1. Maximum displacement and stress at point A

		W_{max} (m)	σ_x^{max} (N/m^2)	σ_y^{max} (N/m^2)
Cases	***Nonlinear***	0,00096	4,302.10^7	0,8305.10^7
	Linear	0,00077	3,540.10^7	0,6470.10^7
Compare (%)		*24,68*	*21,53*	*28,36*

5.2.2. Effects of particles ratio

To consider the influence of particles ratio, the author proceeds to solve the problem with 3 cases of different particles ratio.

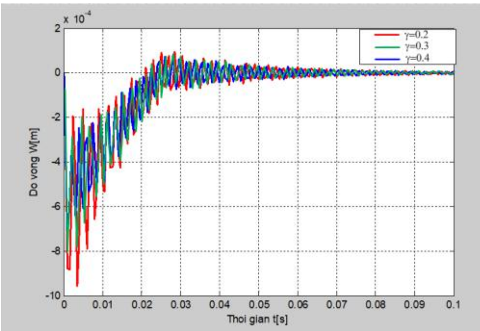


Fig. 9: Vertical displacement variation at point A with different particles ratio

The time response of displacement at the shell point in the cases of different particles ratio is different in both amplitude and period. In which, the deflection response values in the case of 0.2 is the largest, in the case of 0.4 is the smallest in the 3 cases. This shows that in the reasonable range of the ratio between the particles and the matrix, when increasing the ratio of the particles, the response to the maximum displacement will decrease.

5.2.3. Effect of load amplitude

Investigate the problem with load amplitude pmax varying from 0.5,105N/m^2 to 1,66,105N/m^2. The results of the variation of the maximum values of displacement and stress at point A of the shell are shown in graphs in Fig.10, Fig.11, Fig.12.

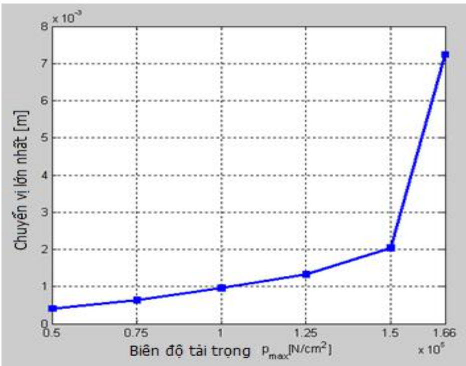


Fig. 10: Displacement W_{max} by amplitude pmax of load

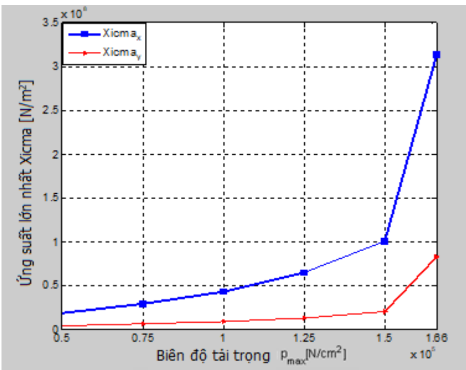


Fig. 11: Maximum stress by amplitude pmax of load

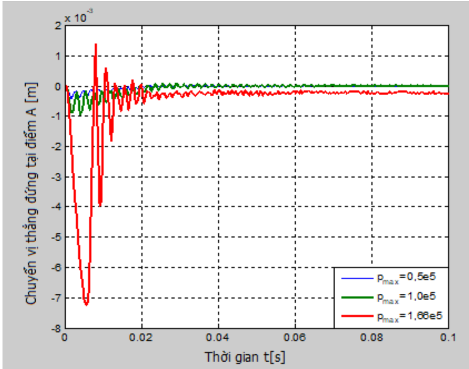


Fig. 12: Time history response of vertical displacement W at point A

The load amplitude has a great influence on the dynamic response of the shell. For the survey problem, when the load amplitude increases, both the maximum displacement and the maximum stress at point A increase nonlinearly. The rate of increase of the maximum values of displacement and stress is large when pmax is greater than 1.25.105N/m2. When the load amplitude pmax = 1,66,105N/m2, both displacement and stress increase very strongly, the shell oscillates and balances to another position, with hysteresis. According to the dynamic stability criterion of Budiansky, B and Roth, R.S [1], the shell is destabilized and the critical amplitude value.

VI. CONCLUSIONS

This paper presents a finite element method to analyze the dynamic response of a 3-phase composite cylindrical shell under the action of dynamic loads. The formulations are based on classical multilayer shell theory taking into account geometric nonlinearity. With the numerical survey results on the problem classes with the change of many parameters, it is the basis for the selection of reasonable parameters for the 3-phase composite cylindrical shell structure subjected to dynamic loads.

REFERENCES

[1] Budiansky B., and Roth R.S., (1962). Axisymmetric dynamic buckling of clamped shallow spherical shells in Collected papers on instability of shell structures. Technical Note NASA TN D-1510, Langley Research Center, Langley Station, VA.

[2] Vanin G.A., (1985). Micro-Mechanics of composite materials. “Nauka dumka”, Kiev. Rusian.

[3] Jonghwi Lee and Albert F. Yee (2001). Fracture Behavior of Glass Bead Filled Epoxies: Cleaning Process of Glass Beads. © 2000 John Wiley & Sons, Inc. J Appl Polym Sci 79: 1371–1383, 2001.

[4] Randall M. German (2016). Particulate Composites Fundamentals and Applications. Springer International Publishing Switzerland. EBook ISBN 978-3-319-29917-4, DOI 10.1007/978-3-319-29917-4.
[5] Roger N. Rothon (2003). Particulate-Filled Polymer Composites. Rapra Technology Limited Shawbury, Shrewsbury, Shropshire, SY4 4NR, UK. ISBN: 1-85957-382-7.
[6] Rothon, R . Particulate-filled Polymer Composites; Longman Scientific & Technical: Essex, U.K., 1995.
[7] Vanin, G. A., and N. D. Duc (1996a). The theory of spherofibrous composite.1: The input relations, hypothesis and models. Mechanics of Composite Materials, 32(3), pp. 291-305.
[8] Vanin, G. A. and N. D. Duc (1996b). The theory of spherofibrous composite.2: The fundamental equations. Mechanics of Composite Materials, 32 (3), pp. 306-316.