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Sinusoidal Water Wave Dispersion Equation Formulated Using the Total Velocity Potential Equation

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Abstract— *In this study, the dispersion equation was formulated with the same procedure as the linear wave theory dispersion equation formulation procedure, but in this study, the total velocity potential was used. The dispersion equation obtained has the same form as the linear wave theory dispersion equation. There is a difference in its coefficients which causes the obtained wavelength to be half of the wavelength dispersion equation of the linear wave theory.*

I. INTRODUCTION

The solution to the Laplace equation with the variable separation method is in the form of a superposition of two functions, namely $cosine$, and $sine$ (Dean, 1991). In Dean (1991), the formulation of the linear wave theory dispersion equation is done by using a single component, namely the $cosine$ only. In this study, the velocity potential equation was carried out at the characteristic point where the value of the $cosine$ function is the same as the value of $the\ sine$ function. Therefore, the total velocity potential is in the form of $cosine$. The function obtained at this characteristic point will apply to the $cosine$ and $sine$.

The formulation of the dispersion equation was carried out with the same procedure as the formulation of the linear wave theory dispersion equation, which is to formulate the water level equation using the Bernoulli (Milne-Thomson, 1960), carried out on the water surface. Then, with the water level equation obtained, the equation was formulated dispersion using linearized Kinematic Free Surface Boundary Condition.

Considering that the same velocity potential was used and carried out with the same procedure, a dispersion equation with the same shape but a different coefficient was obtained. The dispersion equation was analyzed using the wavenumber conservation equation. It was found that the dispersion equation obtained was not a function of water depth.

In deep water, wave steepness was calculated using the maximum wave height from Wiegel (1949,1964). Wave steepness was compared with the critical wave steepness of Michell (1893) and Toffoli et all (2010).

The dispersion equation in shallow water is formulated using the wave number and the energy conservation equation. With those equations, equations for the change in wave height and wavenumber due to water depth change were formulated. Even though it does not aim to develop a shoaling model, it produces a simple shoaling equation.

II. GENERAL SOLUTION OF LAPLACE'S EQUATION

The Laplace's equation(Milne-Thomson, 1960)on a two-dimensional axis system, namely(x, z)in the following form,

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial z^2}=0 \quad ....(1)$$

$\phi=\phi(x,z,t)$is the velocity potential,$x$is the horizontal axis located on the still water surface;$z$is the vertical axis;$t$is time. The properties of the velocity potential(Milne-Thomson (1960))are as follows,

$$\frac{\partial\phi}{\partial x} = -u$$
$$\frac{\partial\phi}{\partial z} = -w$$

$u$is the velocity of the water particles in the -x horizontal direction. Meanwhile,$w$is the velocity of the particles in the -z vertical direction.

The solution (1) is analytically carried out using a well-known method, namely the Variable Separation Method, where the solution is considered to be a multiplication of three functions(Bland, 1961), namely,

$\phi(x,z,t) = X(x)\,Z(z)\,T(t)$ ....(2)

$X(x)$is a function$x$only, $Z(z)$is a function$z$only and$T(t)$is a function$t$only, which gives the general solution

$\Phi(x,z,t) = (Acoskx + B\sin kx)$
$(Ce^{kz} + De^{-kz})sin(\sigma t)$….(3)

This equation is a total general solution of Laplace's equation, which consists of two components, namely the $\cos kx$and the$\sin kx$ components.

2.1. Analysis of the Constants in the Solution of the Laplace Equation

In (3), there are several constants, namely A, B, C, and D, which need to be formulated. These constants can be obtained by working on boundary conditions, where the Laplace equation is known as a boundary value problem where the specific solution was obtained by working on boundary conditions.

Working on the kinematic bottom boundary condition on the flat bottom, as done by Dean (1991), was used to formulate the constants. The kinematic bottom boundary conditions on the flat bottom are:

$w_{-h} = 0$….(4)

Using (3), the velocity of the water particles in the -z vertical direction is

$$w = -\frac{\partial\phi}{\partial z} = -k(Acoskx + B\sin kx)$$
$$(Ce^{kz} - De^{-kz})sin(\sigma t)$$

Then

$$w_{-h} = -k(Acoskx + B\sin kx)$$
$$(Ce^{-kh} - De^{kh})sin(\sigma t)$$

Substitution to (4),

$-k(Acoskx + B\sin kx)(Ce^{-kh} - De^{kh})sin(\sigma t) = 0$

This condition can be occured in

$Ce^{-kh} - De^{kh} = 0$

Where,

$C = De^{2kh}$ ……(5)

Substituting this equation into (3) will get the equation,

$\Phi(x,z,t) = 2De^{kh}(Acoskx + B\sin kx)$
$\cosh k(h+z)\,sin(\sigma t)$ ……(6)

To get the constants A and$B$,it was done that on a wave as a whole there is only one single characteristic, where$A = B$,

$$\Phi(x,z,t) = 2ADe^{kh}(coskx + \sin kx)$$
$$\cosh k(h+z)\,sin(\sigma t)$$

Defined $G = 2ADe^{kh}$,

$\Phi(x,z,t) = G(coskx + \sin kx)$
$\cosh k(h+z)\,sin(\sigma t)$….(7)

Proving that there is only one wave characteristic. Then, an analysis was carried out on G through the velocity equation, for example, the vertical velocity w.

$$w(x,z,t) = -Gk(coskx + \sin kx)$$
$$\sinh k(h+z)\,sin(\sigma t)$$

The particle velocity$w$has units(m/sec)while the wavenumber $k$has units(m^{-1}). Then, the unit of$G$ is$(m.m/sec)$, indicating that$G$is the rate of transfer of wave energy, which must be a single value, implying that$A = B$.

The formulation can be done with other procedures, namely (6) done at the characteristic point where $\cos kx = \sin kx$, then (6) becomes,

$\Phi(x,z,t) = 2De^{kh}(A+B)\cos kx$

$\cosh k(h+z)\sin\sigma t$…(8)

For the$cosine$ component. As for the$sine$ component,

$\Phi(x,z,t) = 2De^{kh}(A+B)\,sin\,kx$
$\cosh k(h+z)\sin\sigma t$…(9)

Both in (9) and (10), can be defined:

$G = 2De^{kh}(A+B)$ ……(10)

Then, (8) and (9) become:

$\Phi(x,z,t) = G\,coskx\,\cosh k(h+z)\,sin(\sigma t)$ …(11)

$\Phi(x,z,t) = G\,sin\,kx\,\cosh k(h+z)\,sin(\sigma t)$ …(12)

This proves the singularity of the value of G. However, it also indicates that the analysis of wave dynamics using (11) or (12)uses$G$which is a combination of two energies.

2.2. The Wavenumber Conservation Equation

In solving the Laplace equation with the variable separation method, there is no flat bottom assumption. The flat bottom assumption is only in the formulation of the constant. Furthermore, if (7) is done on a sloping bottom, there will be values of $\frac{dh}{dx}$, $\frac{dk}{dx}$, and $\frac{dG}{dx}$.In the following section, an analysis of the $\frac{dk}{dx}$ was done.

It has been mentioned that in solving the Laplace equation with the Variable Separation Method, it is assumed that the flow potential consists of 3 components, as presented in (2).

In this equation,

$X(x) = coskx$, isa function of$x$only,

$Z(z) = \cosh k\,(h+z)$, is a function of$z$only

$T(t) = sin\sigma t$, is a function of$t$only.

Given the nature of the function, it must be

$$\frac{\partial Z(z)}{\partial x} = 0$$

Given $Z(z) = \cosh k\,(h+z)$, then

$\frac{\partial k(h+z)}{\partial x} = 0$.......(13)

At$z = \frac{A}{2}$,where$A$is the wave amplitude

$\frac{\partial k\left(h+\frac{A}{2}\right)}{\partial x} = 0$.......(14)

At$z = 0$,

$\frac{\partial kh}{\partial x} = 0$ ....(15)

III. FORMULATION OF THE DISPERSION EQUATION

The dispersion equation will be formulated using velocity potential (11) and on a flat bottom. The velocity of the particle in the $x$horizontal direction and the $z$vertical direction is,

$u = -\frac{\partial \Phi}{\partial x} = Gksinkxcoshk(h+z)sin\sigma t$ ...(16)$w = -\frac{\partial \Phi}{\partial z} = -Gkcos\,kx \sinh k(h+z) \sin \sigma t$ ...(17)

1.1. Water Surface Equation$\eta(x,t)$

To get the dispersion equation, first, the water surface equation is formulated (Dean (1991)). The equation was formulated using the Bernoulli equation which is carried out on the surface, namely

$-\frac{\partial \phi_\eta}{\partial t} + \frac{1}{2}\left(u_\eta^2 + w_\eta^2\right) + g\eta + \frac{p_\eta}{\rho} = C(t)$..(18)

The η index shows that the relevant variable is applied to the surface of the wave, g is the acceleration due to gravity, p is the pressure acting on the fluid particles, ρ is the density of the fluid, and $C(t)$ is a constant that can be used zero for the periodic function (Dean (1991)).

$p_\eta$is the pressure on the surface, i.e., atmospheric pressure, by using atmospheric pressure as the reference pressure,$p_\eta = 0$.After entering the dynamic free surface boundary condition$p_\eta = 0$, (18) divided by$g$

$-\frac{\partial \phi_\eta}{g\partial t} + \frac{1}{2g}\left(u_\eta^2 + w_\eta^2\right) + \eta = 0$ ....(19)

The 2nd term is kinetic energy, while the 3rd term is potential energy. At a small wave amplitude, the 2nd term will be much smaller than the 3rd term. Therefore, it can be ignored, the Bernoulli equation becomes,

$$-\frac{\partial \phi_\eta}{g\partial t} + \eta = 0$$

The obtained water level equation is:

$$\eta(x,t) = \frac{1}{g}\frac{\partial \phi_\eta}{\partial t}$$

Substituting the flow potential equation,

$\eta(x,t) = \frac{G\sigma\, coshk(h+\eta)}{g} \cos kx\, cos\,\sigma\, t$...(20)

For a periodic function, $\frac{G\sigma\, coshk(h+\eta)}{g}$is a constant. Defined

$$A = \frac{G\sigma coshk(h+\eta)}{g}$$

However, it has been mentioned that the use of a single velocity potential (11), implies that G is the sum of the energies of the two waves, therefore the wave amplitude equation must be divided by 2,

$A = \frac{G\sigma coshk(h+\eta)}{2g}$ ......(21)

$A$is the wave amplitude. But, G can not be calculated using this equation. G must be calculated using

$$G = \frac{gA}{2\sigma coshk(h+\eta)}$$

This will be discussed in the next paper.

The surface water level equation becomes

$\eta(x,t) = Acoskx\, cos\,\sigma\, t$....(22)

3.2. Dispersion Equation

The dispersion equation is formulated using the Kinematic Free Surface Boundary Condition (KFSBC),

$w_\eta = \frac{\partial \eta}{\partial t} + u_\eta \frac{\partial \eta}{\partial x}$ .....(23)

For long waves and small amplitudes, $u_\eta$and$\frac{\partial \eta}{\partial x}$are small numbers. Thus,$u_\eta \frac{\partial \eta}{\partial x}$becomes a very small and negligible number, andKFSBC become,

$w_\eta = \frac{\partial \eta}{\partial t}$.....(24)

Substituting (17) and (22)into (24) and the same terms on the left and right sides of the equation cancel each other out, the following equation is obtained

$2\sigma^2 = gk\, tanhk(h+\eta)$ ....(25)

In (25) there is anηvariable whose value needs to be known. From the beginning, the formulation was carried out with a single flow potential, namely (11), where the formulation was carried out at the characteristic point where$coskx = \sin kx$, on the time variable, also used the characteristic point where$cos\,\sigma\, t = sin\,\sigma\, t$. Thus, a characteristic point

that is complete is the point where$cos kx = \sin kx == \frac{1}{2}\sqrt{2}andcos\,\sigma\, t = sin\,\sigma\, t = \frac{1}{2}\sqrt{2}$.
The solution obtained at this point is valued for bothsinandcos. At this point, the water level elevation of (22) is,

$$\eta = \frac{A}{2}$$

Then (25) becomes,

$$2\sigma^2 = gktanhk\left(h + \frac{A}{2}\right) .....(26)$$

Then, the wave number conservation equation (14) was reviewed.

$$\frac{dtanhk\left(h + \frac{A}{2}\right)}{dx} = \frac{1}{cosh^2 k\left(h + \frac{A}{2}\right)} \frac{dk\left(h + \frac{A}{2}\right)}{dx} = 0$$

It was found that

$$tanhk\left(h + \frac{A}{2}\right) = c \quad .....(27)$$

where c is a constantnumber.

Equation (26) must apply to both deep and shallow water. In deep water

$$tanhk\left(h + \frac{A}{2}\right) = 1 .....(28)$$

Then the constant c in (27) is 1. The dispersion equation becomes,

$$2\sigma^2 = gk \text{ ....(29)}$$

Therefore, the dispersion equation is only for deep water.

Equation (29) gives a wavelength $L = \frac{2\pi}{k}$half of the well-known linear wave theory wavelengths in the deepwater of the form

$$\sigma^2 = gk \text{ ....(30)}$$

In Table (1), the comparison between wavelengths of (29), L_{29}, and wavelengths of (30), L_{30}, where$L_{29} = 0.5L_{30}$. The wave steepness of each wavelength is calculated using the maximum wave height ofWiegel (1949,1964), namely

$$H_{0,max} = \frac{gT^2}{15.6^2} \text{....(31)}$$

Table.1: Comparison of wavelengths (29) and (30)

T (sec)	L_{30} (m)	L_{29} (m)	$\frac{H_{0,max}}{L_{30}}$	$\frac{H_{0,max}}{L_{29}}$
6	56.207	28.104	0.026	0.052
7	76.504	38.252	0.026	0.052
8	99.924	49.962	0.026	0.052
9	126.466	63.233	0.026	0.052
10	156.131	78.066	0.026	0.052
11	188.919	94.459	0.026	0.052
12	224.829	112.414	0.026	0.052
13	263.861	131.931	0.026	0.052
14	306.017	153.008	0.026	0.052
15	351.295	175.647	0.026	0.052

Wave steepness $\left(\frac{H}{L}\right)$generated by the two dispersion equations in Table (1), is very small, much smaller than the critical wave steepness ofMichell's criteria (1893),

$$\frac{H}{L} = 0.142 \quad \text{......(32)}$$

And criteria for Toffoli et al (2010)

$$\frac{H}{L} = 0.170 \quad \text{......(33)}$$

From this, although (29) produces a wavelength that is much smaller than (30), it still needs further development in order to achieve a wavelength that gives a critical wave steepness according to the criteria of Michell (1893)or Toffoli et al (2010).

In order to obtain a critical wave steepness following the Mitchell or Toffoli criteria, the left side (29) is multiplied by a coefficientγ,

$$2\gamma\sigma^2 = gk \text{....(34)}$$

$\gamma$is a coefficient that is greater than 1. By trial and error, the value of $\gamma = 2.75$is obtained which produces a wavelength with a wave steepness corresponding to the critical wave steepness of Michell (1893), with the calculation results presented in Table (2).

Table.2: Wavelength and wave steepness at $\gamma = 2.75$

T (sec)	L (m)	$\frac{H_{0,max}}{L}$
6	10.219	0.142
7	13.91	0.142
8	18.168	0.142
9	22.994	0.142
10	28.387	0.142
11	34.349	0.142
12	40.878	0.142
13	47.975	0.142
14	55.639	0.142
15	63.872	0.142

Meanwhile, if Toffoli et al criteria are used, it will obtain$\gamma = 3.285$.

However, (29) and (34), need more intensive analytical study , to fulfill the kinematic free surface boundary condition.

3.3. Dispersion equation in shallow water

To obtain the dispersion equation in shallow water, the wave number conservation equation is carried out. First, define the value of $k_0\left(h_0+\frac{A_0}{2}\right)$ in deep water where $tanhkk_0\left(h_0+\frac{A_0}{2}\right)=1$,

$k_0\left(h_0+\frac{A_0}{2}\right)=\theta\pi$.......(35)

where $\tanh(\theta\pi)=1$. Index 0 indicates the variable in deep water. The value of$\theta$is a positive number whose value is greater than or equal to 1. SPM (1984) uses the value of$\theta=1$. According to the law of conservation of wavenumbers (14),

$$k_h\left(h+\frac{A_h}{2}\right)=k_0\left(h_0+\frac{A_0}{2}\right)=\theta\pi$$

Then

$k_h=\frac{\theta\pi}{\left(h+\frac{A_h}{2}\right)}$ ......(36)

The index h indicates the variable at the water depth h, smaller than h_0. In this equation, there are two unknowns, namely k_h and A_h. One more equation of the relation between k_h and A_h is needed. The available equation is the energy conservation equation. The wave energy at one wavelength (Dean (1991)) is as follows,

$E=\frac{1}{8}\rho g\,H^2L$......(37)

g is the gravitational force, and ρ is the mass density of water. Assuming there is no loss of wave energy, then the relationship should be as follows,

$H_h^2L_h=H_0^2L_0$.....(38)

where H_0 and L_0 are wave height and wavelength in deep water, while H_h and L_h are wave height and wavelength at shallower water depth h. Equation (38) can be expressed as the equation for the wave number k_h,

$k_h=\frac{2\pi\,H_h^2}{E_0}$....(39)

Where $E_0=H_0^2L_0$, substitution (39) to (36) assuming a sinusoidal wave where $A_h=0.5H_h$, the equation for H_h is obtained.

$\frac{H_h^3}{4}+hH_h^2-\frac{\theta E_0}{2}=0$.....(40)

H_h is calculated by (40), then k_h is calculated by (39).

For example, the calculation of wavelength in shallow water used waves with a wave period of 8 sec. deep water wave height H_0 is calculated by (31), deep water wavelength L_0 is calculated by (29) or by (34) with $\gamma=1$. The calculation is done by using $\theta=1$, and ignoring breaking, the calculation results are compared with the linear wave theory dispersion equation, namely,

$\sigma^2=gk\tanh kh$ ....(41)

The calculation results are presented in Fig (1) below.

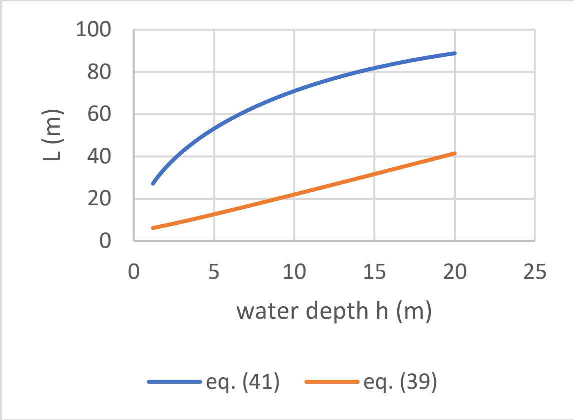

Fig.1: Wavelength of (41) and (39).

In Fig (2), the change in wave height to water depth (shoaling) is presented in the form of a nonlinear line. In the figure, the calculation is stopped when $\frac{H}{h}=0.80$ which is considered breaking. At that point, the wave height is $H=4.80$ m, the water depth is h= 5.99 m, and the wavelength is $L=14.40$ m. From the breaking conditions, it was found that the shoaling that occurred was too large.

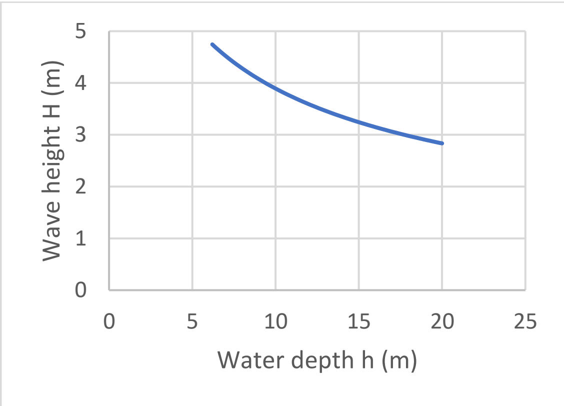

Fig.2: Shoaling on a wave period of 8 sec.

The magnitude of the resulting shoaling is because the wave energy is too large. After all, the wavelength L_0 calculated by (29) is too long. When used (34) using $\gamma=3.285$ where the value of γ is the result of critical wave steepness adjustment with the criteria of Toffoli et al (2010), it is obtained that the deep-water wavelength L_0 is shorter. Thus, the wave energy decrease, and the resulting shoaling is also not too large.

In Table (3), the results of the shoaling calculation are presented where L_0 is calculated by (34) using $\gamma=3.285$,and water depth coefficient $\theta=1$.The wave height at $\frac{H}{h}=0.78$ was compared with the average breaker height of the five-breaker height index (BHI) from a number of previous studies conducted by Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992), and Rattana Pitikon and Shibayama (2000).

Komar and Gaughan (1972)

$$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \qquad \text{..............(42)}$$

Larson and Kraus (1989),

$$\frac{H_b}{H_0} = 0.53\left(\frac{H_0}{L_0}\right)^{-0.24} \qquad \text{...............(43)}$$

Smith and Kraus (1990),

$$\frac{H_b}{H_0} = (0.34 + 2.74m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88m} \text{........(44)}$$

Gourlay (1992),

$$\frac{H_b}{H_0} = 0.478\left(\frac{H_0}{L_0}\right)^{-0.28} \qquad \text{.................(45)}$$

Rattana Pitikon and Shibayama (2000) :

$$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55)\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$$

.......(46)

In these BHI equations, $H_0$is deep water wave height, L_0 is deep water wavelength (calculated using linear wave theory, $k_0 = \frac{\sigma^2}{g}, L_0 = \frac{2\pi}{k_0}$), m is the bottom slope and H_b is breaker height. In this study, bottom slope $m = 0$ is used.

Breaker depth h_b in Table (3) is calculated by the breaker depth equation from SPM (1984), that is,

$$\frac{h_b}{H_b} = \frac{1}{b - \left(\frac{aH_b}{gT^2}\right)} \text{or} h_b = \frac{H_b}{b - \left(\frac{aH_b}{gT^2}\right)} \text{......(47)}$$

$$a = 43.75(1 - e^{-19.0m}) b = \frac{1.56}{1 + e^{-19.5m}}$$

Calculation with bottom slope $m = 0$ obtained $\frac{H_b}{h_b} = 0.78$.

Table.3: Comparison of equations (39+40) with BHI.

T (sec)	Eq. (39+40)		BHI	
	H_h (m)	h (m)	H_b (m)	h_b (m)
6	1.805	2.314	1.721	2.207
7	2.457	3.149	2.343	3.003
8	3.209	4.114	3.06	3.923
9	4.061	5.206	3.873	4.965
10	5.014	6.427	4.781	6.129
11	6.067	7.777	5.785	7.417
12	7.22	9.256	6.885	8.826
13	8.473	10.863	8.08	10.359
14	9.827	12.598	9.371	12.014
15	11.281	14.462	10.757	13.791

Therefore, Table (3) mentioned the value of $\frac{H}{h} = \frac{H_b}{h_b} = 0.78$. It is found that H is close to H_b and h is also close to h_b. This indicates that the shoaling of (39+40) can be improved by shortening the deep wáter wavelength and the wavelength L_0 calculated by (29) is still too long.

IV. CONCLUSION

A review of the critical wave steepness in deep water results that the wavelengths obtained using the total velocity potential are better than the wavelengths formulated using a single velocity potential, likewise with the results of studies in shallow water. However, it still needs to be shortened again.

In short, it can be done in a simple way, by increasing the coefficients in the dispersion equation. However, it is necessary to investigate the origin of these coefficients analytically based on hydrodynamic equations, especially the kinematic free surface boundary condition and the Bernoulli equation.

Considering that the formulation of the dispersion equation in deep water in this study was carried out following the procedure for formulating a linear wave dispersion equation and producing a dispersion equation in the same form as the linear wave theory dispersion equation, the dispersion equation obtained can also be referred to as a linear wave dispersion equation.

To conclude, the analysis of wave dynamics using the velocity potential solution of the Laplace equation with the Variable Separation Method should use the total velocity potential.

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