International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-9, Issue-7; July, 2022
Journal Home Page Available: https://ijaers.com/
Article DOI: https://dx.doi.org/10.22161/ijaers.97.41

Geometrical Model of bearing Capacity for a Safe Rectangular Vertically Loaded Shallow Pad Foundation on a Cohessionless (Sandy) Soil with Minimum Angle of Friction

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Received: 25 Jun 2022,

Received in revised form: 17 Jul 2022,

Accepted: 23 July 2022,

Available online: 31 July 2022

***Keywords* — Structural Footing, Bearing Capacity, Depth Factor, Inclination Factor, Geometry**

***Abstract*— *Provision of adequate structural footing in building infrastructures is sensitive and highly challenging. Thus to alleviate the challenges of the subject task, a model was developed for simulating behavior of a pad rectangular footing on a cohessionless soil under the action of a vertical load. The approach is purely mathematical techniques of modeling, concept and application. A governing equation was theoretically formulated based on a specified condition, theories and applicable parameters. This was experimentally simulated to describe the relationship between variable parameters and to predict the geotechnical behaviour (bearing capacity) of the sandy soil underneath a shallow foundation. The geometry is specified by footing length, L, breadth, B and the placement depth, D_f of the element. The model clearly established the interrelationship between the dominant parameters of the soil and footing geometry for determining bearing capacity to help engineering decisions in the design of a safe footing on a sandy soil environment.*

I. INTRODUCTION

1.1 General Background: The earth crust consists of the rock and soil. Rock is simply the hard solid mineral that forms part of the earth, while soil can be described from engineering point of view as all materials of the earth crust, organic and inorganic, overlying the rock. Engineering structures, like buildings, bridges and retaining walls are designed to be supported by the earth.

If a building is to be constructed on an outcrop of sound rock, no foundation is required. In the contrast, the foundations are provided in the designed to serve as a remedy for the deficiencies of whatever nature (whimsical) has provided for the support of the structure at a particular site. Thus, foundation is a crucial component, it is that part of the structure which is in direct contact with the soil and transmits loads to it. Therefore, Terzaghi, in one of his comments, describes foundation as a ‘‘necessary evil” (Murthy, 2008). However, the stability of such engineering structures depends upon the capability (at design) and stability (at service live) of the supporting soil. The significant of foundation in the design and construction of engineering projects remains a determinant for functionality and so a sensitive challenge to the engineers.

From civil engineering point of view, the adequacy of a structural footing is majorly a function of its geometry. However, the geometry of the footing is dictated by the strengths of the supporting types of soil underneath the foundation. That is to say, different types of soil require different features of foundation geometry under natural condition and a set of specified structural materials grade.

Foundation is therefore, an integral part of a structure, which may be shallow or deep. A foundation is customarily regarded as shallow if the depth of the foundation is less than or equal to the width of the foundation (Terzaghi, 1943). Later, investigator suggested that foundations with depth, D equals to 3 to 4 times their width may be defined as shallow foundations.

Shallow foundations generally are designed to satisfy two criteria: **bearing capacity** and **settlement**. The bearing capacity criterion ensures that there is adequate safety against possible bearing capacity failure within the underlying soil. This is done through provision of an adequate factor of safety of about 3. In other words, shallow foundations are designed to carry a working load of one-third of the failure load. For raft foundations, a safety factor of 1.7 to 2.5 is recommended (Bowles 1996). The settlement criterion ensures that settlement is within acceptable limits. For example, pad and strip footings in cohesionless soils generally are designed to settle less than 25 mm.

Unless a shallow foundation can be founded on strong rock, some noticeable settlement will occur. Design of shallow foundations should ensure that there is an adequate factor of safety against bearing failure of the ground, and that the settlements, including total and differential settlement, are limited to allowable values. Distress experienced on structures founded on sand is not uncommon, especially for sub-soils containing large amounts of cobbles. Excessive settlement or sometimes even shear failure can take place when there is a sudden change of the water table elevation.

For shallow foundations founded on cohesionless soils, the allowable load is usually dictated by the allowable settlement, except where the ultimate bearing capacity is significantly affected by geological or geometric features. Examples of adverse geological and geometrical features are weak seams and sloping ground respectively.

Stability of a structure depends upon the stability of the supporting soil underneath the foundation for which the following two guiding principles are considered (Murthy, 2008);

(a) The foundation must be stable against shear failure of the supporting soil.
(b) The foundation must not settle beyond a tolerable limit to avoid damage to the structure.

Hence, an engineer must perform three major steps for effective design of the type of foundations

(i) The site of the proposed structure must be located.

(ii) Obtain information concerning the superstructure and the loads to be transmitted to the foundation.

(iii) Obtain the subsurface soil condition.

1.2 Ultimate Bearing Capacity of the Soil: The case of a shallow foundation at simplest consideration is the one subjected to a central vertical load, Q when the footing is founded at a depth, D below the ground surface. The settlement, S of the footing is recorded and plotted against the applied load, Q to obtain the load settlement curves similar to that of stress-strain curve, Fig. 1.

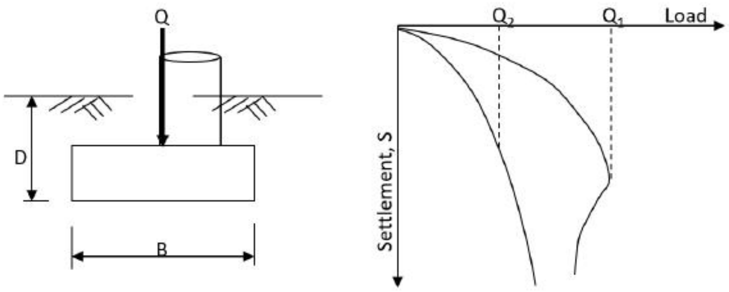

Fig. 1: Typical Shallow Footing and load-settlement curve (Murthy, 2008)

However, the test by Vesic (1967) revealed that the shape of the curve depends on the dimension (geometry) of the footing, the composition of the supporting soil, the character, rate and frequency of loading.

II. METHODOLOGY

2.1 Determination of Ultimate Bearing Capacity, UBC of Soil: Three methods are adoptable in determining UBC of soils;

(i) General shear failure theory of Terzaghi

(ii) Theoretical solution presented by Meyerhof, Brinc Hansen and Vesic.

(iii) Solution based on in-situ tests soils as PLT, SPT and CPT.

However, Terzaghi's method is more popular and the same is considered in this model. Terzaghi (1943) used theoretical equation proposed by Prandtl (1921) for determining the bearing capacity of soil and extended his theory to take account the weight of soil and the effect of soil above the base of the footing on the bearing capacity. He made some assumptions for developing equation for determining UBC, q_d for c-ϕ soil with a Strip Footing, that of which was modified for other types of foundation by introducing shape factors as follows;

i) Strip Footing: $q_d = Q_d/B = cN_c + \gamma D_f N_q + ½\gamma BN_\gamma$ (i)

where, q_d = ultimate bearing load per unit length of footing

c = unit cohesion

γ = effective unit weight of soil

D_f = depth of foundation

N_c, N_q and N_γ = bearing capacity factors that are functions of friction, ϕ.

The cohesion, c and angle of internal friction, ϕ determined using triaxial apparatus, Fig. 2.15.

ii) Square Footings: $q_d = 1.3cN_c + \gamma D_f N_q + 0.4\gamma BN_\gamma$ (ii)

iii) Circular Footing: $q_d = 1.3cN_c + \gamma D_f N_q + 0.3\gamma BN_\gamma$ (ii)

iv) Rectangular Footing: $q_d = (1+0.3B/L)cN_c + \gamma D_f N_q + (1–0.2B/L)½\gamma BN_\gamma$(iii)

2.2 The Governing Equation: Considering the ultimate bearing capacity as presented by Das (2007) in equations (i), (ii), and (iii) which are for continuous, square, and circular foundations only; they do not address the case of rectangular foundations (0 < B/ L < 1). Also, the equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation. To account for all these shortcomings, Meyerhof (1963) suggested the understated general bearing capacity equation, also documented by Das (2007).

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + ½ \gamma BN_\gamma F_{\gamma S}F_{\gamma d}F_{\gamma i} \quad \text{(iv)}$$

where,

c' = cohesion

q = effective stress at the level of the bottom of the foundation

γ = unit weight of soil

B = width/diameter of foundation

$N_cN_qN_\gamma$ = bearing capacity factors

$F_{cs}F_{qs}F_{\gamma S}$ = shape factors

$F_{cd}F_{qd}F_{\gamma d}$ = depth factors

$F_{ci}\ F_{qi}F_{\gamma i}$ = load inclination factors

The shape, depth and load inclination factors are empirical factors based on experimental.

With cohessionless soil, c = 0

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + ½\ \gamma BN_\gamma F_{\gamma S}F_{\gamma d}F_{\gamma i} \quad \text{(v)}$$

2.3 The Inclination Factors: Meyerhof (1963), Hanna and Meyerhof (1981) documented the following as inclination factors;

$$F_{ci} = F_{qi} = (1 – \beta^o/90^o)^2 \quad \text{(vi)}$$

$$F_{\gamma i} = (1 – \beta /Ø')^2 \quad \text{(vii)}$$

Since the load is non-inclined i.e vertically loaded, inclination factors not applicable. Thus equation (v) reduced to;

$$q_u = qN_qF_{qs}F_{qd} + ½\ \gamma BN_\gamma F_{\gamma S}F_{\gamma d} \quad \text{(viii)}$$

2.4 The Bearing Capacity Factors (BCF): These are factors that are non-dimensional and are functions only of the soil friction angle, Ø'. The variations of the BCF are given in **Table 1**.

Table 1: Bearing capacity factors

Ø'	N_c	N_q	N_γ	**Ø'**	N_c	N_q	N_γ
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0,01	27	29.24	15.90	11.60

2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8,26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.8
25	25.13	12.72	8.34				

Source: Kumbhojkar (1993) and Das (2007)

For cohesionless (sandy) soil, the angle of friction, Ø' usually ranges from 26° to 45° (Das, 2007). At minimum value, Ø' = 26°, from Table 1, $N_q = 14.21$ and $N_\gamma = 9.84$.

Substituting the value, equation (viii) becomes

$$q_u = 14.21 q F_{qs} F_{qd} + 4.92\ \gamma B F_{\gamma s} F_{\gamma d} \qquad \text{(ix)}$$

2.5 The Shape Factors: Expressions for the shape factors F_{cs}, F_{qs} *and* $F_{\gamma s}$ were given by De Beer (1970) as;

$$F_{cs} = 1 + (B/L)(N_q/N_c) \qquad \text{(x)}$$

$$F_{qs} = 1 + (B/L)\tan Ø' \qquad \text{(xi)}$$

and

$$F_{\gamma s} = 1 - 0.4(B/L) \qquad \text{(xii)}$$

Where L = length of the foundation (L > B)

Since c = 0, equation (x) not applicable and with Ø' = 26°, equation (xi) becomes;

$$F_{qs} = 1 + 0.4877(B/L) \qquad \text{(xiii)}$$

Put (xii) and (xiii) in (ix),

$$q_u = 14.21\{1 + 0.4877(B/L)\} q F_{qd} + 4.92\{1 - 0.4(B/L)\}\ \gamma B F_{\gamma d}$$

Thus, (ix) becomes;

$$q_u = \{14.21 + 6.9302(B/L)\} q F_{qd} + \{4.92 - 1.968(B/L)\}\ \gamma B F_{\gamma d} \qquad \text{(xiv)}$$

2.6 The Depth Factors: As presented by Hansen (1970), the following equations are for the depth factors;

$$F_{cd} = 1 + 0.4\,(D_f/B) \qquad \text{(xv)}$$

$$F_{qd} = 1 + 2\tan Ø' (1 - \sin Ø')^2 (D_f/B) \qquad \text{(xvi)}$$

$$F_{\gamma d} = 1 \qquad \text{(xvii)}$$

Where D_f is the depth of foundation of embedment of the footing.

Note, equations (viii) and (ix) are applicable for $D_f/B \leq 1$. For a depth $D_f/B > 1$, the following expressions are applicable;

$F_{cd} = 1 + 0.4 \tan^{-1}(D_f/B)$ (xviii)

$F_{qd} = 1 + 2 \tan Ø'(1 - \sin Ø')^2 \tan^{-1}(D_f/B)$ (xix)

$F_{\gamma d} = 1$ (xx)

However, the factor $\tan^{-1}(D_f/B)$ in (xi) and (xii) above must be in radians (Das, 2007).

Recall c = 0 and Ø' = 26°, then (xv) not applicable and (xvi) becomes

$F_{qd} = 1 + 0.3076(D_f/B)$ (xxi)

Substituting (xvii) and (xxi) in (xiv),

$q_u = 14.21q\{1 + 0.3076(D_f/B)\} + 6.9302(B/L)q\{1 + 0.3076(D_f/B)\} + 4.92\gamma B(1) - 1.968(B/L)\gamma B(1)$

Hence, (xiv) becomes

$q_u = 14.21q + 4.3710q(D_f/B) + 6.9302(B/L)q + 2.1320q(D_f/L) + 4.92\gamma B - 1.968(^1/_L)\gamma$ (xxii)

Also substituting effective stress, $q = \gamma D_f$ in equation (xxii).

$q_u = 14.21\gamma D_f + 4.3710\gamma D_f^2/B + 6.9302\gamma D_f(B/L) + 2.1320\gamma D_f^2(^1/_L) + 4.92\gamma B - 1.968(^1/_L)\gamma$ (xxiii)

Equation (xxiii) is therefore the **model governing equation**

III. RESULTS

Equation (xxii) indicates that the bearing capacity of a shallow pad foundation on a cohessionless soil is a function of the effective stress at the level of the bottom of the foundation, the unit weight of the soil at the location and the footing geometry.

Thus, the model tested, considering the bearing capacity, q_u of a rectangular pad foundation required to support a particular total vertical load at depths, D_f of 0.40m, 0.60m, 0.80m and 1.00m with footing length, L of 1.00m at a varying breadth, B of 1.00m, 1.20m, 1.40m, 1.60m and 1.80m on a dense sandy material of 18kN/m^3 unit weight with water table located at a depth, d > B.

Substituting L = 1.00m and γ = 18kN/m^3 in the governing equation (xxiii)

$q_u = 255.7800D_f + 78.6780D_f^2/B + 3.8651D_fB + 19.188D_f^2 + 88.56B - 17.424$ ………..(xxiv)

Details of the bearing capacities, q_u at varying breadth, B and depth, D are as shown in **Table 2** below.

Table 2: Bearing capacity at varying breadth and depth

B (m) 1	D_f (m) 2	D_f^2 3	D_f^2/B 4	D_fB 5	Constant (k) 6	q_u (kN/m^2) 7	Diff. & Ave (kN/m^2) 8
1.00	0.40	0.16	0.16	0.40	17.424	190.650	-
1.20			0.13	0.48		206.309	15.66
1.40			0.11	0.56		222.754	16.45
1.60			0.10	0.64		239.986	17.23
1.80			0.08	0.72		256.443	16.46 (16)
1.00	0.60	0.36	0.36	0.60	17.424	262.156	-
1.20			0.30	0.72		275.609	13.45
1.40			0.26	0.84		290.636	15.03
1.60			0.23	0.96		306.449	15.81
1.80			0.20	1.08		322.273	15.82 (15)
1.00	0.80	0.64	0.64	0.80	17.424	341.481	-
1.20			0.53	0.96		351.155	9.67
1.40			0.46	1.12		363.976	12.82
1.60			0.40	1.28		377.584	13.61
1.80			0.36	1.44		392.776	15.19 (13)

1.00	1.00	1.00	1.00	1.00	17.424	428.647	-
1.20			0.83	1.20		433.754	5.11
1.40			0.71	1.40		442.796	9.04
1.60			0.63	1.60		454.985	12.19
1.80			0.56	1.80		467.970	12.99 (10)

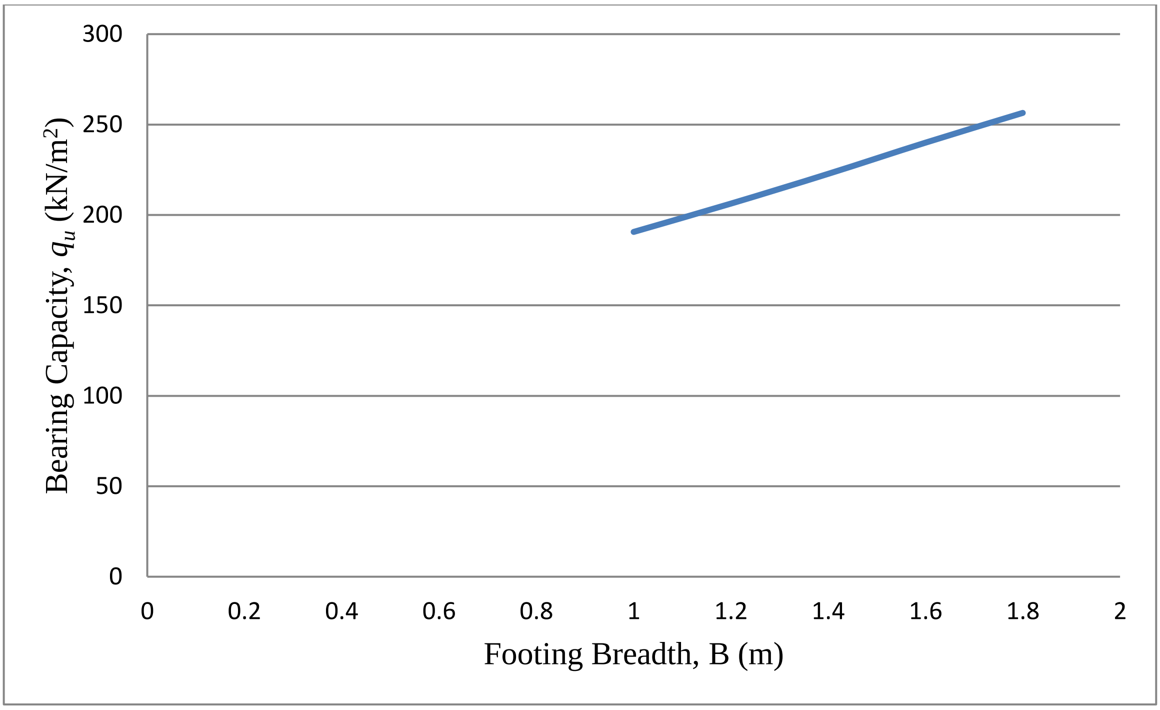

Fig. 2: Bearing capacity at 0.40m depth, 1.00m length and varying breadth

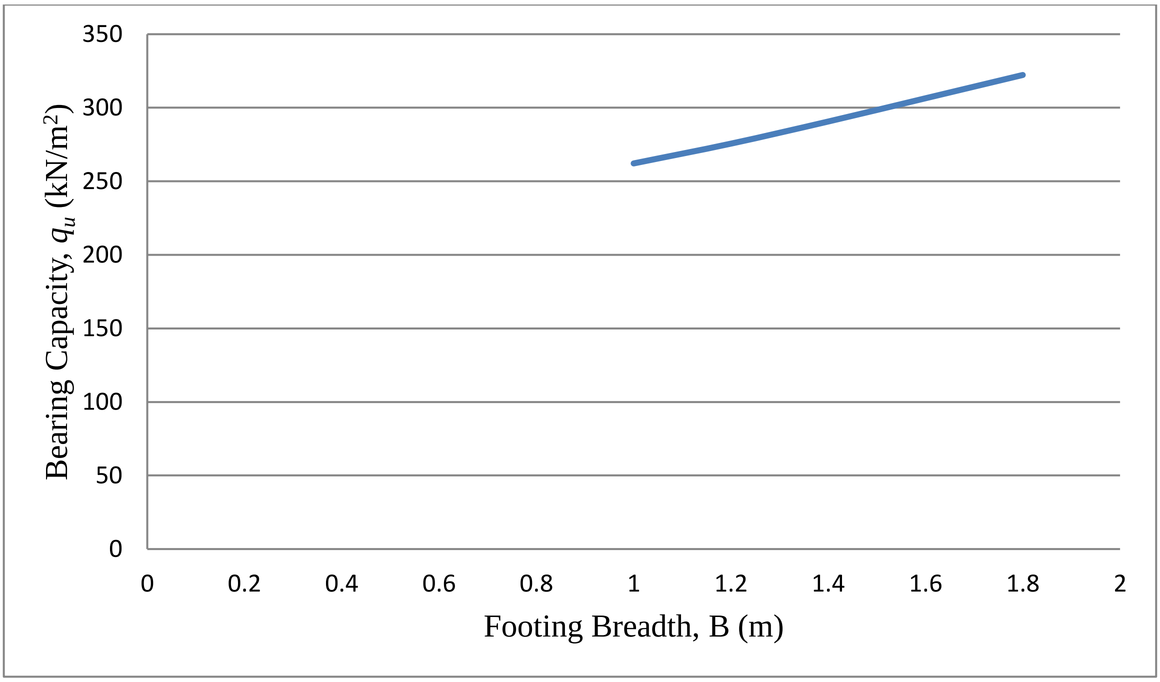

Fig. 3: Bearing capacity at 0.60m depth, 1.00m length and varying breadth

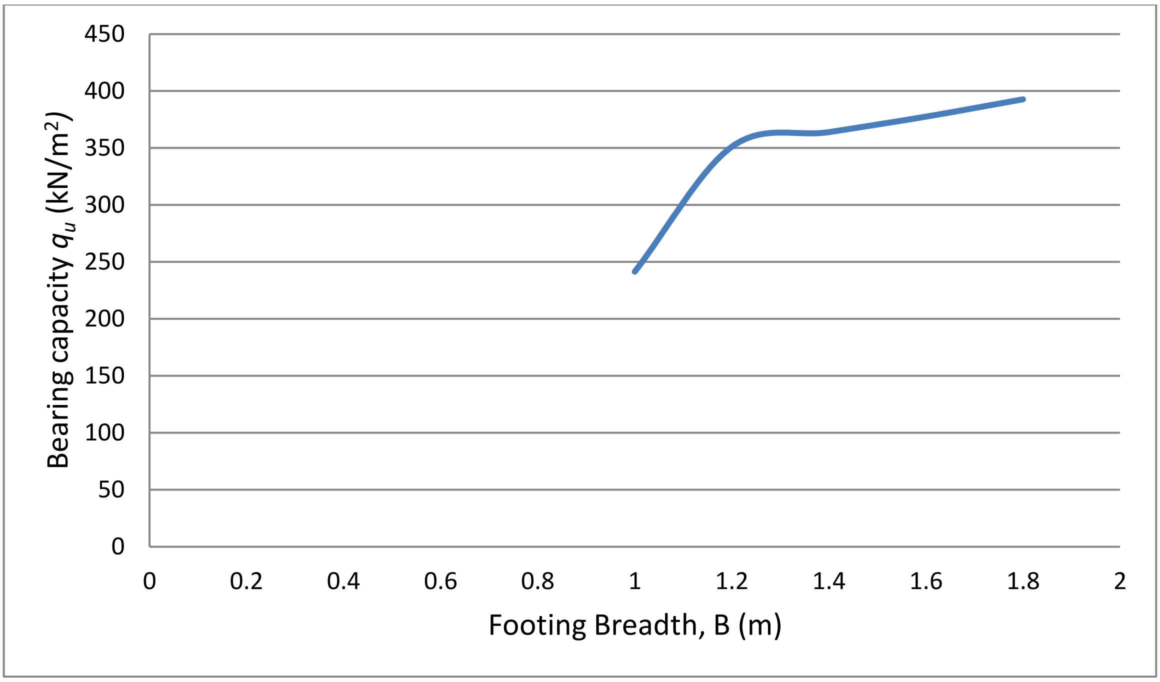

Fig. 4: Bearing capacity at 0.80m depth, 1.00m length and varying breadth

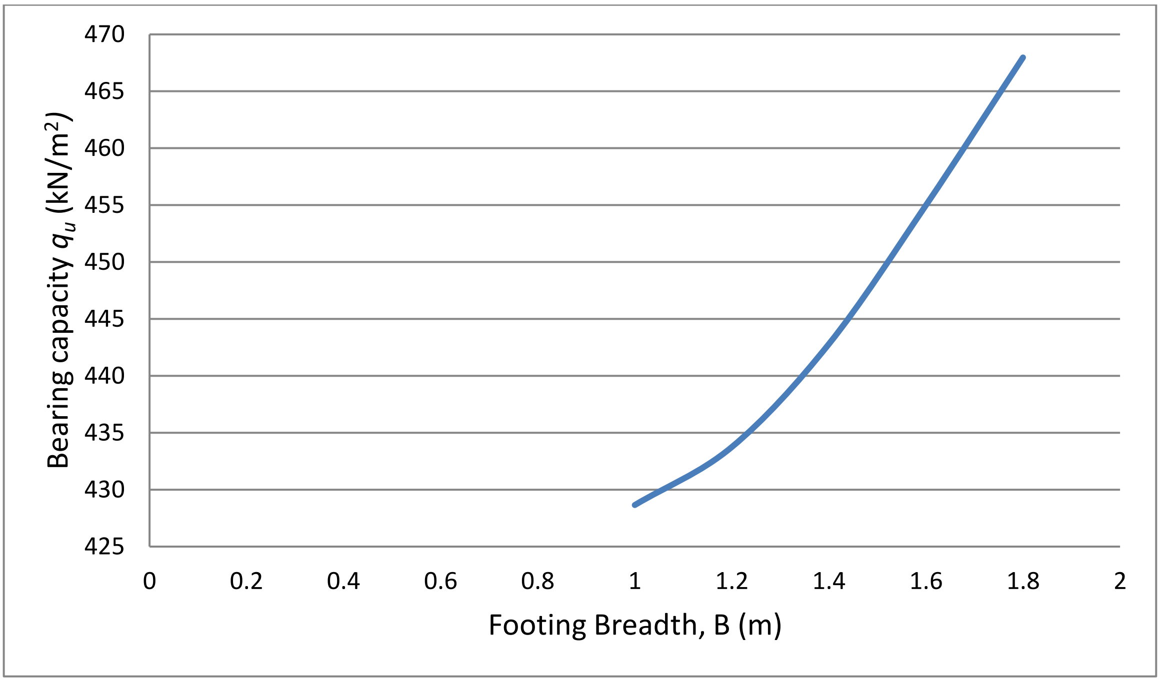

Fig. 5: Bearing capacity at 1.00m depth, 1.00m length and varying breadth

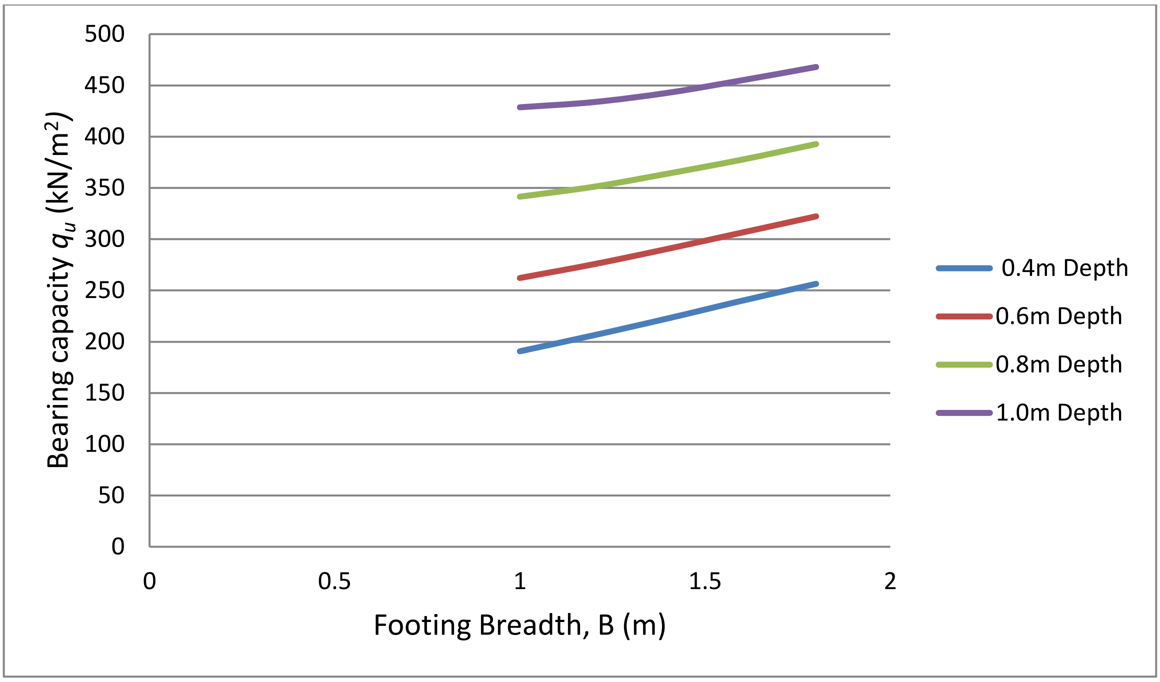

Fig. 6: Summary of the Bearing capacity (BC) at 1.00m length and varying breadth and depth of a shallow foundation footing

IV. DISCUSSION

Considering the bearing capacity (BC) details generated in Table 2 above as obtained from the model equation derived by incorporating existing registered data of the earth crust material of a cohessionless/sandy soil under dry condition and at minimum angle of friction, 26^o. The three dimensional geometry parameters of the rectangular foundation footing were iterated at 1m length and varying breadth from 1m and varying depth from 0.4m respectively, to obtain the behavior at an increasing trend of 0.2m.

The bearing capacity data in Table 02, column 12 and 13 reveals that an increase 0.2m in the breadth of a rectangular foundation footing at a particular depth results in an increase in the load BC of a sandy soil at average value ranging from 16 to 10kN/m^2. It is also noted that the increase trend reduces as depth increases ranging from being directly proportional generating straight line curve to an indirectly proportional curve line nature, Figures 2 to 5.

However, it was also discovered that increase in the depth of the footing at a fixed length and breadth produced a progressive increase in the BC of a sandy soil. Considering 1.00m length by 1.00m breadth at the varying depth for instance, Table 3.

It should be recalled that the test by Vesic (1967) documented that the shape of the curve of footing settlement, S plotted against the applied load, Q is similar to that of stress-strain curve which depends on the dimension (geometry) of the footing, the composition of the supporting soil, the character, rate and frequency of loading.

The nature of the curves in Figures 2 to 6 predicts the geotechnical behavior of a rectangular foundation footing on a cohessionless (sandy) soil which stand as a useful information in foundation engineering practice.

Table 03: Bearing capacity at varying depth

Footing Length, L (m)	**Footing Breadth, B (m)**	**Footing Depth, D_f (m)**	**q_u (kN/m^2)**	**Diff. (kN/m^2)**
1.00	1.00	0.40	190.650	-
		0.60	262.156	71.506
		0.80	341.481	79.325
		1.00	428.647	87.166

V. CONCLUSION

Foundations are provided in the designed to serve as a remedy for the deficiencies of whatever nature has provided to support of the structure at a particular location and loading condition. The model thus predicts the geotechnical behavior of a rectangular foundation footing on cohessionless (sandy) soil and also clearly established the interrelationship between the dominant parameters of the soil and footing geometry for determining bearing capacity. The knowledge of this will therefore go a long way to help engineering decisions in the design of a safe footing on a sandy soil environment.

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