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Wavelength and Wave Period Relationship with Wave Amplitude: A Velocity Potential Formulation

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Abstract*— *In this study, the equation that expresses the explicit relationship between the wave number and wave amplitude, as well as wave period and wave amplitude are established. The wave number and the wave period are calculated solely using the input wave amplitude. The equation is formulated with the velocity potential of the solution to Laplace's equation to the hydrodynamic conservation equations, such as the momentum equilibrium equation, Euler Equation for conservation of momentum, and by working on the kinematic bottom and free surface boundary condition.

I. INTRODUCTION

The relationship between wave period and wave height has long been recognized. Wiegel (1949, 1964) formulated the relationship through field observations. Silvester (1974) formulated this relationship based on the Pierson-Moskowitz spectrum. The Pierson-Moskowitz spectrum (1964) relates the wave period and wave energy, while wave energy correlates with wave height.

Dean (1991) formulated the dispersion equation of the linear wave theory relating the wave number to the wave period. Just like the fifth order Stokes proposed by Skjelbreia (1960), Stokes' waves of the second order (1847) describe the relationship between wave period and wave number.

From the two relationships describing the relationship between wave period and wave height as well as the relationship between wave number and wave period, it could be hypothesized that in velocity potential, there is a direct relationship between wave number and wave height as well as wave period and wave height.

In this study, the constant of velocity potential for the solution to Laplace's equation is obtained by working on the velocity potential on the kinematic bottom boundary condition (Dean (1991) and the momentum equilibrium equation.

After the constant of the solution to Laplace's equation is obtained, the potential velocity is calculated on the kinematic boundary condition. Thus, the relationship between the wave number and the wave amplitude is obtained

The velocity potential is done on Euler Equation for conservation of momentum and the wave number is substituted by the relationship between the wave number and the wave amplitude. Therefore, the relationship between wave period with the wave amplitude is obtained.

II. TAYLOR SERIES ON UNSTEADY FLOW

Taylor series is a statement of the function value at the points around it using the differential of the function. When a function changes over time, the spatial differential likewise changes over time in addition to the function itself. Variables in unsteady flow such as water particle velocity, change across time and space. As a result, the Taylor series for a variable in an unsteady flow should also account for how the differential function changes over time.

The formulation of conservation equations, including conservation of mass and conservation of momentum in fluid flow is formulated using the first-order Taylor series approximation. As a result, in this study, the first-order Taylor series is formulated to account for the differential variable's change over time.

The first order Taylor series (Arden & Astill), for a function $f(x,t)$ at time $t = t$ is,

$$f(x+\delta x,t) = f(x,t) + \delta x \left(\frac{\partial f}{\partial x}\right)^{t}$$

At time $t = t + \delta t$,

$$f(x+\delta x,t+\delta t) = f(x,t+\delta t) + \delta x \left(\frac{\partial f}{\partial x}\right)^{t+\delta t}$$

However, it is incorrect if $\frac{\partial f}{\partial x}$ is calculated only at $t+\delta t$, besides that this equation is an implicit equation. Then, the mean value is used.

$$f(x+\delta x,t+\delta t) = f(x,t+\delta t) + \delta x \left(\mu_1 \left(\frac{\partial f}{\partial x}\right)^{t} + \mu_2 \left(\frac{\partial f}{\partial x}\right)^{t+\delta t}\right)$$

μ_1 and μ_2 are contribution coefficients, where $(\mu_1 + \mu_2) = 1$. In the first term of the right-hand side of the last equation, the Taylor series is done,

$$f(x+\delta x,t+\delta t) = f(x,t) + \delta t \frac{\partial f}{\partial t} + \delta x \left(\mu_1 \left(\frac{\partial f}{\partial x}\right)^{t} + \mu_2 \left(\frac{\partial f}{\partial x}\right)^{t+\delta t}\right) \qquad \ldots(1)$$

This equation is still an implicit equation. As seen from the function $g(x,t) = \frac{\partial f}{\partial x}$.

$$g(x,t+\delta t) = g(x,t) + \delta t \frac{\partial g}{\partial t}$$

$$\left(\frac{\partial f}{\partial x}\right)^{t+\delta t} = \left(\frac{\partial f}{\partial x}\right)^{t} + \delta t \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right)^{t}$$

Substitute to (1)

$$f(x+\delta x,t+\delta t) = f(x,t) + \delta t \frac{\partial f}{\partial t} + \delta x \left(\mu_1 \left(\frac{\partial f}{\partial x}\right)^{t} + \mu_2 \left(\left(\frac{\partial f}{\partial x}\right)^{t} + \delta t \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right)^{t}\right)\right)$$

Considering $(\mu_1 + \mu_2) = 1$,

$$f(x+\delta x,t+\delta t) = f(x,t) + \delta t \frac{\partial f}{\partial t} + \delta x \left(\frac{\partial f}{\partial x}\right)^{t} + \mu_2\, \delta x \left(\delta t \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right)^{t}\right)$$

The first term on the right-hand side is moved to the left and the equation is divided by δx.

$$\frac{f(x+\delta x,t+\delta t) - f(x,t)}{\delta x} = \frac{\delta t}{\delta x}\left(\frac{\partial f}{\partial t}\right)^{t} + \left(\frac{\partial f}{\partial x}\right)^{t} + \left(\mu_2\, \delta t \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right)^{t}\right)$$

For very small δt and δx close to zero, the third term on the right-hand side will be very small close to zero and can be ignored, obtaining:

$$\frac{Df}{dx} = \frac{\delta t}{\delta x}\frac{\partial f}{\partial t} + \left(\frac{\partial f}{\partial x}\right)^{t}$$

The equation is a total spatial derivative. This equation is substituted for $\left(\frac{\partial f}{\partial x}\right)^{t+\delta t}$ on (1)

$$f(x+\delta x,t+\delta t) = f(x,t) + \delta t \frac{\partial f}{\partial t} + \delta x \left(\mu_1 \left(\frac{\partial f}{\partial x}\right)^{t} + \mu_2 \left(\frac{\delta t}{\delta x}\frac{\partial f}{\partial t} + \left(\frac{\partial f}{\partial x}\right)^{t}\right)\right)$$

Or,

$$f(x+\delta x,t+\delta t) = f(x,t) + (1+\mu_2)\, \delta t \left(\frac{\partial f}{\partial t}\right)^{t} + \delta x \left(\frac{\partial f}{\partial x}\right)^{t}$$

The time index t is omitted and defined as $\gamma_2 = 1 + \mu_2$. Thus, the first order Taylor series for space and time functions is:

$$f(x+\delta x,t+\delta t) = f(x,t) + \gamma_2\, \delta t \frac{\partial f}{\partial t} + \delta x \frac{\partial f}{\partial x} \qquad \ldots(2)$$

In the same way for a function with three variables $f(x,z,t)$, the Taylor series is:

$$f(x+\delta x,z+\delta z,t+\delta t) = f(x,z,t) + \gamma_3\, \delta t \frac{\partial f}{\partial t} + \delta x \frac{\partial f}{\partial x} + \delta z \frac{\partial f}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Where,

$\gamma_3 = 1 + 2\mu_2$

For example, if $\mu_2 = 0.6$, then $\mu_1 = 0.4$, which gives greater weight to $\left(\frac{\partial f}{\partial x}\right)^{t+\delta t}$, So $\gamma_2 = 1.6$ and $\gamma_3 = 2.2$.

III. CONTINUITY EQUATION FOR UNSTEADY FLOW

Unsteady flow describes the flow of water in a water wave that changes with time. Therefore, a continuity equation that accounts for the change in velocity with time is required.

3.1.Equation of Conservation of Mass

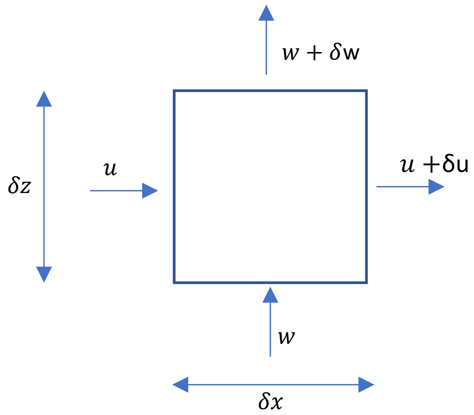

Fig.1 Control Volume to Formulate the Equation of Conservation of Mass

The equation of conservation of mass is formulated using a control volume located in a fluid flow as depicted in Figure 1. The horizontal velocity only changes on the horizontal axis, while the vertical velocity only changes on the vertical axis. x is the horizontal axis and z is the vertical axis. Figure 1 presents the velocity of the particle in the horizontal direction is u, while the velocity of the particle in the vertical direction is w.

Input-output occurs in the control volume:
Input,
$I = \rho u\,\delta z + \rho w\,\delta x$
Output,
$O = \rho(u + \delta u)\delta z + \rho(w + \delta w)\delta x$
Due to the input and output, at the time interval δt there is a change in fluid mass at the control volume of:
$\delta m = (I - O)\delta t$
The input and output equations are substituted in the equation of mass change, and both sides of the equation are divided by $\delta t\,\delta x\,\delta z$,

$$\frac{\delta m}{\delta t \delta x\,\delta z} = -\rho\,\frac{\delta u}{\delta x} - \rho\,\frac{\delta w}{\delta z}$$

For a constant control volume, the mass change in the control volume is

$$\delta m = \delta\rho\,\delta x\,\delta z$$

An equation is formulated:

$$\frac{\delta\rho}{\delta t} = -\rho\,\frac{\delta u}{\delta x} - \rho\,\frac{\delta w}{\delta z}$$

For incompressible flow $\frac{\delta\rho}{\delta t} = 0$, then

$$\frac{\delta u}{\delta x} + \frac{\delta w}{\delta z} = 0 \qquad \text{.......(4)}$$

This equation is the equation of conservation of mass for incompressible flow

3.2. Continuity and Momentum Equilibrium Equation

In the case of the Taylor series, where the horizontal velocity only changes on the horizontal axis and the vertical velocity only changes on the vertical axis, the (2) will be,

$$u(x + \delta x, z, t + \delta t) = u(x, z, t) + \gamma_3\,\delta t\frac{\partial u}{\partial t} + \delta x\frac{\partial u}{\partial x}$$

$$w(x, z + \delta z, t + \delta t) = w(x, z, t) + \gamma_3\,\delta t\frac{\partial w}{\partial t} + \delta z\frac{\partial w}{\partial z}$$

γ_3 is used considering $u = u(x, z, t)$ and $w = w(x, z, t)$. By moving the term to the right-hand side to the left, then

$$\delta u = \gamma_3\,\delta t\frac{\partial u}{\partial t} + \delta x\frac{\partial u}{\partial x} \qquad \text{...........(5)}$$

$$\delta w = \gamma_3\,\delta t\frac{\partial w}{\partial t} + \delta z\frac{\partial w}{\partial z} \qquad \text{...........(6)}$$

Substitute (5) and (6) to (4),

$$\frac{\gamma_3\,\delta t\frac{\partial u}{\partial t} + \delta x\frac{\partial u}{\partial x}}{\delta x} + \frac{\gamma_3\,\delta t\frac{\partial w}{\partial t} + \delta z\frac{\partial w}{\partial z}}{\delta z} = 0$$

The last equation is multiplied by δz, $\frac{\delta z}{\delta x} = \gamma_z$, is used

$$\gamma_z\left(\gamma_3\,\delta t\frac{\partial u}{\partial t} + \delta x\frac{\partial u}{\partial x}\right) + \gamma_3\,\delta t\frac{\partial w}{\partial t} + \delta z\frac{\partial w}{\partial z} = 0$$

This equation is divided by δt at a very small δt close to zero,

$$\gamma_z\left(\gamma_3\,\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial uu}{\partial x}\right) + \gamma_3\,\frac{\partial w}{\partial t} + \frac{1}{2}\frac{\partial ww}{\partial z} = 0$$

This is the continuity equation for unsteady flow. In this study, the value of γ_z has no effect, thus, it requires no further explanation. This equation is equal to zero if,

$$\gamma_3\,\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial uu}{\partial x} = 0 \qquad \text{.......(6)}$$

$$\gamma_3\,\frac{\partial w}{\partial t} + \frac{1}{2}\frac{\partial ww}{\partial z} = 0 \qquad \text{.....(7)}$$

Equation (6) explains that the horizontal momentum input in the control volume only causes a change in the horizontal velocity, while the vertical momentum input only causes a change in the vertical velocity. These two equations are named as the equilibrium equation

IV. VELOCITY POTENTIAL EQUATION.

The Velocity Potential Equation resulted from the solution to Laplace's equation using the variable separation method (Dean (1991)),

$$\phi(x, z, t) = A cos kx(Ce^{kz} + De^{-kz})sin(\sigma t)$$
$$+B sin kx(Ce^{kz} + De^{-kz})sin(\sigma t) \qquad \text{......(8)}$$

Where ϕ is the velocity potential, A, B, C and D are the constants for a solution that need to be determined in the form of the equation. k is the wave number and $\sigma = \frac{2\pi}{T}$ is angular frequency, and T is wave period. Even though their respective values are not constant, these two variables are referred to as wave constants.

Based on velocity potential, the horizontal water particle velocity is:

$$u(x, z, t) = -\frac{\partial\phi}{\partial x} \qquad \text{.....(9)}$$

While vertical water particle velocity is:

$$w(x, z, t) = -\frac{\partial\phi}{\partial z} \qquad \text{......(10)}$$

(8) shows that velocity potential consists of two components of $\cos kx$ and $\sin kx$. Both sinusoidal functions have a point of intersection where the two functions have the same value. Henceforward, the point of intersection is referred to as the characteristic point. By determining the constants of the solution A, B, C dan D at the characteristic point, the values obtained will apply to both velocity potential components.

The first step to getting the equations of these constants is to do the kinematic boundary condition on the flat bottom (Dean (19 91). The kinematic bottom boundary condition is,

$$w_{-h} = -u_{-h}\frac{dh}{dx} \qquad \text{........(11)}$$

Where w_{-h} and u_{-h} respectively are bottom vertical and horizontal water particle velocity at $z = -h$ where h is the water depth to still water level while $\frac{dh}{dx}$ is the bottom slope which is zero at flat bottom. Thus, the kinematic boundary condition on the flat bottom is:

$$w_{-h} = 0 \quad \text{....(12)}$$

Substituting (10) to (12) will obtain:

$$C = De^{2kh} \quad \text{.....(13)}$$

Substituting (13) to (8), velocity potential equation as defined by $A = 2A$ and $B = 2B$,

$$\Phi(x,z,t) = ADe^{kh} coskx\, coshk(h+z) sin(\sigma t)$$

$$+BDe^{kh} sinkx\, coshk(h+z) sin(\sigma t) \quad \text{.......(14)}$$

In (14) there are the constants of the solution that still need to be determined, they are A, B and D. Substitute (14) to (6) will obtain,

$$A = B \quad \text{........(15)}$$

Velocity potential equation will be,

$$\phi(x,z,t) = ADe^{kh}(coskx + \sin kx)\, coshk(h + z) sin(\sigma t) \quad \text{.......(16)}$$

Substitute(16) to (7) to obtain,

$$ADe^{kh} = \frac{\gamma_3 \sigma}{k^2 coshk(h+z)} \quad \text{.........(17)}$$

Velocity potential is done at the characteristic point, and a new constant $G = 2ADe^{kh}$ is defined. Then, the velocity potential equation will be:

$$\Phi(x,z,t) = G\, coskx\, coshk(h+z) sin(\sigma t) \quad \text{...(18)}$$

Where,

$$G = 2ADe^{kh} = \frac{2\gamma_3 \sigma}{k^2 coshk(h+z)} \quad \text{..........(19)}$$

G has a double value, or G is the sum of the energies of the two waves.

V. THE RELATION BETWEEN WAVE NUMBER k AND WAVE AMPLITUDE A

Equation (2) is done to formulate water surface elevation equation $\eta(x,t)$,

$$\eta(x+\delta x, t+\delta t) = \eta(x,t) + \gamma_2\, \delta t \frac{\partial \eta}{\partial t} + \delta x \frac{\partial \eta}{\partial x}$$

The first term on the right-hand side is moved to the left and divided by δt for δt close to zero,

$$\frac{D\eta}{dt} = \gamma_2 \frac{\partial \eta}{\partial t} + u_\eta \frac{\partial \eta}{\partial x}$$

This equation is the total surface vertical water particle velocity, it can be written as,

$$w_\eta = \gamma_2 \frac{\partial \eta}{\partial t} + u_\eta \frac{\partial \eta}{\partial x} \quad \text{......(20)}$$

This equation is a weighted kinematic free surface boundary condition, where w_η is the vertical surface water particle velocity and u_η is the horizontal surface water particle velocity.

Substitute (19) to (20) and done at the characteristic point,

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma_2}\left(tanhk(h+\eta) + \frac{\partial \eta}{\partial x}\right) coshk(h+\eta) coskx\, sin(\sigma t)$$

For a periodic function:

$$\frac{Gk}{\gamma_2}\left(tanhk(h+\eta) + \frac{\partial \eta}{\partial x}\right) coshk(h+\eta) = constant$$

Thus, the integration of $\frac{\partial \eta}{\partial t}$ is done by integrating $sin(\sigma t)$.

$$\eta(x,t) = \frac{Gk}{\gamma_2 \sigma}\left(tanhk(h+\eta) + \frac{\partial \eta}{\partial x}\right) coshk(h+\eta) coskx\, cos(\sigma t)$$

Wave amplitude A is defined as,

$$A = \frac{Gk}{\gamma_2 \sigma}\left(tanhk\left(h + \frac{A}{2}\right) + \frac{\partial \eta}{\partial x}\right) coshk\left(h + \frac{A}{2}\right)$$

(19) shows that G is a superposition of two wave energies. Then, for one wave component:

$$A = \frac{Gk}{2\gamma_2 \sigma}\left(tanhk\left(h + \frac{A}{2}\right) + \frac{\partial \eta}{\partial x}\right) coshk\left(h + \frac{A}{2}\right)$$

This equation is the wave amplitude function equation. In deep water, where,

$$k\left(h + \frac{A}{2}\right) = \theta \pi \quad \text{.....(21)}$$

θ is a positive number, where

$$tanhk\left(h + \frac{A}{2}\right) = tanh(\theta \pi) = 1 \quad \text{.....(22)}$$

Considering conservation law of the wave number (Hutahaean (2020)), (21) and (22) apply to pada shallow water. The wave amplitude function equation will be,

$$A = \frac{Gk}{2\gamma_2 \sigma}\left(1 + \frac{\partial \eta}{\partial x}\right) cosh\,(\theta \pi)$$

The water surface elevation equation will be,

$$\eta(x,t) = A\, coskx\, cos(\sigma t) \quad \text{......(23)}$$

At the characteristic point of space and time on $\cos kx = \sin kx$ and $\cos \sigma t = \sin \sigma t$,

$$\frac{\partial \eta}{\partial x} = -\frac{kA}{2} \quad \text{.....(24)}$$

The wave amplitude function equation will be,

$$A = \frac{Gk}{2\gamma_2 \sigma}\left(1 - \frac{kA}{2}\right) coshk(\theta \pi) \quad \text{......(25)}$$

Substitute (19) for $z = \frac{A}{2}$, the same numerator and denominator cancel each other out, the relationship between wave number k and wave amplitude A is obtained.

$$k = \frac{\gamma_3}{\left(\gamma_2 + \frac{\gamma_3}{2}\right) A} \quad \text{...........(26)}$$

The wave number k is only determined by wave amplitude A. Even though the equation is formulated in deep water, since kA= constant or $\frac{\partial kA}{\partial x}=0$, this equation applies to shallow water. Therefore, the wave number can be calculated if the wave amplitude can be determined in shallow water.

Considering $k=\frac{2\pi}{L}$, it is obtained:

$$L=\frac{2\pi\left(\gamma_2+\frac{\gamma_3}{2}\right)A}{\gamma_3} \quad \text{.......(27)}$$

This equation is the relationship between wavelength L and wave amplitude A. Thus, wavelength is only determined by the wave amplitude. In shoaling, water depth changes wave amplitude. Indirectly, wavelength is determined by water depth.

VI. THE RELATIONSHIP BETWEEN WAVE PERIOD *T* AND WAVE AMPLITUDE *A*

The relation between wave period T and wave amplitude A is formulated using Euler's momentum equation.

By working on (3) on the horizontal water particle velocity $u(x,z,t)$ and the vertical water particle $w(x,z,t)$, the Euler equation in the horizontal and vertical directions are:

$$\gamma_3\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\gamma_3\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g$$

In both equations, the irrotational flow properties are done, where $\frac{\partial u}{\partial z}=\frac{\partial w}{\partial x}$,

$$\gamma_3\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial}{\partial x}(uu+ww)=-\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \text{......(28)}$$

$$\gamma_3\frac{\partial w}{\partial t}+\frac{1}{2}\frac{\partial}{\partial z}(uu+ww)=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g \quad \text{....(29)}$$

Equation (29) is multiplied by dz, integrated to the vertical axis. The dynamic free surface boundary condition was done where the pressure on the water surface $p_\eta=0$. The pressure equation is:

$$\frac{p}{\rho}=\gamma_3\int_z^\eta\frac{\partial w}{\partial t}dz+\frac{1}{2}\left(u_\eta u_\eta+w_\eta w_\eta\right)$$
$$-\frac{1}{2}(uu+ww)+g(\eta-z)$$

This equation is differentiable at the horizontal axis. The driving force obtained is in the horizontal direction. Next, the driving force is substituted to the right-hand side (28), where the same terms on the left and right-hand sides of the equation cancel each other out,

$$\gamma_3\frac{\partial u}{\partial t}=-\gamma_3\frac{\partial}{\partial x}\int_z^\eta\frac{\partial w}{\partial t}dz-$$
$$\frac{1}{2}\frac{\partial}{\partial x}\left(u_\eta u_\eta+w_\eta w_\eta\right)-g\frac{\partial\eta}{\partial x} \quad \text{......(30)}$$

By working (18) on the second term on the right-hand side, it is revealed that in deep water, the term is zero. With the conservation law of the wave numbers (Hutahaean, 2020) and (21) the second term on the right-hand side is also zero in shallow waters, so (30) becomes:

$$\gamma_3\frac{\partial u}{\partial t}=-\gamma_3\frac{\partial}{\partial x}\int_z^\eta\frac{\partial w}{\partial t}dz-g\frac{\partial\eta}{\partial x}$$

$\frac{\partial}{\partial x}\int_z^\eta\frac{\partial w}{\partial t}$ solved by substituting velocity potential (18) to obtain:

$$\frac{\partial}{\partial x}\int_z^\eta\frac{\partial w}{\partial t}=\frac{\partial u_\eta}{\partial t}-\frac{\partial u}{\partial t}$$

Resulting,

$$\gamma_3\frac{\partial u_\eta}{\partial t}=-g\frac{\partial\eta}{\partial x} \quad \text{.......(31)}$$

This is the horizontal surface water particle velocity equation. Substitute (18) and (23) to (31), where the equal terms between the left and right sides cancel each other obtaining:

$$\gamma_3 G\sigma\cosh k(h+\eta)=gA$$

Wave amplitude A on the right-hand side is substituted by (25) to obtain:

$$\sigma^2=\frac{g}{2\left(\gamma_2+\frac{\gamma_3}{2}\right)^2 A}$$

Considering $\sigma=\frac{2\pi}{T}$,

$$T=\sqrt{\frac{8\pi^2\left(\gamma_2+\frac{\gamma_3}{2}\right)^2 A}{g}}\ (sec) \quad \text{.......(32)}$$

Wiegel (1949,1964) formulate the relationship between wave period T and the wave height H,

$$T_{Wieg}=15.6\sqrt{\frac{H}{g}}\quad sec) \quad \text{.....(33)}$$

Wave height H is in meter, $g=9.81\ m/sec^2$. Silvester (1974) describes the relationship between wave period T and wave height H,

$$T_{Silv}=2.43\sqrt{\frac{H}{0.3048}}\ (sec) \quad \text{........(34)}$$

Wave height H in meter.

VII. EQUATION RESULTS

Table (1) represents the results of (27), (32), (33), and (34), with input wave amplitude, assuming a sinusoidal wave, the wave height $H=2A$. The calculation is done using $\mu_1=\mu_2=0.5$, where $\gamma_2=1.50$ and $\gamma_3=2.00$.

Table (1) Shows the Calculation Results Using (27), (32), (33), and (34)

A (m)	L (m)	$\frac{H}{L}$	T (sec)	T_{Wieg} (sec)	T_{Silv} (sec)
0.2	1.571	0.255	3.172	3.15	2.784
0.4	3.142	0.255	4.486	4.455	3.937
0.6	4.712	0.255	5.494	5.456	4.822
0.8	6.283	0.255	6.344	6.3	5.567
1	7.854	0.255	7.093	7.044	6.225
1.2	9.425	0.255	7.769	7.716	6.819
1.4	10.996	0.255	8.392	8.334	7.365
1.6	12.566	0.255	8.971	8.91	7.874
1.8	14.137	0.255	9.516	9.45	8.351
2	15.708	0.255	10.03	9.961	8.803

Calculation of wavelength with (27) produces a wavelength with wave steepness $\frac{H}{L} = 0.255$, where this wave steepness exceeds the critical wave steepness of Michell (1893) and Toffoli et al (2010). According to Michell (1893, the critical wave steepness is,

$$\frac{H}{L} = 0.142 \qquad \ldots\ldots(35)$$

According to Toffoli et al. (2010),

$$\frac{H}{L} = 0.170 \qquad \ldots\ldots(36)$$

The wave period resulted from (32) is larger for both the wave period of Wiegel T_{Wieg} as well as wave period from Silvestre T_{Silv}, but quite close to T_{Wieg}. Calculations are carried out using $\gamma_2 = 1.50$ dan $\gamma_3 = 2.00$.

Next, $\mu_2 = 0.4$ is used, where $\gamma_2 = 1.40$ and $\gamma_3 = 1.80$, which means that a contribution coefficient of $\mu_2 = 0.4$ for $\left(\frac{\partial f}{\partial x}\right)^{t+\delta t}$ in (1).

With this weighting coefficient, the wave steepness decreases to $\frac{H}{L} = 0.249$. Wave period also decreases, smaller that T_{Wieg} but bigger than T_{Silv}, but quite close to T_{Silv}, see Fig. (2).

Hence, despite the resulting equation giving the results from the weighting coefficients γ_2 and γ_3, the equation results are still around T_{Silv} and T_{Wieg}. Silvester (1974) formulates (34) using the Pierson-Moskowitz spectrum, while Wiegel (1949,1964) formulates (33) from field observations.

Table (2) Calculation results using $\gamma_2 = 1.40$ dan $\gamma_3 = 1.80$

A (m)	L (m)	$\frac{H}{L}$	T (sec)	T_{Wieg} (sec)	T_{Silv} (sec)
0.2	1.606	0.249	2.918	3.15	2.784
0.4	3.211	0.249	4.127	4.455	3.937
0.6	4.817	0.249	5.054	5.456	4.822
0.8	6.423	0.249	5.836	6.3	5.567
1	8.029	0.249	6.525	7.044	6.225
1.2	9.634	0.249	7.148	7.716	6.819
1.4	11.24	0.249	7.721	8.334	7.365
1.6	12.846	0.249	8.254	8.91	7.874
1.8	14.451	0.249	8.754	9.45	8.351
2	16.057	0.249	9.228	9.961	8.803

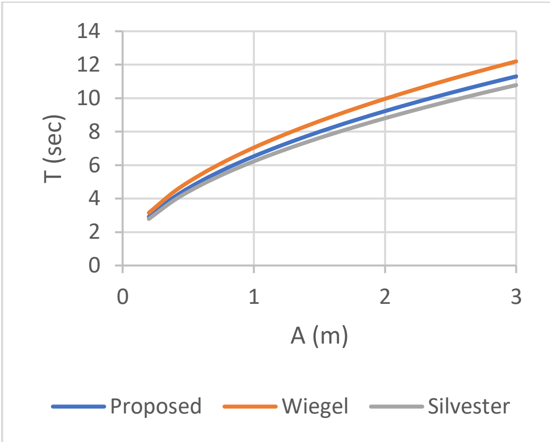

Fig. (2) Relationship between Wave Period and Wave Amplitude

The wave period T is determined by amplitude A. In the shoaling, there is an increase in the wave amplitude until breaking occurs, then there is a decrease in wave amplitude. Therefore, during the shoaling-breaking, there should also be an increase and a decrease in the wave period. By using (32) on the shoaling-breaking model from Hutahaean (2022), for deep water wave amplitude $A_0 = 1.00\,m$ or deep wave height $H_0 = 2.00\,m$, the change of wave period toward water depth h is obtained as depicted in Figure 3. The picture illustrates that when the wave-height increases, the wave period also increases, while when the wave-height decreases, the wave period also decreases

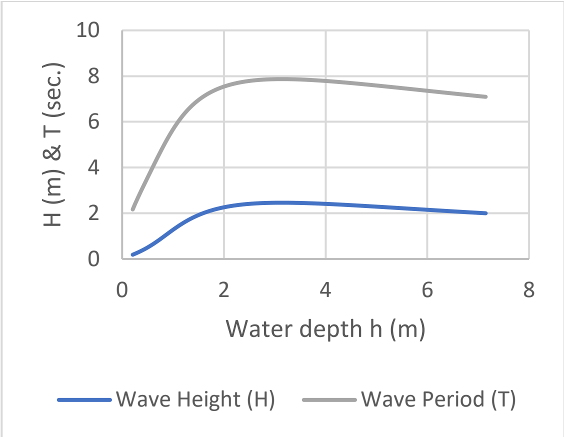

Fig. (3). The Change of Wave Height and Wave Period during Shoaling-Breaking.

VIII. CONCLUSION

This study proves wavelength and wave period explicit relationship with wave amplitude. To summarize, in the velocity potential equation, only the wave amplitude serves as the input for Laplace's equation. Wavelength and wave period can be calculated using the input.

The equation of the relationship between wave period and wave amplitude also applies to shallow water. However, to obtain shallow water wave amplitude, a shoaling-breaking analysis is required. Therefore, wave period analysis in shallow water requires a shoaling-breaking analysis. On the other side, the shoaling-breaking analysis must also account for the possibility of wave period changes.

Changes in the wave period in shallow water should be taken into consideration in a range of wave calculations, such as wave forces, sediment transport by waves, and other related calculation. The wave period in deep water should not be used for these calculations.

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