# Development of Linear Water Wave Dispersion Equation using Critical wave Steepness Criteria

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***Abstract*— *This study begins by examining the wavelength based on criteria of critical steep steepness, both in deep water and shallow water, and it is compared to the wave length of the linear water wave dispersion equation. It was found that the wave length of the linear water wave dispersion equation is too long. The wave length adjustment of the linear water wave dispersion equation can be done by multiplying the left part of the equation with a coefficient greater than one.***

*Furthermore, the analytic dispersion equation is formulated using weighted total acceleration, thus there is a coefficient on the left segment of the dispersion equation. The formulation is done by assuming the small amplitude wave so that the obtained dispersion equation is as same as the dispersion equation of the linear water wave theory, but using a wave length matching the criteria of critical wave steepness.*

*Based on critical wave steepness, it is found that the wave length of the linear wave theory is too long. To shorten it, it can be done by multiplying the left segment of the dispersion equation of the linear wave with a coefficient, where the coefficient can be obtained from critical wave steepness criteria and by analytical.*

***Keywords*— Water Wave, Critical Wave Steepness, Linear Water Wave Theory**.

## I. INTRODUCTION

Wave steepness of water wave is the ratio between wave height $H$ and wavelength $L$ which is $\frac{H}{L}$. In the water wave, there is a value of the wave steepness in which when exceeded, the wavewill break. The value of the wave steepness is called critical wave steepness. The first researcher proposing this critical wave steepness isMichell, J.H. (1893), while the most recent is Toffoli, A. et all (2010). The result of the research by Michell (1893) followed by other researchers formulating breaker index in the form of $\frac{H_b}{L_b}$, in which $H_b$ is breaker height and$L_b$ is breaker length, those researchers are Miche, R. (1944), Battjes, J.A. and Jansen, J.P.F.M, (1978) and Battjes, J.A. and Stive M.J.F. (1985). Actually, there are still many more, but in this study, it was selected they who did not using the bottom slope as their parameter, for practicality.

The study was conducted at two water depth conditions, namely deep water and shallow water. At deep water, research was done by using the criteria of Toffoli,A. et all (2010) and using the Wiegel's equation, Wiegel,R.L. (1949,1964), in which it is an equation stating the relation between the maximum wave height in the wave period. With the equationWiegel,R.L. (1949-1964), the wave height in a wave period was calculated, then wavelength for critical wave steepness was calculated using relation Toffoli, A., et all (2010). With this wave period, the wave length of the linear wave theory is also calculated. Further, the two wavelengths were compared. It was found that the wavelength of dispersion equation of the linear wave theory is too long. The wavelength of dispersion equation of the linear wave theory can be shortened or adjusted to the wavelength of the criteria of critical wave steepness by multiplying the left part of the dispersion equation by a coefficient greater than one.

In shallow water, the wavelength study was carried out using the Miche's equation, Miche, R. (1944), to calculate the breaker length. This calculation requires input of breaker height and breaker depth. The breaker height was calculated by using the equations from Komar, P.D. and Gaughan , M.K. (1972). Furthermore, with the input of breaker height, the breaker depth was calculated by the McCowan, J. (1894). Wave height of deep water was calculated using the Wiegel's equation, Wiegel,R.L. (1949,1964), by setting a wave period. After obtaining breaker height and breaker depth, breaker length was calculated using the Miche's equation Miche, R. (1944). With the obtained breaker length, the breaker steepness was calculated, and the obtained breaker steepness is much smaller than the criteria Toffoli, A., Babanin, A., Onorato, M., and Waseda T. (2010). This result showed that the breaker length of equation Miche, R. (1944) is too long. In order to obtain a wavelength matching the criteria of Toffoli et all, the right segment of equation Miche, R. (1944) was multiplied by a coefficient.

The wavelength at the breaker depth was also calculated using a linear wave dispersion equation, and the longest wavelength is obtained where the breaker wave steepness became too small. To get the results of wavelength matching the criteria of Toffoli, et all of shallow water, the left segment of the dispersion equation from the linear wave theory was multiplied by a coefficient, as was the case with deep water.

Furthermore, analytical linear wave dispersion equation is formulated in which there is a coefficient on the left segment of the equation. In the process of formulation, small amplitude and long wave assumptions were carried out as well as the formulation of linear wave dispersion equations Dean, R.G., Dalrymple, R.A. (1991). The dispersion equation is formulated using the Kinematic Free Surface Boundary Condition (KFSBC) and the Euler momentum equation, wherein the two equations were weighted total acceleration, Hutahaean , S. (2019).

In addition, the wavelength formulated based on the nonlinear wave conditions was reviewed. In the results of dispersion equation, there is a coefficient regulating the wavelength. With this dispersion equation, the shortest wave length was obtained.

## II. STUDY ON DEEP WATER

In this section, a wavelength study will be conducted using criteria Toffoli, et all (2010) and with, equationWiegel,R.L. (1949,1964).

### 2.1. Wave Length Estimation using criteria Toffoli et all (2010).

Criteria ofToffoli et all (2010)for critical wave steepness in deep water is,

$\frac{H}{L} = 0.170$ ........(1)

$H$ wave height and $L$ is wave length. Meanwhile, criteria fromMichell, J.H. (1893), $\frac{H}{L} = 0.142$. With (1), the relation between wave length and wave height can be stated, which is,

$L = \frac{H}{0.170}$ ..........(2)

The wave height in (2) is obtained from equation Wiegel,R.L. (1949,1964), which is the relation between the maximum wave height in a wave period,

$T = 15.6\sqrt{\frac{H_{max}}{g}}$ ..........(3)

This equation can be written into an equation for maximum wave height, i.e.

$H_{max} = \frac{gT^2}{15.6^2}$ ..........(4)

The dispersion equation of the linear wave theory,Dean (1991), is

$\sigma^2 = gktanh\ kh$ ..........(5)

$\sigma$is angular frequency $\sigma = \frac{2\pi}{T}$, $T$ is wave period, $g$ is gravitational acceleration $k$ is wave number where $k = \frac{2\pi}{L}$, $L$ is wavelength and $h$ is water depth. In deep water where $tanh\ kh = 1$, (5) becomes,

$\sigma^2 = gk$ ..........(6)

*Table 1. Comparison of wave length from (2) and (6)*

| $T$ (sec.) | $H_{max}$ (m) | $L$(m) Eq. (2) | Eq.(6) | $\frac{H_{max}}{L_{eq-6}}$ |
|---|---|---|---|---|
| 6 | 1,451 | 8,536 | 56,207 | 0,026 |
| 7 | 1,975 | 11,619 | 76,504 | 0,026 |
| 8 | 2,58 | 15,176 | 99,924 | 0,026 |
| 9 | 3,265 | 19,207 | 126,466 | 0,026 |
| 10 | 4,031 | 23,712 | 156,131 | 0,026 |
| 11 | 4,878 | 28,692 | 188,919 | 0,026 |
| 12 | 5,805 | 34,145 | 224,829 | 0,026 |

The calculations results of wavelength with (1) and (6), presented in Table 1. It showed that the wavelength of the linear wave theory (6) is much longer than the wavelength of the critical steepness wave (1). With the wavelength from (6), it was obtained wave steepness $\frac{H_{max}}{L} = 0.026$. This value is much smaller than Michell,J.H (1893) where $\frac{H_{max}}{L} = 0.142$ and from (2) where $\frac{H_{max}}{L} = 0.170$. From this calculation result, it was obtained that the wavelength of dispersion equation of the linear wave theory is too long, where a very large wave height will be obtained if the wave height with (1) is calculated using the wavelength. To improve the wavelength of dispersion equation of the linear wave theory, the left segment of (5) is multiplied by a coefficient $\gamma^2$,

$\gamma^2\sigma^2 = gktanh\ kh$..........(7)

Where on deep water,

$\gamma^2\sigma^2 = gk$ ..........(8)

With $k = \frac{2\pi}{L}$ and with wavelength stated with (1), which is $L = \frac{H}{0.170}$, (8) can be written into equations for $\gamma^2$, i.e.

$\gamma^2 = \frac{0.170(2\pi g)}{\sigma^2 H}$ ..........(9)

$\sigma = \frac{2\pi}{T}$, where wave height $H$ obtained from (4) with input of wave period $T$ determined, it was obtained value $\gamma = 2.566$ for all wave period, as presented in Table 2.

*Table 2. The value of wave length of deep water with $\gamma = 2.566$*

| $T$ (sec.) | $H_{max}$ (m) | $L$(m) Eq. (2) | Eq.(8) | $\gamma$ |
|---|---|---|---|---|
| 6 | 1,451 | 8,536 | 8,536 | 2,566 |
| 7 | 1,975 | 11,619 | 11,619 | 2,566 |
| 8 | 2,58 | 15,176 | 15,176 | 2,566 |
| 9 | 3,265 | 19,207 | 19,207 | 2,566 |
| 10 | 4,031 | 23,712 | 23,712 | 2,566 |
| 11 | 4,878 | 28,692 | 28,692 | 2,566 |
| 12 | 5,805 | 34,145 | 34,145 | 2,566 |

## III. STUDY ON SHALLOW WATER

In this section, a research on breaker steepness $\frac{H_b}{L_b}$ resulted by breaker index and compared to critical wave steepness (1) will be conducted. There are a number of breaker indexes in the form of $\frac{H_b}{L_b}$, where $H_b$ is breaker height and $L_b$ is breaker length. With input of breaker height $H_b$ and breaker depth $h_b$, breaker length $L_b$ can be calculated.

There are two form of equations of breaker index, which are using the bottom slope as a parameter and do not using the bottom slope. In this study, to make it easier, the breaker index that does not use the bottom slope as a parameter was used. The breaker index equations include,

1. Miche's Breaker Index

$\frac{H_b}{L_b} = 0.142 tanh\left(\frac{2\pi h_b}{L_b}\right)$ ..........(10)

2. Battjes dan Jansen's Breaker Index

$\frac{H_b}{L_b} = 0.14\ tanh\left(\frac{0.8}{0.88}\frac{2\pi h_b}{L_b}\right)$ ..........(11)

3. Battjes dan Stive's Breaker Index

$$\frac{H_b}{L_b} = 0.14\ tanh\left(\left(0.5 + 0.4 tanh\left(33\frac{H_0}{L_0}\right)\right)\frac{2\pi h_b}{0.88 L_b}\right)$$

..........(12)

These three equations are seen as derivatives of the criteria of critical wave steepness of Michell's which is $\frac{H}{L} = 0.142$.

Of the three breaker index equations, (10) will be used. Calculation of breaker length $L_b$ using (10) needed input of breaker height $H_b$ and breaker depth $h_b$. Breaker height $H_b$ was calculated using equation of breaker index from Komar and Gaughan (1972), which is

$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-1/5}$ ..........(13)

$H_0$ is deep water wave height obtained from (4), while $L_0$ is deep water wavelength calculated with (6). Breaker depth $h_b$ was calculated using equation of breaker index from McCowan (1894), which is

$\frac{H_b}{h_b} = 0.78$ ..........(14)

*Table 3. Breaker length $L_b$ calculated with (10)*

| $T$ (sec.) | $H_0$ (m) | $H_b$ (m) | $h_b$ (m) | $L_b$ (m) | $\frac{H_b}{L_b}$ |
|---|---|---|---|---|---|
| 6 | 1,451 | 1,689 | 2,165 | 20,409 | 0,083 |
| 7 | 1,975 | 2,298 | 2,947 | 27,779 | 0,083 |
| 8 | 2,58 | 3,002 | 3,849 | 36,282 | 0,083 |
| 9 | 3,265 | 3,799 | 4,871 | 45,92 | 0,083 |
| 10 | 4,031 | 4,691 | 6,013 | 56,691 | 0,083 |
| 11 | 4,878 | 5,676 | 7,276 | 68,596 | 0,083 |
| 12 | 5,805 | 6,754 | 8,659 | 81,635 | 0,083 |

In the Table 3. it is seen that obtained breaker steepness $\frac{H_b}{L_b} = 0.083$ that is smaller than 0.170. In an effort to achieve critical wave steepness values, (10) is developed by multiplying the right segment with a coefficient of 1.36267, so (10) becomes

$\frac{H_b}{L_b} = (1.36267)(0.142)\ tanh\left(\frac{2\pi h_b}{L_b}\right)$ ..........(15)

In Table 4., the calculation results are presented with (15), which is obtained breaker steepness $\frac{H_b}{L_b} = 0.170$ and breaker length $L_b$ becomes much shorter, which is approximately half of the breaker length of (10).

*Table 4. Breaker length $L_b$ calculated with (15)*

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| $T$ (sec.) | $H_0$ (m) | $H_b$ (m) | $h_b$ (m) | $L_b$ (m) | $\frac{H_b}{L_b}$ |
|---|---|---|---|---|---|
| 6 | 1,451 | 1,689 | 2,165 | 9,933 | 0,17 |
| 7 | 1,975 | 2,298 | 2,947 | 13,52 | 0,17 |
| 8 | 2,58 | 3,002 | 3,849 | 17,658 | 0,17 |
| 9 | 3,265 | 3,799 | 4,871 | 22,349 | 0,17 |
| 10 | 4,031 | 4,691 | 6,013 | 27,591 | 0,17 |
| 11 | 4,878 | 5,676 | 7,276 | 33,385 | 0,17 |
| 12 | 5,805 | 6,754 | 8,659 | 39,731 | 0,17 |

The next study is an attempt to get value $\gamma$ on (7) for shallow water. By using (13) and (14), it is obtained breaker height $H_b$ and breaker depth $h_b$. With the breaker depth, breaker length with (7) is calculated by trial and error namely by changing the value $\gamma$ , and it was obtained value$\frac{H_b}{L_b} = 0.17$ and obtained value $\gamma = 2.23$ with wavelength inTable 5.

*Table 5. Breaker length $L_b$ calculated with (7), $\gamma = 2.23$*

| $T$ (sec.) | $H_0$ (m) | $H_b$ (m) | $h_b$ (m) | $L_b$ (m) | $\frac{H_b}{L_b}$ |
|---|---|---|---|---|---|
| 6 | 1,451 | 1,689 | 2,165 | 9,932 | 0,17 |
| 7 | 1,975 | 2,298 | 2,947 | 13,519 | 0,17 |
| 8 | 2,58 | 3,002 | 3,849 | 17,657 | 0,17 |
| 9 | 3,265 | 3,799 | 4,871 | 22,348 | 0,17 |
| 10 | 4,031 | 4,691 | 6,013 | 27,59 | 0,17 |
| 11 | 4,878 | 5,676 | 7,276 | 33,383 | 0,17 |
| 12 | 5,805 | 6,754 | 8,659 | 39,729 | 0,17 |

In the case of calculation of breaker length with (7), it was used $\gamma = 2.566$ as result of research on deep water, then it was obtained breaker steepness $\frac{H_b}{L_b} = 0.211$ (Table 6). The value of this critical breaker steepness can be hapened, considering the occurrence of wave energy compression that is the occurrence of shoaling and shortening of the wave length when the waves enter shallower waters.

*Table 6. Breaker length $L_b$ calculated with (7),*

$\gamma = 2.566$

| $T$ (sec.) | $H_0$ (m) | $H_b$ (m) | $h_b$ (m) | $L_b$ (m) | $\frac{H_b}{L_b}$ |
|---|---|---|---|---|---|
| 6 | 1,451 | 1,689 | 2,165 | 7,988 | 0,211 |
| 7 | 1,975 | 2,298 | 2,947 | 10,873 | 0,211 |
| 8 | 2,58 | 3,002 | 3,849 | 14,201 | 0,211 |
| 9 | 3,265 | 3,799 | 4,871 | 17,973 | 0,211 |
| 10 | 4,031 | 4,691 | 6,013 | 22,189 | 0,211 |
| 11 | 4,878 | 5,676 | 7,276 | 26,849 | 0,211 |
| 12 | 5,805 | 6,754 | 8,659 | 31,952 | 0,211 |

From the results of research on deep water and shallow water, it is found that for the dispersion equation of the linear wave theory to produce a wavelength meeting the criteria of critical wave steepness, the left segment of the equation must be multiplied by a coefficient $\gamma^2$ as it is with (7). Research on deep water is obtained value $\gamma = 2.566$, while research on shallow water is obtained value $\gamma = 2.23$ , both are on the critical wave steepness of 0.170.

## IV. FORMULATION OF DISPERSION EQUATION ANALYTICALLY

In the following section, the dispersion equation will be formulated analytically where there are coefficients $\gamma^2$ on left segment of dispersion equation.

### 4.1. Characteristic Point

The velocity potential of the Laplace equation solution by the variable separation method consists of two components of the sinusoidal equation, which are component *cosinus* and component *sinus*, i.e.,

$$\Phi(x,z,t) = A\cos kx(Ce^{kz} + De^{-kz})\sin\sigma t + B\sin kx(Ce^{kz} + De^{-kz})\sin\sigma t \quad ..........(16)$$

Hutahaean (2019) has shown that both velocity potential components have the same wave constant, thus it can be written as,

$$\Phi(x,z,t) = G\cos kx\cosh k(h+z)\sin\sigma t + G\sin kx\cosh k(h+z)\sin\sigma t \quad ..........(17)$$

Function $\cos kx$ and $\sin kx$ has a intersection point where both functions have the same value, the point is called the characteristic point. By studying the value of the wave constant at the characteristic point, the constant value applies to all points on the entire wave curve. At the characteristic point of the velocity potential equation, it can be written as,

$\Phi(x,z,t) = Gcoskxcoshk(h+z)sin\sigma t$....(18)

Where in this equation, it is defined a new constant which is $G = 2G$. Constant $G$obtained at the characteristic point must be divided by $\sqrt{2}$ if it will be applied to the complete velocity potential equation (17). However, the important part in this case is that the wave constants $G$ and $k$ can be formulated on the conditions $coskx = sinkx$.

4.2. Weighted Total Acceleration

Hutahaean (2019), formulated the weighted total acceleration equation using the Taylor series where KFSBC became,

$\gamma \frac{\partial \eta}{\partial t} = w_\eta - u_\eta \frac{\partial \eta}{\partial x}$ ..........(19)

while the surface momentum equation becomes,

$\gamma \frac{\partial u_\eta}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\left(u_\eta^2 + \gamma_z w_\eta^2\right) = -g\frac{\partial \eta}{\partial x}$ ..........(20)

Where $\gamma = 3$ and $\gamma_z = 1.63$ ia a coefficient, $\eta = \eta(x,t)$ is water surface elevation equation, $u_\eta$ is particle velocity on the surface water in the horizontal direction-$x$, $w_\eta$ is particle velocity on the surface water in the vertical direction-$z$.

4.3. Water Surface Equation, $\eta(x,t)$

The water surface equation will be formulated usinglinearized KFSBC as it is done in the formulation of the dispersion equation of the linear wave theory, which is by assuming the small amplitude wave and long wave, the second term on the right segment (19) will be very small compared to other terms, so it can be ignored, thus (20) becomes,

$$\gamma \frac{\partial \eta}{\partial t} = w_\eta$$

By using potential velocity of Eq. (18), where $w = -\frac{\partial \Phi}{\partial z}$

$w_\eta = -Gkcoskxsinhk(h+\eta)sin\sigma t$

By working on the assumption of small amplitude wave where $(h+\eta) = h\left(1+\frac{\eta}{h}\right) \approx h$.

$$\gamma \frac{\partial \eta}{\partial t} = -Gkcoskxsinhkhsin\sigma t$$

Integration to time $t$, obtained water level equation .

$\eta(x,t) = \frac{Gk}{\gamma\sigma} sinhkh\ coskxcos\sigma t$ ..........(19)

4.4. Formulation of Linear Dispersion Equations

By working the assumption of small amplitude and long wave, the second term of the left segment of the momentum equation can be considered as a very small number, then the (20) becomes

$\gamma \frac{\partial u_\eta}{\partial t} = -g\frac{\partial \eta}{\partial x}$ ..........(20)

From velocity potential equation of (18), particle velocity in the horizontal direction-$x$ is,

$u_\eta = -\frac{\partial \Phi}{\partial x} = Gksinkxcoshk(h+\eta)sin\sigma t$

By working on the assumption of small amplitude dan long wave,

$u_\eta = Gk\ sinkx\ coshkh\ sin\sigma t$

$\frac{\partial u_\eta}{\partial t} = Gk\sigma\ sinkx\ coshkh\ cos\sigma t$ ..........(21)

From (19),

$\frac{\partial \eta}{\partial x} = -\frac{Gk}{\gamma\sigma} ksinhkh\ sinkx\ cos\sigma t$ ..........(22)

Substitution (21) and (22) to (20)

$$\gamma Gk\sigma\ sinkx\ coshkh\ cos\sigma t = g\frac{Gk}{\gamma\sigma}k\ sinhkh\ sinkx\ cos\sigma t$$

For $sinkx\ cos\sigma t$ was not zero, these elements will eliminate each other between the left and the right segment of the equation,

$\gamma^2\sigma^2 = gktanhkh$ ..........(23)

This equation is the dispersion equation of the linear water wave which is the same as (7) where there is a coefficient $\gamma^2$ on the left segment of the equation. Therefore, it can be shown analytically that there is a linear wave dispersion equation where there is a coefficient in the left segment $\gamma^2$. Analytically (using the Taylor series), were obtained $\gamma = 3$. However, if the $\gamma = 3$is used, a complete momentum equation should be used, Hutahaean (2019).

As an illustration of the wave length produced by (7) or (23), is as shown in Table 7, where a wave period of 8 sec is used.

*Table 7. Wave length on several value $\gamma$, wave period $T = 8$ sec.*

| $h$ (m) | $L_{linear}$ (m) | $L_\gamma$ (m) $\gamma = 2.23$ | $\gamma = 2.57$ | $\gamma = 3.0$ |
|---|---|---|---|---|
| 15 | 81,79 | 20,09 | 15,176 | 11,103 |
| 14 | 79,982 | 20,087 | 15,176 | 11,103 |
| 13 | 78,008 | 20,082 | 15,175 | 11,103 |
| 12 | 75,85 | 20,072 | 15,174 | 11,103 |
| 11 | 73,489 | 20,053 | 15,173 | 11,103 |
| 10 | 70,898 | 20,018 | 15,168 | 11,102 |
| 9 | 68,049 | 19,955 | 15,159 | 11,102 |
| 8 | 64,903 | 19,842 | 15,136 | 11,1 |

| | | | | |
|---|---|---|---|---|
| 7 | 61,409 | 19,643 | 15,087 | 11,095 |
| 6 | 57,501 | 19,301 | 14,979 | 11,078 |
| 5 | 53,082 | 18,736 | 14,753 | 11,028 |
| 4 | 48,006 | 17,831 | 14,299 | 10,885 |
| 3 | 42,031 | 16,419 | 13,443 | 10,505 |
| 2 | 34,691 | 14,227 | 11,9 | 9,595 |
| 1 | 24,794 | 10,648 | 9,088 | 7,561 |

In the Table 7., $L_{linear}$ is wavelength with dispersion equations of the linear wave theory or $\gamma = 1$, while$L_{\gamma}$ is the wavelength calculated by values $\gamma = 2.23, \gamma = 2.566$ and $\gamma = 3.0$. What needs attention is that there is a huge difference between $L_{linear}$ and $L_{\gamma}$, in which it confirms that further research is needed on the wavelength of the water waves.

4.5. Nonlinear Dispersion Equations in Deep Water

In the case of non-linear assumptions, namely the assumption of small amplitude and long wave, the nonlinear term equation in the momentum equation of (20) is also done. The formulation is not written here because the limited space, where the formulation can be seen in Hutahaean (2019). The nonlinear dispersion equation in deep water is,

$$\gamma^2\sigma^2\left(1-\frac{kA}{2}\right)+\frac{\gamma^2\sigma^2}{4}(1-\gamma_z)kA = gk\left(1-\frac{kA}{2}\right)^2$$
..........(24)

where $A$ is wave amplitude, $\gamma = 3.0$ while $\gamma_z = 1.63$. Actually $\gamma_z$ is function of $\gamma$, Hutahaean (2019), where for $\gamma = 3$then $\gamma_z = 1.63$will be obtained.

Comparison between the wave length of (8) and the wave length of Eq. (24), where in (8) used the value $\gamma = 2.566$ while in (24) used the value $\gamma = 3.0$ with $\gamma_z = 1.63$ is as follows,

*Table 8. Comparison between wave length (8) and wave length (24)*

| $T$ (sec.) | $H$ (m) | $L_{eq-8}$ (m) | $L_{eq-24}$ (m) | $\frac{H}{L_{eq-24}}$ |
|---|---|---|---|---|
| 6 | 1,451 | 8,536 | 5,933 | 0,245 |
| 7 | 1,975 | 11,619 | 8,076 | 0,245 |
| 8 | 2,58 | 15,176 | 10,548 | 0,245 |
| 9 | 3,265 | 19,207 | 13,349 | 0,245 |
| 10 | 4,031 | 23,712 | 16,481 | 0,245 |
| 11 | 4,878 | 28,692 | 19,942 | 0,245 |
| 12 | 5,805 | 34,145 | 23,732 | 0,245 |

In Table 8, it can be seen that the wave length of (8) with $\gamma = 2.566$ is longer than wave length (24) with $\gamma = 3.0$ and $\gamma_z = 1.64$, where$\frac{H}{L_{eq-24}} = 0.245$ while $\frac{H}{L_{eq-8}} = 0.17$ which is the criteria of the critical steepness wave (1). The values $\gamma = 3.0$ and $\gamma_z = 1.63$ is entirely an analytic result, while the value $\gamma = 2.566$ is derived from the criteria (2) which is the experimental result. Based on this condition, another opinion needs to be reviewed, namely the criteria of critical waves from Michell,J.H (1894),$\frac{H}{L} = 0.142$, which will produce value $\gamma = 2.345$. Comparison with Michell's criteria, presented in Table (9).

*Table 9. Comparison of the wave length of different criteria.*

| $T$ (sec.) | $H$ (m) | $L_{eq-8}$ (m) $\gamma = 2.345$ | $\gamma = 2.566$ | $L_{eq-24}$ (m) |
|---|---|---|---|---|
| 6 | 1,451 | 10,22 | 8,536 | 5,933 |
| 7 | 1,975 | 13,91 | 11,619 | 8,076 |
| 8 | 2,58 | 18,168 | 15,176 | 10,548 |
| 9 | 3,265 | 22,994 | 19,207 | 13,349 |
| 10 | 4,031 | 28,388 | 23,712 | 16,481 |
| 11 | 4,878 | 34,349 | 28,692 | 19,942 |
| 12 | 5,805 | 40,878 | 34,145 | 23,732 |

From the wave length comparison in Table 9, it is actually still quite difficult or further study is needed to determine the value $\gamma$ in (8). From the results of the comparison, $\gamma$ a moderate value can be determined, that is, the value used $\gamma = 2.566$. However, the results of studies using the Toffoli's criteria on shallow water, produce a value $\gamma = 2.23$ which is quite close to 2,345 which is value $\gamma$ from Miche;;'s criteria. Thus, as a temporary conclusion is that it is better to use the value $\gamma = 2.345$ in (8) both in deep water and shallow water so that the analysis of water waves is not done in critical wave conditions.

## V. CONCLUSIONS

In this study, it was found that the wave length of dispersion equation of the linear wave theory is too long to reach the criteria of critical wave steepness. The wave length of dispersion equation of the linear water wave can be shortened by multiplying the left segment of the

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equation by a coefficient. Therefore, the first conclusion is that the dispersion equation of the linear water wave theory can be corrected by multiplying the left segment of the equation by a coefficient.

Coefficient values can be obtained from the criterion equation of critical steepness wave from both in deep water and shallow water. The coefficient value obtained by the results of the study on deep water is different from that on shallow water. It is also different from the results of analytical research, but it can still be estimated the best value among the coefficients obtained with the consideration that the analysis of water waves is not carried out in critical wave conditions. However, further research is needed to obtain a better single coefficient.

Wavelength and wave number are important parameters in modeling the dynamics of water waves. With a better wave number, the water wave model will produce a phenomenon that is closer to the natural water wave phenomenon.

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