

# Bootstrap Method in Price Analysis in Reverse Logistics of Solid Waste from Commercial Restaurants

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**Abstract**— Reverse logistics is defined as the practices and processes established to organize the returns of products from point of sales to the manufacturer in order to repair, recycle or dispose of these items in the most economical way possible. There's a close connection between reverse logistics and environmental protection because it focuses on the management of products, components and materials that have been used and disposed of, and of which the manufacturer has some kind of responsibility. Its purpose is to reuse these products and, where possible, reduce the final amount of waste. To demonstrate the economic viability of reverse logistics, robust statistical procedures are required. Among these procedures, we can highlight the bootstrap method, which is a resampling procedure used to estimate statistics of a population by sampling a set of data with repetition. Therefore, the objective of this study was to use the bootstrap method to estimate the ideal selling price of solid waste in a network of commercial restaurants in the metropolitan region of Recife, in order to obtain information on the profit from the operation. The results showed that the bootstrap method was an efficient statistical tool in the analysis of data from reverse logistics operations and can be used in decision making on the variation of solid waste sales prices in the chain of restaurants studied. Therefore, the network of restaurants studied can certainly profit from the reverse logistics operation, in addition to sustainable business marketing that attracts a larger group of customers.

**Keywords**— Computational Statistics, Mathematical Modeling, Operational Analysis, Statistical Analysis, Sustainable Economy.

## I. INTRODUCTION

Reverse logistics (RL) has the power to create tangible and intangible value to the used and/or returned goods, extracting the maximum value from them. This method avoids wastage with labor, time and with the raw material itself, whose purchase is minimized. In addition, it adds value by increasing product lifecycles, which takes into account after sales, thus generating customer satisfaction and loyalty. Based on this, buyer feedback is needed so that one can think of continuous improvement and analyze the reasons for product returns, which should lead to future improvements or new product designs [1].

Scholars from around the world have defined four reverse logistics network standards: established by the companies themselves, outsourcing to third parties, integration of direct and reverse logistics and joint construction by related companies. All of these standards

have their own advantages and disadvantages, but for companies that build the reverse logistics network alone, which can directly contact customers and obtain more accurate information, have the cost of high deployment [2].

The food industry can benefit from one of the reverse logistics models, so it's necessary to assess the financial viability of selling, donating or disposing of the waste produced locally. There's a need for rapid movement of food products along the supply chain, which requires not only management, but also the development of a backward product flow management system. For this reason, RL is a good choice for dealing with this kind of problem as it deals with the return and management of waste products along its reflux supply chain. In addition, the company receives tax benefits and advantage over the competition.

The obstacles posed to LR are the questions about the level of adoption of reverse logistics by restaurant owners, the rate of consumer return of food products to restaurants, the way in which the waste is managed, the role of the firm at the level of reverse logistics adoption and which determinants influence the level of adoption of reverse logistics by retailers in the context of Brazil. In fact, many managers question the advantages of implementing reverse logistics. Erroneously, they believe that this is just a cost-generating factor for the company. In fact, the adoption of reverse logistics is not a matter of choice. With the emergence of the National Solid Waste Policy, companies must adapt and implement waste management systems and reverse logistics, under penalty of being penalized with fines, forced to paralyze their operations, have financial losses and damages the image of the company.

On August 2, 2010, Law 12.305 was introduced, which defined reverse logistics as the "Instrument of economic and social development characterized by a set of actions, procedures and means to enable the collection and restitution of solid waste to the sector recycling, in its cycle or in other productive cycles, or other environmentally adequate final destination" [3].

Sustainable and green initiatives can aid in the planning of operations and the traceability of products, which favors the adoption of reverse logistics by the industries. Their adoption not only supports the triage of operations, but also the greening of the supply chain, in line with global government sustainability regulations, in addition to generating profit.

It's appropriate to know the cost and price of all the products or services involved in the RL, and to balance these two factors in order to make a profit, it's necessary for the service to go through a pricing process, which is considered one of the problems of the organizations. However, when done correctly, pricing becomes a powerful lever to foster growth with profitability and to achieve strategic business goals [4].

It's known that the bootstrap method shows a great practicality and accuracy in relation to other classical statistical methods in relation to the continuous variables that a study can contain, so it was chosen as the method of analysis in this study [5]. Therefore, the bootstrap method is a robust statistical tool capable of assisting in decision making on the ideal sale price of solid waste formed in the researched restaurants.

## II. METHODOLOGY

### 2.1. Sampling

The research involved 32 commercial restaurants in the à la carte fast food chain, all located in the

Metropolitan Region of Recife, which used reverse logistics in their favor during the year 2018. The survey was conducted during the month of May to November 2018. The prices of each recyclable product were surveyed through quotations with cooperatives, selective collection programs, old iron, industries and estimates provided by market professionals, whose information is periodically updated on the CEMPRE website ([www.cempre.org.br](http://www.cempre.org.br)). In the period of the survey the price of the pound of cardboard was in R\$ 0.26, plastic R\$ 0.60 and paper at R\$ 0.30. However, in the present study, only the prices of plastic and paper were considered, since they were the most important recyclables in the reverse logistics of the studied restaurants. For the analysis, it was necessary to quantify the weight of the discarded recyclables monthly. To do it so, a place was separated in each of these restaurants to accomplish this task. All teams involved received training on how to properly dispose of and how to perform the weighing.

### 2.2. Statistical analysis of data by the bootstrap method

Efron and Tibshirani (1993) presented the basic underlying ideas in the bootstrap method, in the classic scope of inference of statistics, as the following. A bootstrap sample is a sample composed by  $x^* = (x_1^*, x_2^*, L, x_n^*)$  that is obtained in a random form with repositioning from the experimental sample  $x = (x_1, x_2, L, x_n)$ , also designated bootstrap population. Here, the asterisks denote that  $x^*$  it is a randomized version, or resampling of  $x$ , rather than a new group of real data, the bootstrap sample consists of corresponding members of  $x$ . In each bootstrap resampling procedure of size  $n$ , we obtained the arithmetic mean  $\bar{X}_i^*$  which was calculated by Equation 1.

$$\bar{X}_i^* = \sum_{i=1}^n \frac{X_{in}^*}{n} \quad 1$$

For a number  $m$  resampling, the bootstrap arithmetic mean was calculated by Equation 2. The standard deviation was determined by Equation 3.

$$\bar{X}_m^* = \sum_{i=1}^m \frac{\bar{X}_i^*}{m} \quad 2$$

$$\sigma_m = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (\bar{X}_i^* - \bar{X}_m^*)^2} \quad 3$$

The bootstrap probability distribution is obtained by the following sequence:

$$X_1^* = [x_{11}^*, x_{12}^*, L, x_{1n}^*]$$

$$X_2^* = [x_{21}^*, x_{22}^*, L, x_{2n}^*]$$

M

$$X_m^* = [x_{m1}^*, x_{m2}^*, L, x_{mn}^*]$$

In practice, the bootstrap distribution is constructed by the Monte-Carlo method with a number of repetitions,  $m$ , that are sufficiently large. In this case, bootstrap arithmetic means converge to the normal probability distribution (Manly, 1997). Its convergence is guaranteed by the great numbers law, because,  $(x_1^*, x_2^*, L, x_n^*)$  are nothing more than a sample of independent random variables and are identically distributed. Operationally, the bootstrap procedure consists of a resampling of the same size and with repositioning of the experimental sample data, and

$$CI_{Boot-z}(\bar{x}_m^*, 100(1-\alpha)\%) = \left[ \bar{x}_m^* - z_{\alpha/2} \sigma_{mboot}; \bar{x}_m^* + z_{\alpha/2} \sigma_{mboot} \right] \quad 4$$

### III. RESULTS AND DISCUSSION

To test the feasibility of applying the bootstrap method on the ideal sale price of solid waste formed in the researched restaurants, was used data from one restaurant that served as a model for the others. In this case, the prices of plastic and paper were used, since they are the recyclables of greater weight in the reverse logistics of said restaurant. Due to anonymity, the commercial establishment was classified as restaurant 1. In this case, the number of recyclables produced during the year 2018 are presented in Table 1. The average selling price of waste (R\$/kg) for the paper and plastic was R\$ 0.30 and R\$ 0.60, respectively. Table 2 shows the prices of recyclables in the year 2018. These values were obtained by multiplying the quantity produced by the respective values in R\$/kg.

Table 1. Production of recyclables per month (kg) in restaurant 1.

Month	Recyclables	
	Plastic	Paper
01	43.4	21.7
02	35.9	18.2
03	32.4	14.4
04	42.0	23.4
05	36.7	18.5
06	44.2	22.3
07	41.5	21.5
08	36.2	17.9
09	36.7	16.7

calculus of statistic interest for each resampling, called pseudo-values.

The following algorithm was built by the Monte Carlo method to optimize the average price in reverse logistics: (1) the experimental sample was selected by using a generator of random numbers. The  $n$  values with repositioning to form bootstrap samples of the same size as the original; (2) the arithmetic average was computed in each procedure of resampling; (3) step (2) was repeated a  $m$  number of times, this way obtaining  $m$  values of the statistic in question and (4) the mean  $\bar{X}_m^*$  was obtained.

The mean  $\bar{X}_m^*$  was used to estimate the average price.

The confidence interval ( $CI_{Boot-z}$ ) for the estimator  $\bar{X}_m^*$  is calculated by Equation 4. The simulation process was carried out using a programme developed in the C++ language, with a generator of random numbers. In the present work 2,000 bootstrap interactions were carried out.

10	41.9	23.9
11	39.2	21.7
12	44.1	25.4

Table 2 shows a significant variability in the values of recyclable prices. These variations may cause fluctuations in the estimates of profit figures from sales of recyclables. This makes it difficult to find the true profit margin, since in this case the average price varies within a certain confidence interval. Figures 1 and 2 show the histograms of plastics and paper prices, respectively.

Table 2. Price of recyclables (R\$) per month in restaurant 1.

Month	Recyclables	
	Plastic	Paper
01	26.0	6.5
02	21.5	5.5
03	19.4	4.3
04	25.2	7.0
05	22.0	5.6
06	26.5	6.7
07	24.9	6.5
08	21.7	5.4
09	22.0	5.0
10	25.1	7.2
11	23.5	6.5
12	26.4	7.6

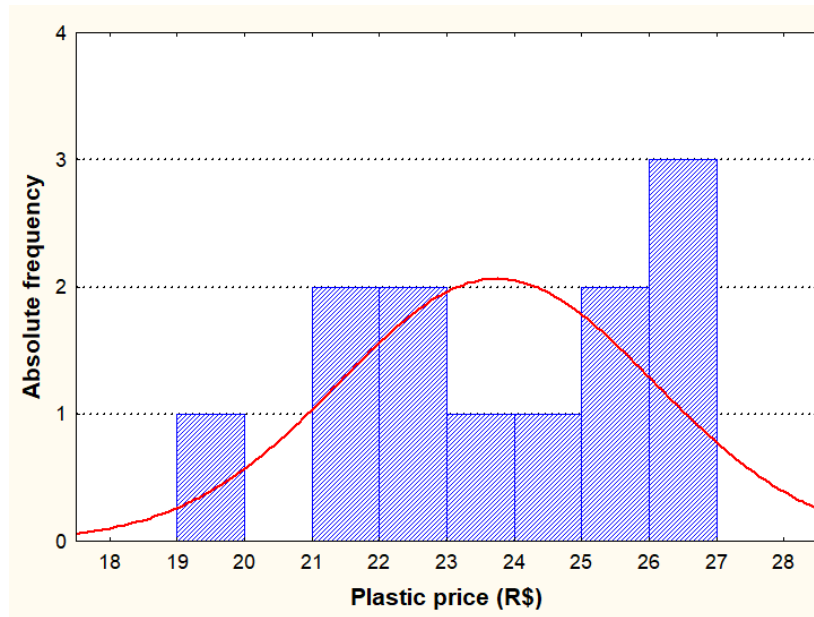


Fig. 1. Histogram of plastic prices

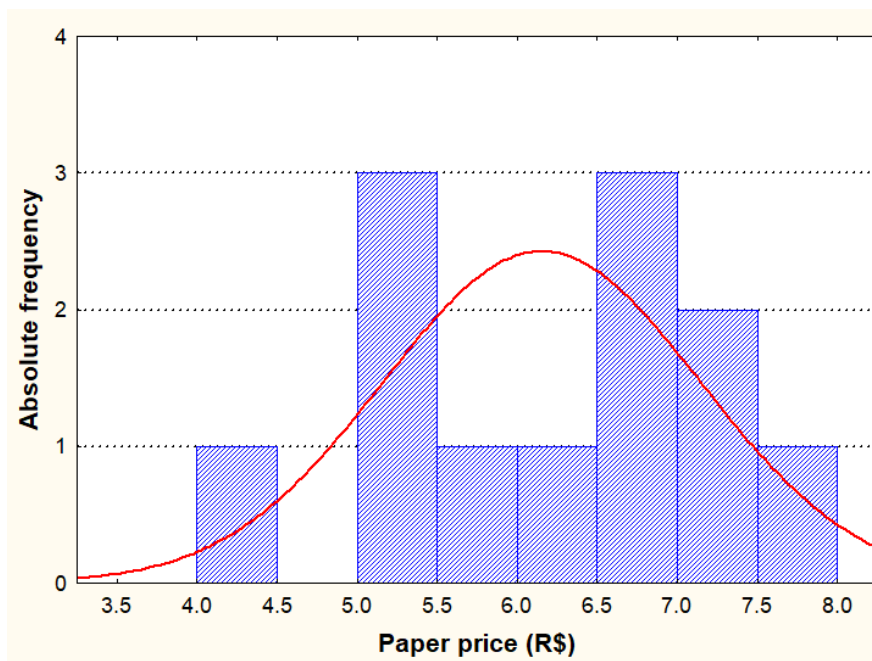


Fig. 2. Histogram of paper prices

However, many evaluations in sampling theory has limited use when the data set is small (Figures 1 and 2). For example, the estimate of the standard deviation for  $n$ , normally distributed values, is very sensitive to fluctuations when  $n$  is small (Table 2) than when it's large [8]. However, to solve this problem, it's necessary to collect a large number of samples, in order to reduce statistical fluctuations in the data set. On the other hand, often due to financial issues and limitations of the collection conditions, it's not always possible to make collections of a large number of samples. The Big

Numbers law guarantees that, when  $n = 30$ , there's a probability that the data will have a normal distribution [9]. However, Silva et al., 2007 [10] found that even for  $n > 30$ , the normality condition isn't guaranteed when there are anomalous values, and the arithmetic mean is inadequate to represent the data set. In case of the data presented in Table 2, it's observed that the sample amount was less than 30. It makes a very difficult statistical analysis adequate, often limited only to the discussion of range of values, or in the very one estimate using the median.

On the other hand, there's a big statistical problem that needs to be solved. The median doesn't adequately represent a set of data since it's a measure of central tendency lower than the arithmetic mean [11]. This would cause an underestimate, as the extreme values are discriminated by the median. In this case, the median doesn't adequately represent the values in the data set. The bootstrap method is a fairly viable alternative to solve this problem, since the Central Limit Theorem guarantees

that when  $n$  is sufficiently large, the mean and standard deviation estimators are normally distributed and converge to their true values [8]. However, using the bootstrap method, it's possible to obtain an arithmetic mean that's resistant to fluctuations in sales prices of recyclables (Table 2), and consequently a more robust confidence interval. Figures 3 and 4 show the bootstrap histograms of plastics and paper price values for 2000 interactions.

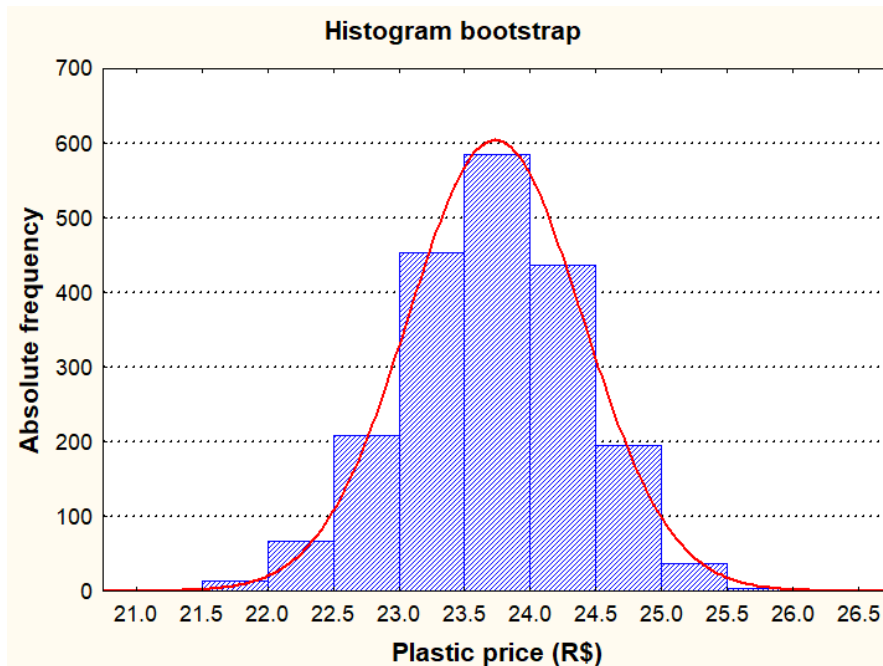


Fig. 3: Histogram bootstrap of plastics prices

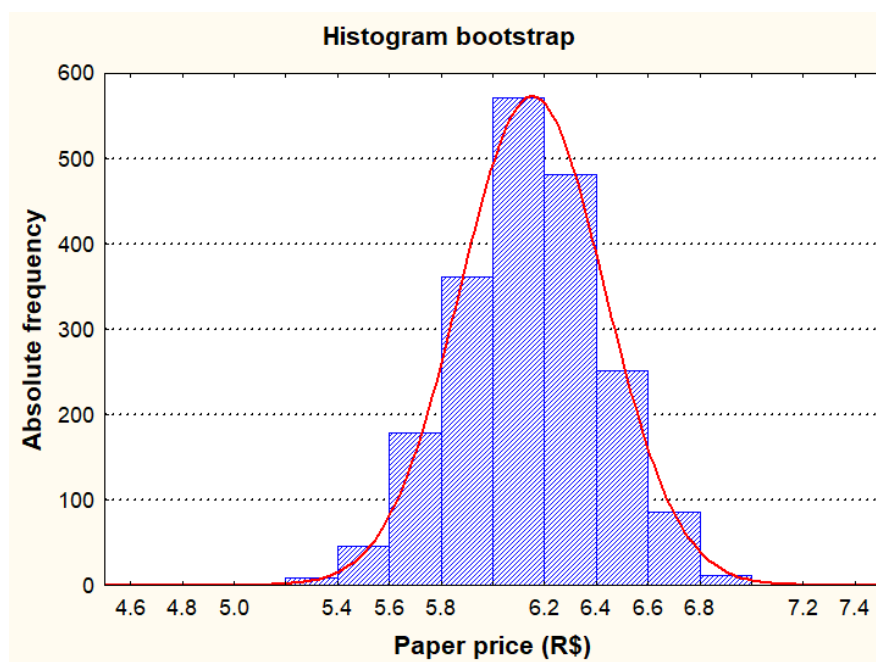


Fig. 4: Histogram bootstrap of paper prices

The Kolmogorov-Smirnov test showed that the values of plastic prices and bootstrap resampling papers has a normal distribution (Figures 5 and 6). Figures 5 and 6 show that fluctuations in recyclables prices have been significantly attenuated, thus providing reliable statistical

analysis. Thus, statistical analyzes based on the bootstrap distribution are notoriously robust, as observed by Efron and Tibshirani, 1993 [6]. Table 3 shows the values of central tendency measurements for the conventional normal distribution and bootstrap of recyclables prices.

Table 3. Conventional arithmetic mean and bootstrap prices of recyclables.

Recyclables	Conventional arithmetic mean (R\$)	Bootstrap arithmetic mean (R\$)
Plastic	23.7 ± 2.3	23.7 ± 0.5
Paper	6.1 ± 1.0	6.2 ± 0.2

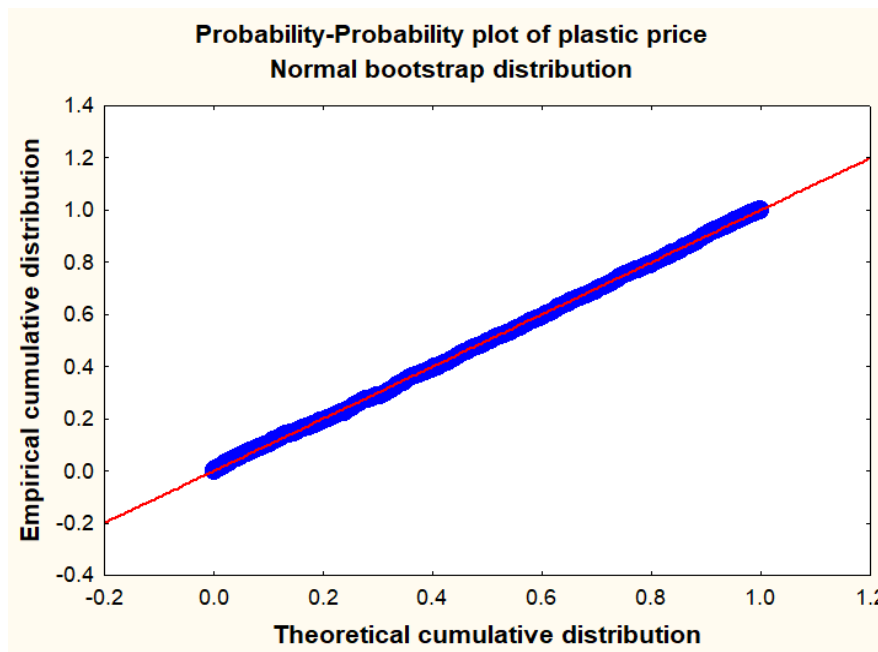


Fig. 5: Probability-probability graph of the normal bootstrap distribution of plastics prices.

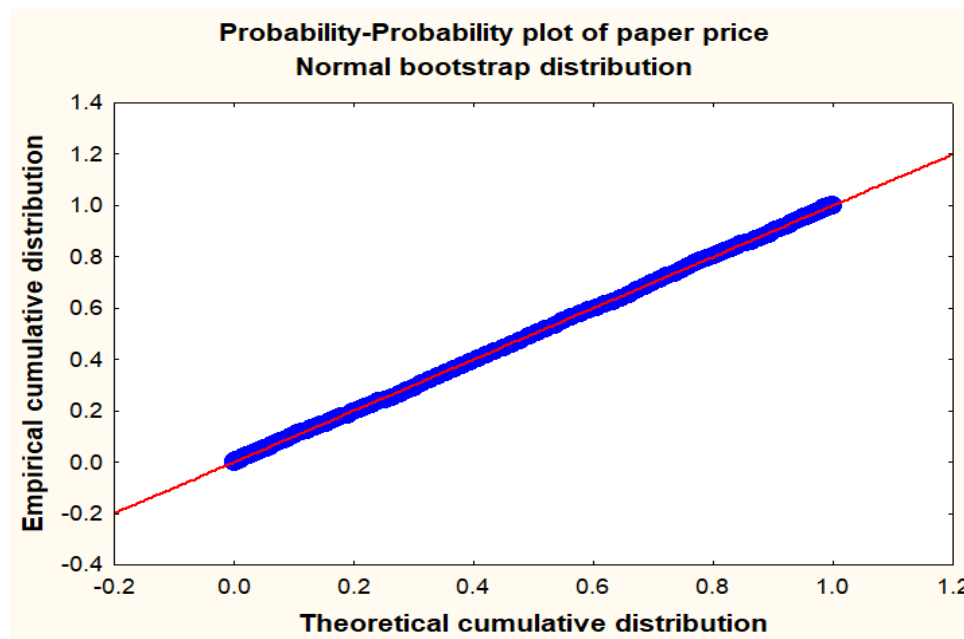


Fig.6. Probability-probability chart of the normal bootstrap distribution of paper prices.



There was a significant reduction in the dispersion of values around the bootstrap arithmetic mean of recyclables prices (Table 3). On the other hand, there was no change in arithmetic mean values (Table 3). Therefore, the bootstrap arithmetic mean was the most adequate to represent the data set in Table 2. With this, it

was possible to obtain more representative confidence intervals in the decision on the average sales price of the recyclables. Table 4 shows the values of the lower and upper limits for the conventional arithmetic mean and bootstrap of the recyclable prices.

Table 4. Confidence intervals for experimental arithmetic mean and bootstrap

Recyclables	Confidence interval of conventional arithmetic mean (R\$)	Confidence interval of the arithmetic mean bootstrap (R\$)
Plastic	[22.2; 25.2]	[22.4; 24.7]
Paper	[5.5; 6.7]	[5.6; 6.6]

The confidence interval of the conventional arithmetic mean is wider than that of the bootstrap arithmetic mean (Table 4). Therefore, the confidence interval of the average bootstrap is more adequate to represent amplitude in the average prices of recyclables. Thus, the insertion of the bootstrap method into the reverse logistics in a restaurant network was an important statistical tool in decision making within the context of the profitability of the sale of recyclables. According to Efron, 1993 [6], when a set of data is formed by small samples and has fluctuations without occurrence of anomalous values, as is the case of the data presented in Table 2, the values of bootstrap resampling with 2,000 iterations are symmetrical with respect to the distribution (Figures 3 and 4).

#### IV. CONCLUSION

The bootstrap method proved to be quite efficient in the statistical analysis of the estimate of the sales price of recyclables in restaurant chains. It could be used to decrease fluctuations in values, but also proved to be ideal for constructing a confidence interval in reverse logistics.

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