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Water Wave Profile at Breaker Point

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Abstract*— *In this research, a study was done on wave profile and type at breaker point. The study was done using shoaling-breaking model. Output of the shoaling-breaking model becomes the input at water wave surface equation to obtain wave profile. By obtaining the wave profile, the calculation of breaker height becomes more accurate.
***Keywords*— shallow water wave type, cnoidal wave type.**

I. INTRODUCTION

Wilson (1963) obtained that there are four types of water wave, i.e. Airy's waves, Stoke's waves, Cnoidal waves and Solitary waves. Airy's type, where wave crest is symmetrical with wave trough and wave height is twice of that wave amplitude. This type of wave is found only in a wave with a very small amplitude. Next is Stoke's type, formed in a wave with wave amplitude bigger than wave amplitude of the Airy's type. In this Stoke's type, the wave profile is somewhat asymmetricand the wave height is still close to twice the wave amplitude. In a bigger wave amplitude, cnoidal type of wave is formed. In cnoidal type, the wave profile is asymmetric where the distance between wave trough and neutral line or still water level is very much different with the distance between wave crest and still water level. For a wave with bigger wave amplitude a perfect Cnoidal profile will be formed with wave trough that almost coincides with still water level. This type of wave is also called Solitary wave. Either Cnoidal type or Solitary wave type shows that almost all part of the wave is above the still water level, nevertheless wave height that is twice wave amplitude might occur.

In shoaling-breaking model, some use wave amplitude as the variable and some other use wave height as the variable. For this second type, the formulation was done with an assumption that wave height has a value twice that of wave amplitude. With those four types of wave, then the output of shoaling breaking model that uses wave amplitude as the variable or the model that uses half of the wave heightcan not confirm the wave height. To obtain a certainty of the wave height as the result of shoaling-breaking analysis, the analysis of water wave surface profile needs to be done to obtain a better wave height value.

This research used water wave surface equation of Hutahaean (2019a) to conduct wave profile analysis at breaker point. The equation used wave constant G, wave number $k$and wave amplitude Aas the parameter. The three wave parameters were obtained from shoaling-breaking analysis of Hutahaean (2019b).

At the same time, this research is also a revision on Hutahaean (2019b), where at the research the wave amplitude as the result of shoaling-breaking model was considered has a value of half wave height. Even though it provides a result that is quite close to breaker height from breaker height index equation as the result of laboratory analysis, the result is less than accurate.

II. SEVERAL TYPES OF WAVE

The general form of water wave profile is shown on Fig.1. The form is assymetrical between wave crest and wave trough. If the wave crest elevation against still water level is called η_{max}; whereas the wave trough elevation is called η_{min}, then there is an asymmetry where $\eta_{max} > |\eta_{min}|$, whereas wave height is$H = \eta_{max} - \eta_{min}$.

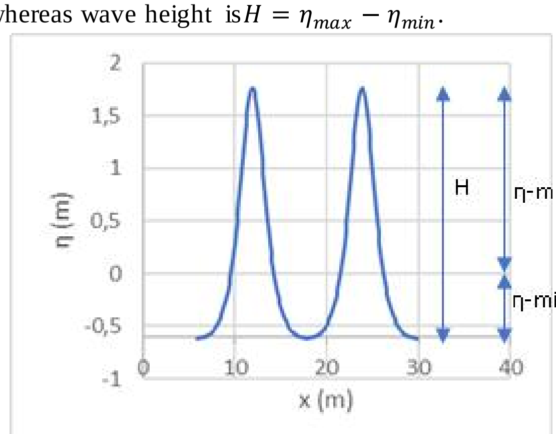

Fig.1. General profile of water wave surface

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2.1. Wilson Criteria (1963)

Wilson (1963) classified wave based on the value of $\frac{\eta_{max}}{H}$ (Table (1)). Type Airy's waves have the value of $\frac{\eta_{max}}{H} < 0.505$, which shows that the wave form is asymmetrical, where $\eta_{max} \approx |\eta_{min}|$. The second type is Stoke's wave type that is still quite close with the Airy's waves characters. Furthermore, cnoidal waves type is absolutely asymmetrical and the last is solitary waves type, where $\frac{\eta_{max}}{H} = 1.0$.

Table.1: Wilson Criteria (1963)

Wave Type	$\frac{\eta_{max}}{H}$
Airy waves	< 0.505
Stoke's waves	< 635
Cnoidal waves	$0.635 < \frac{\eta_{max}}{H} < 1$
Solitary waves	$= 1$

In the next sections the analysis of wave profile in the deep water is presented, includes the calculation of η_{max}, η_{min}, wave height H and wave height ratio $\frac{H}{A}$ and the depiction of the wave profile using equation (1). The result of the calculations shows a compatibility with Wilson criteria (1963).

2.2. Airy's Waves Type

Airy's waves type have symmetrical profile where $\eta_{max} = |\eta_{min}|$ and wave height $H = 2A$. This type of wave is found only in a wave with a very small amplitude. Table (2) shows the measurement of Airy's wave profile in the deep water for several wave periods with a very small wave amplitude A, where $\frac{\eta_{max}}{H} = 0.503$ was obtained, which shows that it is not really symmetrical, but $H = 2A$ was obtained. The profile of Airy's wave can be seen in Fig. 2.

Table.2: Wave amplitude for sinusoidal wave, in the deep water

T (sec.)	A (m)	$\frac{\eta_{max}}{H}$	H (m)	$\frac{H}{A}$
8	0,017	0,503	0,035	2
9	0,02	0,503	0,041	2
10	0,023	0,503	0,047	2
11	0,027	0,503	0,053	2
12	0,03	0,503	0,06	2
13	0,033	0,502	0,067	2
14	0,037	0,502	0,073	2
15	0,04	0,502	0,081	2

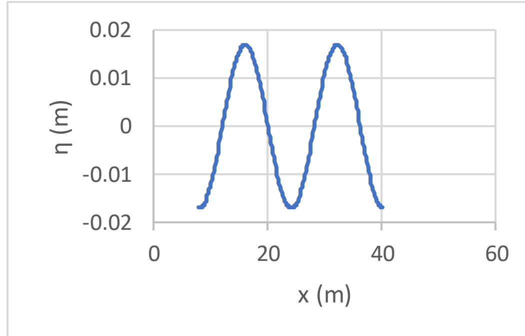

Fig.2. Airy's wave $T = 8$ sec., $A = 0.017$ m, in the deep water.

2.2. Type Stoke's wave.

Table (3) shows wave profile in the deep water with a quite large wave amplitude, with $\frac{\eta_{max}}{H} = 0.635$ that according to Wilson criteria is the maximum limit of Stoke's wave type. The value of $\frac{H}{A} = 2.099$, is still quite close to 2. Therefore, the characteristics of Stoke's wave is that wave profile is asymmetric but $H \approx 2A$, although it is at its maximum limit.

Table.3: Wave profile of Stoke's wave at its maximum limit

T (sec.)	A (m)	$\frac{\eta_{max}}{H}$	H (m)	$\frac{H}{A_{max}}$
7	0,47	0,635	0,986	2,099
8	0,614	0,635	1,288	2,099
9	0,777	0,635	1,631	2,099
10	0,959	0,635	2,013	2,099
11	1,161	0,635	2,436	2,099
12	1,381	0,635	2,899	2,099
13	1,621	0,635	3,402	2,099
14	1,88	0,635	3,946	2,099
15	2,158	0,635	4,529	2,099

2.3. Cnoidal Wave Type.

Table (4) is wave profile in deep water with large amplitude where $\frac{\eta_{max}}{H} = 0.803$ was obtained with the value of $\frac{H}{A} = 2.726 > 2$. Therefore, in this cnoidal wave type the wave profile is asymmetric and $H \neq 2A$. If it is approximated with Airy's waves type there will be a quite large error, in this case is 36.2 %. For a wave with bigger wave amplitude, there will be even bigger error

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Table.4: Cnoidal wave profile

T (sec.)	A (m)	$\frac{\eta_{max}}{H}$	H (m)	$\frac{H}{A}$
7	0,899	0,803	2,449	2,726
8	1,174	0,803	3,199	2,726
9	1,485	0,803	4,049	2,726
10	1,834	0,803	4,998	2,726
11	2,219	0,803	6,048	2,726
12	2,641	0,803	7,198	2,726
13	3,099	0,803	8,447	2,726
14	3,594	0,803	9,797	2,726
15	4,126	0,803	11,246	2,726

From the illustration of the three waves, for a wave with large wave amplitude, $H = 2A$ can not be confirmed although $\frac{H}{A} = \frac{\eta_{max} - \eta_{min}}{A} \leq 2$ could happen and although the profile is asymmetrical.

III. WATER WAVE-SURFACE EQUATION

In this section, the formulation of water wave surface is a revision of the formulation of Hutahaean (2019a), where some typographical errors occurred. The water wave surface equation is obtained by integrating KFSBC,

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\sigma coskx sin\sigma t$$
$$-\frac{Gk}{\gamma\sigma}\beta(\eta)\sigma sinkx sin\sigma t\frac{\partial \eta}{\partial x} \quad ....(1)$$

$\beta_1(\eta) = sinhk(h + \eta)$ ....(2)

$\beta(\eta) = coshk(h + \eta)$ ......(3)

$\gamma$is weighting coefficient at weighted total acceleration equation with a value around 2.202-3.0. This research uses $\gamma = 2.483$ (Hutahaean (2019c,d)).

(1) was integrated against time-t where acccording to wave number conservation, i.e. at $z = \eta$, $\frac{\partial k(h+\eta)}{\partial t} = 0$(Huatahean (2019a)), which means that $\beta_1(\eta)$and$\beta(\eta)$have constant values against time t, thus the integration of the first term right side of the equation was completed by integrating $sin\sigma t$,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)coskx cos\sigma t$$
$$-\frac{Gk}{\gamma\sigma}\beta(\eta)\sigma sinkx\int sin\sigma t\frac{\partial \eta}{\partial x}dt$$

The integration of the second term right side of the equation will be completed using partial integration method, as follows

Assume a function of $f = cos\sigma t\frac{\partial \eta}{\partial x}$. This $cos\sigma t$function was used so when it was differentiated against time t, $sin\sigma t\frac{\partial \eta}{\partial x}$ will be formed, i.e.

$$\frac{\partial f}{\partial t} = -\sigma sin\sigma t\frac{\partial \eta}{\partial x} + cos\sigma t\frac{\partial^2 \eta}{\partial t\partial x}$$

This differential equation was multiplied with $dt$and integrated against time t,

$$f = -\sigma\int sin\sigma t\frac{\partial \eta}{\partial x}dt + \int cos\sigma t\frac{\partial^2 \eta}{\partial t\partial x}dt$$

Substitute $f$and the first term right side was moved to the left and $f$was moved to the right and both equations were divided by σ,

$$\int sin\sigma t\frac{\partial \eta}{\partial x}dt8 = -\frac{1}{\sigma}cos\sigma t\frac{\partial \eta}{\partial x} + \frac{1}{\sigma}\int cos\sigma t\frac{\partial^2 \eta}{\partial t\partial x}dt$$

The integration of the second term right side of the equation can be completed the same way, but with an assumption that $\frac{\partial^3 \eta}{\partial t^2 \partial x}$ is a very small number, the integration can be completed by integrating just the$cos\sigma t$ element.

$$\int sin\sigma t\frac{\partial \eta}{\partial x}dt = -\frac{1}{\sigma}cos\sigma t\frac{\partial \eta}{\partial x} + \frac{1}{\sigma^2}sin\sigma t\frac{\partial^2 \eta}{\partial t\partial x}$$

Substitute the result of integration,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)coskx cos\sigma t$$
$$-\frac{Gk}{\gamma\sigma}\beta(\eta)sinkx\left(-cos\sigma t\frac{\partial \eta}{\partial x} + \frac{1}{\sigma}sin\sigma t\frac{\partial^2 \eta}{\partial t\partial x}\right)$$

Working on an assumption that $\frac{1}{\sigma}sin\sigma t\frac{\partial^2 \eta}{\partial t\partial x}$is a very small number and can be ignored,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)coskx cos\sigma t + \frac{Gk}{\gamma\sigma}\beta(\eta)sinkx cos\sigma t\frac{\partial \eta}{\partial x}$$

The equation was differentiated against horizontal-x axis.

$$\frac{\partial \eta}{\partial x} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx cos\sigma t$$
$$+\frac{Gk}{\gamma\sigma}\beta(\eta)kcoskx cos\sigma t\frac{\partial \eta}{\partial x}$$
$$+\frac{Gk}{\gamma\sigma}\beta(\eta)sinkx cos\sigma t\frac{\partial^2 \eta}{\partial x^2}$$

or

$$\frac{\partial \eta}{\partial x} = \left(-\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx + \frac{Gk}{\gamma\sigma}\beta(\eta)kcoskx\frac{\partial \eta}{\partial x} + \frac{Gk}{\gamma\sigma}\beta(\eta)sinkx\frac{\partial^2 \eta}{\partial x^2}\right)cos\sigma t \quad .....(4)$$

In accordance with the provision at the velocity potential equation where there is $t$function only,$x$function onlyand$z$function only, then at the water wave surface using variable from velocity potential equation, water surface

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equation also has variable that is $t$function only an$x$function only, so it can be stated that the general form of (4) is,

$$\frac{\partial \eta}{\partial x} = f(x)cos\sigma t$$

In this equation $f(x)$is just a function of x. $f(x)$in (4) is,

$$f(x) = \left(-\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx + \frac{G\,k}{\gamma\sigma}\beta(\eta)kcoskxcos\sigma tf(x) + \frac{Gk}{\gamma\sigma}\beta(\eta)sinkxcos\sigma t\,\frac{\partial f}{\partial x}\right)$$

Bearing in mind$\beta_1(\eta)$and$\beta(\eta)$should be constant against time $t$and against horizontal x axis. In the second and third terms right side of the equation there is a function of time t, i.e.$cos\sigma t$, then the term shouldn't be there. Therefore$f(x)$is,

$$f(x) = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx$$

Thereby

$$\frac{\partial \eta}{\partial x} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\,ksinkxcos\sigma t$$

Substitute this equation to (1),

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\sigma coskxsin\sigma t + \left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta)\beta_1(\eta)\sigma ksin^2kxsin\sigma tcos\sigma t$$

is integrated against time t. Integration will be completed by bearing in mind wave number conservation , thus integration was done by integrating only sinusoidal element and bearing in mind that $sin\sigma tcos\sigma t = \frac{1}{2}sin2\sigma t$,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)\,coskxcos\sigma t - \frac{1}{4}\left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta)\beta_1(\eta)ksin^2kxcos2\sigma t$$

......(5)

To determine the values of $\beta_1(\eta)$, $\beta(\eta)$, an approach was done that water surface equation is sinusoidal, i.e.

$\eta_0(x,t) = Acoskxcos\sigma t$ ....(6)

Therefore, the approach of hyperbolic function is $\beta_1(\eta) = \beta_1(\eta_0)$, $\beta(\eta) = \beta(\eta_0)$.

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta_0)coskxcos\sigma t - \frac{1}{4}\left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta_0)\beta_1(\eta_0)ksin^2kxcos2\sigma t$$

......(7)

IV. THE CALCULATION OF G, k AND A AT BREAKER POINT

The calculation of G, k and$A$at breaker point was done using shoaling-breaking model (Hutahaean (2019b)). In principle, the model is a transformation analysis of G, $k$and$A$from the deep water to breaker point.

4.1. The calculation of $G$and$k$in Deep Water

To calculate G and $k$two equations were needed. KFSBC and surface momentum equation, with the formulation can be seen in Hutahaean (2019b).

Wave number $k$in the deep water was calculated using the following equation,

$\gamma^2\sigma^2 = gk - \frac{gk^2A}{2}$.............(8)

$g$is gravitation velocity, whereas $A$is wave amplitude. This equation is a quadratic equation of wave number $k$where$k$can be calculated with a simple equation, i.e. finding the root of the equation. Wave amplitudemaximum at (8) is (Hutahaean (2019a))

$A_{max} = \frac{0.91g}{2\gamma^2\sigma^2}$ ....(9)

Therefore, wave amplitude in (8) should be smaller or less than A_{max}.

Furthermore, deep water depth should be determined which is the starting point of shoaling-breaking analysis where wave amplitude $A$is known as input of the model. Deep water depth is calculated with,

$h_0 = \frac{1.65\pi}{k_0} = 1.65\,\pi A_{max}$ ....(10)

The coefficientof 1.65 was obtained from the calibration of the breaker depth of the shoaling-breaking model against breaker depth of SPM (1984), Hutahaean (2019b).

After wave number k was obtained, wave constant$G$was calculated in the deep water with the following equation

$\gamma\sigma G\beta\left(\frac{A}{2}\right) = gA$ ....(11)

Where wave amplitude A is known.

4.2. Shoaling-Breaking Model Equations

In the shoaling-breaking analysis, there are 3 (three) variables that change along with the change in the depth, i.e.k, $A$and$G$where there is a dependency among the changes of the three variables.

To calculate the three variables, three conservation equations were used, and the formulation of shoaling-

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breaking equations in this section can be seen in Hutahaean (2019b).

The first equation is the result of the substitution of KFSBC and energy conservation equation to wave number conservation equation, i.e.:

$$\left(\left(h+\frac{A}{2}\right)+\frac{Gkf}{2\gamma\sigma}\right)\frac{\partial k}{\partial x}=-k\frac{\partial h}{\partial x}+\frac{\mu G k^3 f}{2\gamma\sigma}$$

......(12)

$$f=\left(\beta_1\left(\frac{A_0}{2}\right)-(1+\mu)\beta\left(\frac{A_0}{2}\right)\frac{kA}{2}\right)$$

.......(13)

(12) and (13) were formulated using velocity potential at the sloping bottom, where in this case,

$$\beta(\eta_0)=\beta\left(\frac{A_0}{2}\right)=\alpha_0 e^{k_0\left(h_0+\frac{A_0}{2}\right)}+e^{-k_0\left(h_0+\frac{A_0}{2}\right)}$$

.....(14)

$$\beta_1(\eta_0)=\beta_1\left(\frac{A_0}{2}\right)=\alpha_0 e^{k_0\left(h_0+\frac{A_0}{2}\right)}-e^{-k_0\left(h_0+\frac{A_0}{2}\right)}$$

....(15)

$$\alpha_0=\frac{1}{2}\left(\frac{1+\frac{\partial h}{\partial x}}{1-\frac{\partial h}{\partial x}}+\frac{1-\frac{\partial h}{\partial x}}{1+\frac{\partial h}{\partial x}}\right) \quad ....(16)$$

$\frac{\partial h}{\partial x}$is bottom slope with negative value. After $\frac{\partial k}{\partial x}$is knownthe second equation is done, i.e. energy conservation equation

$$\frac{\partial G}{\partial x}=-\mu G k \quad ....(17)$$

$$\mu=\frac{2\frac{\partial k}{\partial x}}{\left(3\frac{\partial k}{\partial x}+4k^2\right)} \quad ....(18)$$

After$\frac{\partial G}{\partial x}and\frac{\partial k}{\partial x}$were known $\frac{\partial A}{\partial x}$ can be calculated using the third equation, i.e. KFSBC,

$$\gamma\sigma\frac{\partial A}{\partial x}=\left(\frac{\partial G}{\partial x}k+G\frac{\partial k}{\partial x}\right)f \quad ......(19)$$

The calculation of$\frac{\partial k}{\partial x}$, with(12), was done iteratively.At the beginning of the calculation, μin (18), is not known since $\frac{\partial k}{\partial x}$has not been known, as initial approximation equation$\frac{\partial k}{\partial x}=-\frac{k}{h}\frac{\partial h}{\partial x}$was used andμwas calculated with (18) and by obtaining the new $\frac{\partial k}{\partial x}$from (12), μwas recalculated with (18). This calculation is repeated until a stable $\frac{\partial k}{\partial x}$is obtained where, in general, stability is achieved after 5 times iteration. After stable$\frac{\partial k}{\partial x}$andμare obtained, $\frac{\partial G}{\partial x}$is calculated with (17) and$\frac{\partial A}{\partial x}$with (19). Furthermore, wave number, wave constant and wave amplitude are calculated at the point $x+\delta x$using Taylor series, i.e.

$$k_{x+\delta x}=k_x+\delta x\frac{\partial k}{\partial x}$$

$$G_{x+\delta x}=G_x+\delta x\frac{\partial G}{\partial x}$$

$$A_{x+\delta x}=A_x+\delta x\frac{\partial A}{\partial x}$$

V. THE RESULT OF THE MODEL

5.1 Illustration of the Result of Shoaling-Breaking Model

As an illustration of the result of shoaling-breaking model, shoaling and breaking analysis was done for the wave with wave period of 8 sec, with deep water wave amplitude $A_0=1.056$ m, obtained from (6). Sea bed slope $\frac{dh}{dx}=-0.005$,the result of the model is presented in Fig.3., where at the beginning there is shoaling, i.e. the enlargement of wave - amplitude A, followed by breaking. Output of the model at breaker point includes: breaker amplitude $A_b=1.402$ m, breaker depth $h_b=3.15$ m, breaker wave number $k_b=1.4355$, wave constant $G_b=0.00139$.

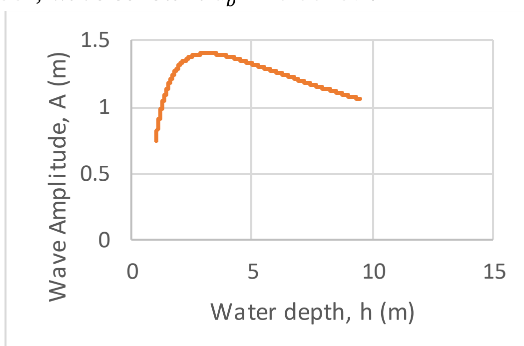

Fig. 3. Wave amplitude as the result of shoaling-breaking model

5.2. The Calculation of Breaker Height H_b

As has been stated before, the output of shoaling-breaking model is breaker wave amplitude $A_b$that must be changed into breaker height H_b.

The calculation of $H_b$was done using water wave surface equation (1). With wave parameter input at breaker point, i.e. A_b, h_b, $k_b$andG_b, the elevation of wave crest η_{max}, the elevation of wave trough η_{min}were calculated, then$H_b=\eta_{max}-\eta_{min}$. From the example of the result of shoaling-breaking in fig.3, breaker height $H_b=2.522$ m was obtained, with the value of$\frac{H_b}{A_b}=1.798$, where$H_b\neq 2A_b$. The wave profile at breaker point is presented in Fig.4.

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which shows the almost perfect cnoidal shaped of wave profile.

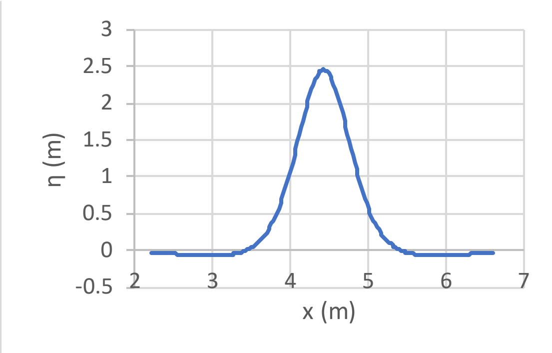

Fig. 4. Wave profile at breaker point

The value of $\frac{H_b}{A_b} = 1.798$is quite close to $H_b = 2A_b$. Therefore in Hutahaean (2019b), H_b model was found to be quite close with the breaker from empirical equation formulated from laboratory observation.

5.2. Calibration and Adjustment of the Result of the Model

As a comparison of the result of the model, breaker height index equations from 5 (five) researchers were used and for the comparator breaker height, the average value of breaker height produced by the five breaker height index was used. The breaker height index (BHI) equations that were used are equations from Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992) and Rattana Pitikon and Shibayama (2000). As a comparator of the result of breaker depth, equation from SPM (1984) was used.

The adjustment of breaker height from the result of the model with the result of breaker height index equation was done by changing the value of wave constant G_b, and that in order for the breaker height to be correspond to breaker height from breaker height index, G_b from the result of shoaling-breaking model must be multiplied by 0.76.

Whereas breaker depth of the output of the shoaling-breaking model was also determined by the choice of deep water depth or the initial depth that was used. The result of the calibration of breaker depth model with breaker height form SPM (1984) shows that by using deep water depth $h_0 = \frac{1.65\pi}{k_0}$breaker depth that corresponds to SPM (1984) was obtained.

5.3. Some of the Model Results

In this section, model was done for several wave periods, with deep water wave amplitude from (6), and the result of the calculation is in Table (5) and Table (6). Adjustment with the result of breaker height index (BHI) equation was done with a process that has been discussed in sub-chapter (5.2), i.e. by multiplying wave constant G_b with 0.76. The result is that breaker height model is very close with breaker height from BHI, where breaker height from BHI is the average values of 5 breaker height index equations as stated in (5.2).

Whereas breaker depth, the result of the model is very close to breaker depth from SPM (1984), that was obtained using deep water depth $h_0 = \frac{1.65\pi}{k_0}$

Table (6) shows that the model produces the value $\frac{H_b}{h_b} = 0.801$for all wave period, which is very close to the result of BHI equation, i.e. $\frac{H_b}{h_b} = 0.802$.

Table.5: The Result of Shoaling-breaking Calculation

T (sec.)	A_0 (m)	H_b (m)		h_b (m)	
		model	BHI	Model	SPM
7	0,809	1,93	1,929	2,411	2,406
8	1,056	2,522	2,52	3,15	3,142
9	1,337	3,194	3,19	3,988	3,977
10	1,65	3,943	3,938	4,922	4,909
11	1,997	4,77	4,765	5,956	5,94
12	2,377	5,675	5,67	7,09	7,07
13	2,789	6,659	6,655	8,32	8,297
14	3,235	7,722	7,718	9,65	9,622
15	3,714	8,862	8,86	11,08	11,046

Table.6: The Result of Shoaling-breaking Calculation (continued)

T (sec.)	A_b (m)	$\frac{H_b}{h_b}$		$\frac{H_b}{L_b}$	$\frac{H_b}{A_b}$
		Model	BHI		
7	1,073	0,801	0,802	0,569	1,798
8	1,402	0,801	0,802	0,569	1,799
9	1,775	0,801	0,802	0,569	1,8
10	2,191	0,801	0,802	0,569	1,8
11	2,651	0,801	0,802	0,569	1,799
12	3,155	0,8	0,802	0,569	1,799
13	3,702	0,8	0,802	0,569	1,799
14	4,294	0,8	0,802	0,569	1,798
15	4,929	0,8	0,802	0,568	1,798

Wave steepness at breaker point is $\frac{H_b}{L_b} = 0.569 = \frac{1.788}{\pi}$, which is very close to or similar to the analytical result as shown as follows. In (10) there is a breaking characteristic when:

$$f = \left(\beta_1\left(\frac{A_0}{2}\right) - (1+\mu)\beta\left(\frac{A_0}{2}\right)\frac{kA}{2}\right) = 0$$

$\beta_1\left(\frac{A_0}{2}\right)$and $\beta\left(\frac{A_0}{2}\right)$is the value in the deep water where$\frac{\beta_1\left(\frac{A_0}{2}\right)}{\beta\left(\frac{A_0}{2}\right)} \approx tanhk\left(h + \frac{A_0}{2}\right) = 1$, then breaking occurs when

$$\frac{kA}{2} = \frac{1}{1+\mu}$$

or

$$\frac{A_b}{L_b} = \frac{1}{(1+\mu)\pi}$$

The value $\frac{H_b}{A_b} = 1.80$, obtained

$$\frac{H_b}{L_b} = \frac{1.80}{(1+\mu)\pi}$$

With an assumption that the bottom slope is very small,

$$\frac{H_b}{L_b} \approx \frac{1.80}{\pi} = 0.572$$

It shows that the numerical result is very close to or similar to the analytical result which also proved that$\frac{H_b}{A_b} = 1.80$is a good value.

VI. VI CONCLUSION

Wave profile at breaker point is cnoidal-typed wave, where wave crest is not symmetrical with wave trough. Interpreting the result of shoaling-breaking model should perform analysis on water wave surface profile. As an approximation,a criteriathat at breaker point the relation$\frac{H_b}{A_b} = 1.8$ applies with wave steepness $\frac{H_b}{L_b} = 0.572$ can be used.

Water wave surface equation that was used can produce wave types that corresponds to the Wilson criteria (1963), and compatible enough with the shoaling-breaking model that was used. However, further research is still needed because adjustment should still be done on wave contant G of the shoaling breaking model. Further research is still needed in shoaling-breaking model as well as water wave surface equation.

The calculation of wave force in a structure, should take into account the cnoidal-shaped wave profile, so that a better estimation on the location point of wave force is needed. Furthermore, the planning of coastal construction elevation should take into account the asymmetry wave profile so that a parameter as big as half the wave height cannot be used.

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