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Determination of Time Step and Grid Size Based on the Formulation Conditions of the Fundamental Hydrodynamic Equations for Time-Series Numerical Water Wave Modeling

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***Keywords*— conditions for formulating the basic equations of hydrodynamics.**

***Abstract*— *The formulation of the fundamental hydrodynamic equations is based on two key assumptions. The first arises from the derivation of the Euler momentum conservation equation, which assumes that the temporal and spatial increments $(\delta t, \delta x)$ are sufficiently small for the Taylor series expansion to be truncated after the first-order terms. The second assumption is associated with the continuity equation, in which the fluid-particle velocity is assumed to remain constant over the small time-interval δt. Based on these two assumptions, two optimization equations are formulated with two unknowns, time step and grid size $(\delta t, \delta x)$. Solving these equations provides the optimal time step and grid size for numerical simulations. The resulting values are consistent with the assumptions underlying the derivation of the fundamental hydrodynamic equations while also satisfying the requirement of sufficiently small discretization intervals imposed by numerical methods.The proposed approach enables more accurate numerical simulations of challenging wave conditions, particularly during wave breaking and post-breaking propagation.*

I. INTRODUCTION

Time series numerical modeling of water wave using discrete methods, including the Finite Difference Method (FDM) and the Finite Element Method (FEM), requires the selection of sufficiently small time-steps δt and grid sizes δx. However, relatively few studies have examined appropriate values for these two parameters. In general, it is assumed that smaller values produce more accurate results, subject to the accuracy limits of the numerical approximation. For example, quadratic interpolation permits the use of larger grid sizes than linear interpolation while maintaining comparable accuracy.

Courant, Friedrichs, and Lewy (1928) formulated the grid size as a function of the time step, although they did not provide a criterion for determining the time step itself.

Hutahaean (2024) derived the time step using a truncated Taylor series approximation for a sinusoidal function, $f = f(t)$. In this approach, the Taylor series is truncated after the first-order term. The grid size was subsequently derived from a truncated Taylor series for the sinusoidal function $f = f(x,t)$, using the previously determined time step. This formulation is conceptually similar to that of Courant, Friedrichs, and Lewy (1928), in which the time step is specified independently of the grid size. A similar approach was also adopted by Hutahaean (2025a).

In the present study, the time step and grid size are determined simultaneously using a system of two coupled equations, establishing a mutual dependence between these parameters. These equations are developed from the fundamental derivation of the governing hydrodynamic equations. The first equation is derived from the Euler momentum conservation equation, in which the acceleration term is formulated using a truncated Taylor series. The second equation is derived from the continuity equation under the assumption that the fluid particle velocity remains constant over a sufficiently small time-interval. Solving these two equations simultaneously yields coupled expressions for the time step and grid size, demonstrating that each parameter constrains the allowable value of the other.

II. THE FORMULATION OF $(\delta t, \delta x)$ EQUATIONS

a. Derivation from the Truncated Taylor Series

For a function $f(x,t)$, Arden, Bruce W. and Astill Kenneth N. (1970), where x is the horizontal coordinate and t is time, the Taylor series expansion,

$$f(x+\delta x, t+\delta t) = f(x,t) + s_1 + s_2 + s_3 + \cdots \quad (1)$$

Where:

$$s_1 = \delta t \frac{\partial f}{\partial t} + \delta x \frac{\partial f}{\partial x}$$

$$s_2 = \frac{\delta t^2}{2}\frac{\partial^2 f}{\partial t^2} + \delta t \delta x \frac{\partial^2 f}{\partial t \partial x} + \frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2}$$

$$s_3 = \frac{\delta t^3}{3!}\frac{\partial^3 f}{\partial t^3} + \frac{\delta t^2}{2}\delta x \frac{\partial^3 f}{\partial t^2 \partial x} + \delta t \frac{\delta x^2}{2}\frac{\partial^3 f}{\partial t \partial x^2} + \frac{\delta x^3}{3!}\frac{\partial^3 f}{\partial x^3}$$

When both δt and δx are very small, the Taylor series can be truncated after the first-order terms, yielding

$$f = f(x,t) + s_1$$

Within the condition of

$$\left|\frac{s_2 + s_3 + s_4 + \cdots}{s_1}\right| \leq \varepsilon \quad \ldots (2)$$

This expression is referred to as the optimization equation. Because δt and δx are very small, where $(s_3 + s_4 + \cdots) \ll s_2$, therefore the optimized equation is expressed as,

$$\left|\frac{s_2}{s_1}\right| \leq \varepsilon \quad \ldots (3)$$

For instance, using the following formula:

$$f(x,t) = \cos kx \cos \sigma t \quad \ldots (4)$$

$$s_1 = -\sigma \delta t \cos kx \sin \sigma t - k\, \delta x \sin kx \cos \sigma t$$

$$s_2 = -\sigma^2 \frac{\delta t^2}{2} \cos kx \cos \sigma t + \sigma k\, \delta t\, \delta x \sin kx \sin \sigma t - k^2 \frac{\delta x^2}{2} \cos kx \cos \sigma t$$

σ is angular frequency, $\sigma = \frac{2\pi}{T}$, T is wave period and k is wave number, where $k = \frac{2\pi}{L}$, L is wave number

Substituting s_1 and s_2 to (3),

$$\left| \frac{-\sigma^2 \frac{\delta t^2}{2} \cos kx \cos \sigma t + \sigma k\, \delta t \delta x \sin kx \sin \sigma t}{-\sigma \delta t \cos kx \sin \sigma t - k\, \delta x \sin kx \cos \sigma t} - \frac{k^2 \frac{\delta x^2}{2} \cos kx \cos \sigma t}{-\sigma \delta t \cos kx \sin \sigma t - k\, \delta x \sin kx \cos \sigma t} \right| \leq \varepsilon$$

The optimization coefficient ε is a small positive constant satisfying $0 < \varepsilon \ll 1.0$. The final expression is evaluated at the characteristic point, where $\cos kx = \sin kx$ and $\cos \sigma t = \sin \sigma t$

$$\left| \frac{-\sigma^2 \frac{\delta t^2}{2} + \sigma k\, \delta t\, \delta x - k^2 \frac{\delta x^2}{2}}{-\sigma \delta t - k\, \delta x} \right| \leq \varepsilon$$

Or

$$\left| \frac{\sigma^2 \frac{\delta t^2}{2} - \sigma k\, \delta t \delta x + k^2 \frac{\delta x^2}{2}}{\sigma \delta t + k\, \delta x} \right| \leq \varepsilon$$

Because $\varepsilon > 0$ and the denominator is postive, the numerator is required to be positive under the selected conditions. Therefore, the absolute value may be omitted, yielding:

$$\frac{\sigma^2 \frac{\delta t^2}{2} - \sigma k\, \delta t \delta x + k^2 \frac{\delta x^2}{2}}{\sigma \delta t + k\, \delta x} \leq \varepsilon$$

Since the denominator is positive, multiplying both sides by the denominator preserves the inequality. At the limiting condition, the inequality $(\leq)$is replaced by equality $(=)$, hence

$$\sigma^2 \frac{\delta t^2}{2} - \sigma k\, \delta t \delta x + k^2 \frac{\delta x^2}{2} = \varepsilon(\sigma \delta t + k\, \delta x) \quad \ldots (5)$$

Equation (5) contains two unknowns, δt and δx. Therefore, a second independent equation is required to determine both parameters.

Note: In this study, the *characteristic point* is defined as the intersection at which two functions attain the same value (Fig. 1). The parameter values determined at this point satisfy both functions simultaneously.

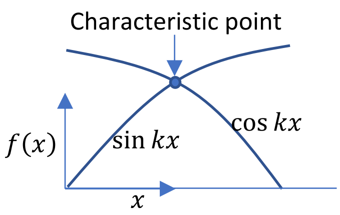

Fig (1). Characteristic point.

b. Starting from the continuity equation.

The continuity equation in the (x,z) plane, where x denotes the horizontal axis and z denotes the vertical axis (Dean, 1991), is expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad \ldots(6)$$

where $u(x,z,t)$ is the fluid particle velocity in the horizontal direction, and $w(x,z,t)$ is the fluid particle velocity in the vertical direction. This equation is derived under the assumption that, over a sufficiently small time-interval δt both u and w remain constant. Furthermore, it is assumed that u varies only along the horizontal direction for a constant elevation $z = constant$, such that $u = u(x,t)$, whereas w varies only along the vertical z direction for a constant horizontal position $x = constant$, such that $w = w(z,t)$.

Seen from the changes in value $f = f(x,t)$,

$$f(x+\delta x, t+\delta t) = f(x,t) + s_1 + s_2 + s_3$$

Transposing the first term on the right-hand side to the left-hand side and dividing both sides by δx, yields

$$\frac{f(x+\delta x, t+\delta t) - f(x,t)}{\delta x} = \frac{s_1+s_2+s_3}{\delta x}$$

With very small δx, δt close to zero, this equation can be written as spatial total derivative as follows.

$$\frac{Df}{dx} = \frac{s_1+s_2+s_3}{\delta x}$$

Where,

$$\frac{s_1}{\delta x} = \frac{\delta t}{\delta x}\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}$$

With very small δx, δt close to zero, therefore $\frac{\delta t}{\delta x}$ has a value that is not zero,

$$\frac{s_2}{\delta x} = \frac{\delta t}{2}\frac{\delta t}{\delta x}\frac{\partial^2 f}{\partial t^2} + \delta t\frac{\partial^2 f}{\partial t\partial x} + \frac{\delta x}{2}\frac{\partial^2 f}{\partial x^2}$$

As δx, δt become infinitesimally small and approach zero, the first and second terms on the right-hand side become negligible because they are proportional to δt, whereas the third term becomes negligible because it is proportional to δx. Consequently, $\frac{s_2}{\delta x}$ approaches zero and can be neglected.

$$\frac{s_3}{\delta x} = \frac{\delta t^2}{3!}\frac{\delta t}{\delta x}\frac{\partial^3 f}{\partial t^3} + \frac{\delta t^2}{2}\frac{\partial^3 f}{\partial t^2 \partial x} + \delta t\frac{\delta x}{2}\frac{\partial^3 f}{\partial t \partial x^2} + \frac{\delta x^2}{3!}\frac{\partial^3 f}{\partial x^3}$$

As δx, δt approach zero, the first and second terms on the right-hand side become negligible because they are proportional to δt^2, the third term becomes negligible because it is proportional to $\delta t \delta x$ and the fourth term becomes negligible because it is proportional to δx^2. Therefore, $\frac{s_3}{\delta x}$ also approaches zero and may be neglected. Likewise, all higher-order terms vanish in the limit. Hence, the total spatial derivative can be approximated as

$$\frac{Df}{dx} = \frac{s_1}{\delta x}$$

$$\frac{Df}{dx} = \frac{\delta t}{\delta x}\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}$$

Using similar method for $f = f(z,t)$ yields,

$$\frac{Df}{dz} = \frac{\delta t}{\delta z}\frac{\partial f}{\partial t} + \frac{\partial f}{\partial z}$$

When t variations are taken into account, Equation (6) should therefore be expressed as

$$\frac{Du}{dx} + \frac{Dw}{dz} = 0$$

Using the total spatial derivative equation,

$$\frac{\delta t}{\delta x}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\delta t}{\delta z}\frac{\partial w}{\partial t} + \frac{\partial w}{\partial z} = 0$$

The equation becomes,

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

with the following requirements to be satisfied,

$$\left|\frac{\frac{\delta t}{\delta x}\frac{\partial u}{\partial t}}{\frac{\partial u}{\partial x}}\right| \le \varepsilon$$

And

$$\left|\frac{\frac{\delta t}{\delta z}\frac{\partial w}{\partial t}}{\frac{\partial w}{\partial z}}\right| \le \varepsilon$$

Where ε is a sufficiently small positive number $0 < \varepsilon \ll 1.0$.

In general, for a function $f = f(x,t)$, $\frac{Df}{dx} \approx \frac{\partial f}{\partial x}$ the following is required

$$\left|\frac{\frac{\delta t}{\delta x}\frac{\partial f}{\partial t}}{\frac{\partial f}{\partial x}}\right| \le \varepsilon \qquad \ldots(7)$$

As an example, consider the function $f(x,t) = \cos kx \cos \sigma t$

$$\left|\frac{-\sigma\frac{\delta t}{\delta x}\cos kx \sin \sigma t}{-k \sin kx \cos \sigma t}\right| \le \varepsilon$$

At the characteristic point, this reduces to,

$$\left|\frac{\sigma\frac{\delta t}{\delta x}}{k}\right| \le \varepsilon$$

Since the expression inside the absolute value is positive, the absolute value signs may be omitted. Replacing the inequality with an equality yields,

$$\frac{\sigma\frac{\delta t}{\delta x}}{k} = \varepsilon$$

$$\delta t = \varepsilon\frac{k}{\sigma}\delta x \qquad \ldots(8)$$

Substituting (8) to (5),

$$\delta x = \frac{2\varepsilon(\varepsilon+1)}{(\varepsilon^2 - 2\varepsilon + 1)k} \qquad \ldots(9)$$

Once δx is determined, δt can be calculated using equation (8). Alternatively, the equation of δt can be formed by substituting δx to (8).

$$\delta t = \varepsilon \frac{k}{\sigma} \frac{2\varepsilon(\varepsilon+1)}{(\varepsilon^2 - 2\varepsilon + 1)k}$$

$$\delta t = \frac{2\varepsilon^2(\varepsilon+1)}{(\varepsilon^2 - \varepsilon + 1)\sigma} \qquad \dots(10)$$

For $f(z,t) = \cosh k(h+z) \cos \sigma t$, hence

$$\left| \frac{-\frac{\delta t}{\delta z} \sigma \cosh k(h+z) \sin \sigma t}{\text{k} \sinh k(h+z) \cos \sigma t} \right| \leq \varepsilon$$

At the characteristic point, where $\cosh k(h+z) = \sinh k(h+z)$ and $\cos \sigma t = \sin \sigma t$ the above expression reduces to,

$$\left| \frac{-\frac{\delta t}{\delta z} \sigma}{k} \right| \leq \varepsilon$$

Removing the absolute value

$$\frac{\frac{\delta t}{\delta z} \sigma}{k} = \varepsilon$$

yields

$$\delta t = \varepsilon \frac{k}{\sigma} \delta z$$

Since this expression has the same form as Equation (8), it concludes that $\delta z = \delta x$.

c.Example calculations for different values of ε.

Table 1 presents the calculated values of δt and δx for a wave with a wave height of $H = 2.4\ m$ a wave period of $T = 7.247$ s, and a wavelength of $L = 17.835\ m$.

Table (1) Results of δt and δx calculations for different ε values

ε	δt (sec)	δx (m)	$\frac{\delta t}{\delta x}$
0.002	0.00001	0.01251	0.00081
0.004	0.00004	0.02517	0.00163
0.006	0.00009	0.03798	0.00244
0.008	0.00017	0.05095	0.00325
0.01	0.00026	0.06407	0.00406
0.012	0.00038	0.07735	0.00488
0.014	0.00052	0.09079	0.00569
0.016	0.00068	0.10439	0.0065
0.018	0.00086	0.11815	0.00731
0.02	0.00107	0.13207	0.00813

As shown in Table 1, increasing ε results in larger values of δt and δx and the ratio $\frac{\delta t}{\delta x}$. For $\varepsilon = 0.02$ the ratio $\frac{\delta t}{\delta x} =$ 0.00813, which is sufficiently small. Therefore, adopting $\varepsilon = 0.02$, provides adequate justification for the assumption that the velocity components u and w remain constant over the time interval δt.

Table 2 presents the values of $\left|\frac{s_2}{s_1}\right|$ and $\left|\frac{s_3}{s_1}\right|$ for the function $f(x,t) = \cos \sigma t \cos kx$, where wave period $T = 7.247$ seconds, with wave length $L = 17.835\ m$ as the previous example. The calculations were performed on $\sigma t = kx = \frac{\pi}{4}$.

Table (2) Examining $\left|\frac{s_2}{s_1}\right|$ and $\left|\frac{s_3}{s_1}\right|$

ε	δt (sec)	$\left\lvert\frac{s_2}{s_1}\right\rvert$	$\left\lvert\frac{s_3}{s_1}\right\rvert$
0.002	0.00081	0.00775	3E-06
0.004	0.00163	0.01549	1.1E-05
0.006	0.00244	0.02323	2.6E-05
0.008	0.00325	0.03096	4.7E-05
0.01	0.00406	0.0387	7.5E-05
0.012	0.00488	0.04644	0.00011
0.014	0.00569	0.05418	0.00015
0.016	0.0065	0.06192	0.0002
0.018	0.00731	0.06966	0.00026
0.02	0.00813	0.07741	0.00033

As shown in Table 2, $\left|\frac{s_2}{s_1}\right| > \varepsilon$ dan $\left|\frac{s_3}{s_1}\right| \ll \varepsilon$. In the finite difference method (FDM), the central difference scheme is generally employed at interior grid points. Under this scheme, the even-order differential terms cancel naturally, while the odd-order higher-order terms are neglected. Consequently, the relatively large value of $\left|\frac{s_2}{s_1}\right|$ does not affect the numerical accuracy. In contrast, $\left|\frac{s_3}{s_1}\right|$ is sufficiently small, and the combined contribution of the remaining odd-order terms, $\left|\frac{s_3+s_5+s_7+\cdots}{s_1}\right|$ is also negligible. Therefore, these higher-order terms can be safely ignored.

At the computational domain boundaries, where forward and/or backward difference schemes are applied, the numerical accuracy can be improved by reducing the spatial size δx while keeping the time step δt fixed at the value obtained from the preceding analysis. This refinement should satisfy the constraint $\left|\frac{\delta t}{\delta x}\right| < \varepsilon$. For example, the spatial size may be reduced to $\delta x = \frac{\delta x}{2}$.

Table 3. Calculated values of δt and δx for selected combinations of (T, L), $\varepsilon = 0.02$

H (m)	T (sec)	L (m)	δt (sec)	δx (m)	$\frac{\delta t}{\delta x}$ Sec/m
0.4	3.24	3.26	0.0004	0.0220	0.0199
0.8	4.58	6.51	0.0006	0.0440	0.0141
1.2	5.61	9.77	0.0008	0.0660	0.0115
1.6	6.48	13.02	0.0009	0.0881	0.0100
2	7.25	16.28	0.0010	0.1101	0.0089
2.4	7.94	19.53	0.0011	0.1321	0.0081
2.8	8.57	22.79	0.0012	0.1541	0.0075
3.2	9.16	26.04	0.0012	0.1761	0.0070
3.6	9.72	29.30	0.0013	0.1981	0.0066
4	10.25	32.56	0.0014	0.2201	0.0063
4.4	10.75	35.81	0.0015	0.2421	0.0060
4.8	11.22	39.07	0.0015	0.2641	0.0058

This result indicates that, for long-wave simulations, a larger optimization coefficient ε may be adopted while still satisfying the approximation requirements.

Both Tables 1 and 3 show that the resulting time step and grid spacing are sufficiently small. Consequently, they satisfy not only the assumptions used in deriving the continuity and momentum conservation equations but also the stability and accuracy requirements of the numerical method.

III. EXAMPLE APPLICATION TO A TIME-SERIES WATER WAVE MODEL

In this section, the numerical water wave model developed by Hutahaean (2025b, 2026) is implemented using the time step and grid size determined in the present study, with a numerical accuracy of $\varepsilon = 0.02$. Spatial derivatives are discretized using the Finite Difference Method (FDM), while the temporal derivatives are solved using a predictor-corrector scheme. In the predictor step, the governing equations are solved using FDM with a central difference scheme. The corrector step employs the Newton-Cotes numerical integration method (Hutahaean, 2024).

The simulation considers a deep-water wave amplitude of $A_0 = 1.2\ m$, corresponding to a deep-water wave height of $H_0 = 2.40\ m$, with a wave period of $T = 7.94\ sec$. The model is applied to a bathymetry with a constant bottom slope of 0.04, extending from an initial deep-water depth of $h_0 = 9.0\ m$ to a final shallow-water depth of $h = 0.10\ m$. The simulation results are presented in Fig. 2. The model successfully propagates the wave to a water depth of 0.8 m, where the wave height reaches 1.60 m. These results demonstrate that the selected time step and grid size enable the numerical model to pass through the critical conditions associated with wave breaking and post-breaking.

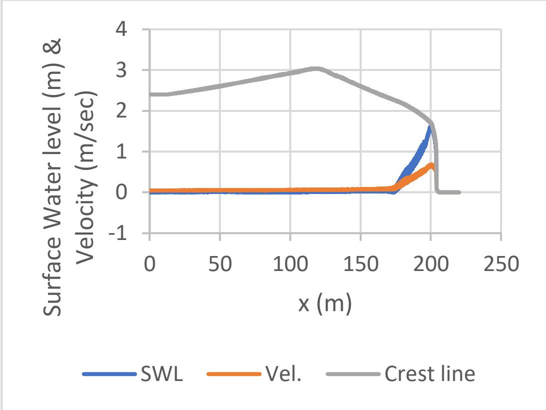

Fig (2) Results of shoaling breaking model.

The relatively large wave height near the end of the simulation is attributed to the relatively steep bottom slope (0.04). In addition, the energy dissipation in shallow water remains insufficient.

By increasing the energy dissipation in shallow water, the model is expected to remain stable at even shallower water depths. However, the objective of the present study is limited to evaluating the effects of time step and grid size on numerical performance.

IV. POTENTIAL EXTENSION OF THE CONTINUITY EQUATION

With the time step and grid size determined, the continuity equation can be further developed by incorporating fluid-particle acceleration, yielding

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \frac{\delta t}{\delta x}\frac{\partial u}{\partial t} + \frac{\delta t}{\delta z}\frac{\partial w}{\partial t} = 0$$

It is assumed that, regardless of how small the ratios $\frac{\delta t}{\delta x}$ and $\frac{\delta t}{\delta z}$ are, with $\delta x = \delta z$, the terms $\frac{\delta t}{\delta x}\frac{\partial u}{\partial t}$ and $\frac{\delta t}{\delta z}\frac{\partial w}{\partial t}$, remain finite. Consequently, the resulting continuity equation more closely represents the physical behavior of unsteady fluid flow.

V. CONCLUSION

Discrete numerical methods, including the Finite Difference Method (FDM) and the Finite Element Method (FEM), require sufficiently small grid sizes and time steps to achieve accurate numerical solutions. However, no general criteria are available for determining appropriate values of these parameters.

By following the assumptions and simplifications adopted in the derivation of the fundamental hydrodynamic equations, this study establishes a set of criteria and governing equations for determining the time step and grid size required for unsteady flow simulations.

The proposed approach provides time-step and grid-size values that are consistent with both the underlying hydrodynamic formulation and the requirements of discrete numerical methods. As a result, errors associated with the assumption of constant velocity over a finite time interval in the continuity equation, as well as truncation errors arising from the Taylor series approximation used in the momentum conservation equation, are reduced. The proposed criteria therefore improve the accuracy and numerical stability of unsteady flow simulations.

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