

# Water Wave Modeling using Wave Constant $G$

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**Abstract**— In this research, analytical method of water wave dynamics was developed using wave constant  $G$  at velocity potential of Laplace equation solution result, where at small amplitude theory the wave constant was eliminated with wave amplitude. The assumption of sinusoidal wave is maintained.

The calculation methods that were formulated include wave number calculation method and shoaling and breaking modeling. Using the wave constant, then equilibrium equation will be met more accurately.

**Keywords**— Wave constant  $G$ , sinusoidal water wave surface, wavelength, shoaling-breaking.

## I. INTRODUCTION

Small amplitude wave theory (Dean (1991)) was formulated with an assumption that wave amplitude is very small so that the elevation of water wave surface at the execution of Bernoulli equations at the surface is considered to coincide with the still water level elevation (zero elevation). Using this method, relation equation between wave constant  $G$  and wave amplitude  $A$  was formulated. Afterwards, wave constant  $G$  at velocity potential equation was eliminated with wave amplitude  $A$ . Bearing in mind that there is no wave amplitude at the velocity potential equation, then the accuracy of wave analysis by eliminating wave constant  $G$  requires accurate relation equation. Particle velocity equation which is a differential of velocity potential equation is a function of wave constant  $G$  where the particle velocity is used in various calculations and at equilibrium equation. Hence, eliminating wave constant  $G$  with wave amplitude requires an accurate relation.

In this research, relation between wave amplitude  $A$  and wave constant  $G$  was formulated without working on the assumption of small and long wave and without eliminating wave constant  $G$ . As a result, an equation where wave constant  $G$  together with wave amplitude  $A$  as its variables was formulated. The relation was formulated using KFSBC together with momentum equation.

To simplify the calculation, the equation was developed by maintaining an assumption that the wave is sinusoidal, where wave height  $H$  is twice wave amplitude or  $H = 2A$ . The existing understanding is that sinusoidal wave is only

for small amplitude wave. This research experienced no obstacle as a result of the working on sinusoidal wave assumption and quite accurate calculation result was obtained.

## II. VELOCITY POTENTIAL FOR SLOPING BOTTOM

In this research, velocity potential at sloping bottom from Hutahaean (2008) was used with initial form as follows

$$\Phi = Ge^{kh}\beta(z)\cos kx\sin\sigma t$$

$G$  is wave constant,  $k$  is wave number,  $\sigma$  is angular frequency,  $\sigma = \frac{2\pi}{T}$ ,  $T$  is wave period,  $z$  is elevation at

vertical axis with  $z = 0$  at still water level surface. In this research, a new constant  $G = Ge^{kh}$  was formed and bearing in mind the separation between  $G$  and  $e^{kh}$  did not provide any benefit at all; therefore, to make it more concise, incorporation was done, then the potential velocity equation becomes:

$$\Phi = G\beta(z)\cos kx\sin\sigma t \quad \dots\dots\dots(1)$$

$$\beta(z) = \alpha e^{k(h+z)} + e^{-k(h+z)} \quad \dots\dots\dots(2)$$

$$\beta_1(z) = \alpha e^{k(h+z)} - e^{-k(h+z)} \quad \dots\dots\dots(3)$$

$$\alpha = \frac{1}{2} \left( \frac{1 + \frac{\partial h}{\partial x}}{1 - \frac{\partial h}{\partial x}} + \frac{1 - \frac{\partial h}{\partial x}}{1 + \frac{\partial h}{\partial x}} \right) \quad \dots\dots\dots(4)$$

$\frac{\partial h}{\partial x}$  is bottom slope. At sloping bottom there will be  $\frac{\partial G}{\partial x}$  and  $\frac{\partial k}{\partial x}$ . Related to the changes in the value of the wave constant, there are two conservation laws in the velocity potential equation, i.e. wave number conservation and energy conservation.

a. Wave Number Conservation

At the execution of Laplace equation with variable separation method, as it is with  $\cosh k(h+z)$  at velocity potential for horizontal bottom, it has been determined that  $\beta(z)$  is just a function of  $z$ , hence:

$$\frac{\partial \beta(z)}{\partial x} = 0 \quad \dots\dots\dots(5)$$

$$\frac{\partial k(h+z)}{\partial x} = 0 \quad \dots\dots\dots(6)$$

At  $z = 0$ , applies

$$\frac{\partial kh}{\partial x} = 0 \quad \dots\dots\dots(7)$$

or

$$\frac{\partial k}{\partial x} = -\frac{k}{h} \frac{\partial h}{\partial x} \quad \dots\dots\dots(8)$$

For  $z = \eta$  where  $\eta$  is the elevation of water wave surface against still water level,

$$\frac{\partial k(h + \eta)}{\partial x} = 0$$

At the characteristic point where  $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t$ , hence  $\eta = \frac{A}{2}$ , where  $A$  is wave amplitude. This will be explained in other part. Wave number conservation equation becomes

$$\frac{\partial k(h + \frac{A}{2})}{\partial x} = 0 \quad \text{.....(9)}$$

or

$$\left(h + \frac{A}{2}\right) \frac{\partial k}{\partial x} + \frac{k}{2} \frac{\partial A}{\partial x} + k \frac{\partial h}{\partial x} = 0 \quad \text{.....(10)}$$

Equation (10) can be called continuity equation or mass conservation equation at waves experiencing changes in water depth, wave number  $k$  and wave amplitude  $A$ . This equation cannot stand by itself, it has to meet certain limitation conditions, in this case Kinematic Free Surface Boundary Condition (KFSBC).

#### b. Energy conservation

Other characteristic contained in (1) is energy conservation that will be formulated as follows.

The velocity of water particle in horizontal  $u$  direction at the direction of axis- $x$  is,

$$u = -\frac{\partial \Phi}{\partial x} = Gk\beta(z) \sin kx \sin \sigma t -$$

$$\frac{\partial G}{\partial x} \beta(z) \cos kx \sin \sigma t \quad \text{.....(11)}$$

Differential of this particle velocity equation in horizontal direction against axis- $x$ , where there are changes in  $G$  and  $k$  is

$$\begin{aligned} \frac{\partial u}{\partial x} = & Gk^2 \beta(z) \cos kx \sin \sigma t \\ & + \frac{\partial Gk}{\partial x} \beta(z) \sin kx \sin \sigma t \\ & + \frac{\partial G}{\partial x} k \beta(z) \sin kx \sin \sigma t - \frac{\partial^2 G}{\partial x^2} \beta(z) \cos kx \sin \sigma t \end{aligned}$$

Particle velocity in vertical direction is,

$$w = -\frac{\partial \Phi}{\partial z} = -Gk\beta_1(z) \cos kx \sin \sigma t \quad \text{.....(12)}$$

The differential of this equation against vertical- $z$  axis is

$$\frac{\partial w}{\partial z} = -Gk^2 \beta(z) \cos kx \sin \sigma t$$

Mass conservation law or continuity equation is  $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$ . Substitute equations  $\frac{\partial u}{\partial x}$  and  $\frac{\partial w}{\partial z}$ , and work on the characteristic point, i.e. a point where  $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t$ ,

$$G \frac{\partial k}{\partial x} + 2 \frac{\partial G}{\partial x} k - \frac{\partial^2 G}{\partial x^2} = 0 \quad \text{.....(13)}$$

The equation can be written as,

$$\frac{\partial^2 G}{\partial x^2} = G \frac{\partial k}{\partial x} + 2 \frac{\partial G}{\partial x} k \quad \text{.....(14)}$$

Either (13) or (14) can be called mass conservation equation, but considering that at  $G$  there is energy

dimension, then it can be called energy conservation equation. At either long wave or small amplitude an assumption of  $\frac{\partial^2 G}{\partial x^2} = 0$  can be done, so the following relation was obtained,

$$\frac{\partial G}{\partial x} = -\frac{\frac{\partial k}{\partial x}}{2k} G \quad \text{.....(15)}$$

or

$$\frac{\partial G}{\partial x} = -\mu Gk \quad \text{.....(16)}$$

$$\mu = \frac{\frac{\partial k}{\partial x}}{2k^2} \quad \text{.....(17)}$$

$\frac{\partial k}{\partial x}$  at (15), (16) and (17) can be approached with (8) i.e.

$\frac{\partial k}{\partial x} = -\frac{k}{h} \frac{\partial h}{\partial x}$ . Therefore, in this case energy conservation equation is connecting changes in  $G$  with changes in water depth  $h$  and wave number  $k$ . In this research energy conservation equation with higher degree of accuracy was used, i.e.  $\frac{\partial^3 G}{\partial x^3} = 0$ . Equation for  $\frac{\partial^3 G}{\partial x^3}$  was obtained by differentiating (14) against horizontal- $x$  axis and at the element  $\frac{\partial^2 G}{\partial x^2}$  substitute with (14), and  $\frac{\partial^2 k}{\partial x^2}$  is ignored since it is very small, a form of energy conservation like (16) was obtained, i.e.  $\frac{\partial G}{\partial x} = -\mu k G$  with different coefficient of change  $\mu$ .

$$\mu = \frac{2 \frac{\partial k}{\partial x}}{\left(3 \frac{\partial k}{\partial x} + 4k^2\right)} \quad \text{.....(18)}$$

With relation (16), hence particle velocity at the direction of horizontal- $x$  axis becomes

$$u = Gk(\sin kx + \mu \cos kx) \beta(z) \sin \sigma t \quad \text{.....(19)}$$

### III. THE METHOD OF WAVELENGTH CALCULATION

The first equation connecting  $G$  and  $k$  is KFSBC:

$$\gamma \frac{\partial \eta}{\partial t} = w_\eta - u_\eta \frac{\partial \eta}{\partial x} \quad \text{.....(20)}$$

$\gamma$  is weighting coefficient at weighted total acceleration equation with a value of 2.784-3.160 (Hutahaeen (2019a)) and 2.483 for  $H_{1/3}$  and 2.202 for  $H_{1/10}$  in Hutahaeen (2019b). This research used the value of  $\gamma = 2.483$ . Substitute (12) and (19) that was done at  $z = \eta$  to (20), where the right side is multiplied with  $\frac{\sigma}{\sigma}$ ,

$$\begin{aligned} \frac{\partial \eta}{\partial t} = & -\frac{Gk}{\gamma \sigma} \left( \beta_1(\eta) \cos kx \sin \sigma t \right. \\ & \left. + (\sin kx + \mu \cos kx) \beta(\eta) \sin \sigma t \frac{\partial \eta}{\partial x} \right) \end{aligned}$$

At the characteristic point, this equation can be written as

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma \sigma} \left( \beta_1(\eta) + (1 + \mu) \beta(\eta) \right) \sigma \cos kx \sin \sigma t$$

Wave amplitude  $A$  is defined as,

$$A = \frac{Gk}{\gamma \sigma} \left( \beta_1(\eta) + (1 + \mu) \beta(\eta) \right) \frac{\partial \eta}{\partial x} \quad \text{.....(21)}$$

$$\frac{\partial \eta}{\partial t} = -A \sigma \cos kx \sin \sigma t \quad \text{.....(22)}$$

$$\eta = A \cos kx \cos \sigma t \dots (23)$$

$$\frac{\partial \eta}{\partial x} = -kA \sin kx \cos \sigma t \dots (24)$$

By working on the assumption at (22), (23) and (24), there is an assumption that water wave surface equation is sinusoidal, where wave height  $H = 2A$ . From (23), at the characteristic point  $\eta = \frac{A}{2}$ , then (21) becomes

$$A = \frac{Gk}{\gamma\sigma} \left( \beta_1 \left( \frac{A}{2} \right) - (1 + \mu) \beta \left( \frac{A}{2} \right) \frac{kA}{2} \right) \dots (25)$$

This equation is the first equation for  $G$  and  $k$  calculation with wave amplitude  $A$  as input,

$$f_1(G, k) = -A + \frac{Gk}{\gamma\sigma} \left( \beta_1 \left( \frac{A}{2} \right) - (1 + \mu) \beta \left( \frac{A}{2} \right) \frac{kA}{2} \right)$$

...(26)

As the second equation is surface momentum equation where convective velocity is ignored.

$$\gamma \frac{\partial u_\eta}{\partial t} = -g \frac{\partial \eta}{\partial x} \dots (27)$$

From (19)

$$u_\eta = Gk (\sin kx + \mu \cos kx) \beta(\eta) \sin \sigma t$$

$$\gamma \frac{\partial u_\eta}{\partial t} = Gk\sigma (\sin kx + \mu \cos kx) \beta(\eta) \cos \sigma t$$

At the characteristic point the second equation was obtained, i.e.

$$f_2(G, k) = \gamma\sigma G(1 + \mu) \beta \left( \frac{A}{2} \right) - gA$$

.....(27)

(27) can be formed into equation for  $G$  and it is substituted to (26), so an equation is formed just for wave number  $k$ . In this research, the two equations are done simultaneously, so the values of  $G$  and  $k$  are obtained which meet KFSBC and momentum equation, with input  $\sigma$ ,  $h$  and wave amplitude  $A$ . The calculation was done using Newton-Raphson method, where this iteration method needs initial price of iteration. As the initial price, for wave number  $k$  equation from Hutahaeen (2019a) was used, i.e.

$$\gamma^2 \sigma^2 = gk \left( 1 - \left( \frac{kA}{2} \right) \right) \dots (28)$$

This equation is a quadratic equation of wave number  $k$  that can be completed using simple method to find the root of a quadratic equation. After the value of wave number was obtained, the initial estimation value of  $G$  can be calculated using (25) where wave amplitude as input,

$$G = \frac{\gamma\sigma A}{k \left( \beta_1 \left( \frac{A}{2} \right) - (1 + \mu) \beta \left( \frac{A}{2} \right) \frac{kA}{2} \right)} \dots (29)$$

### 3.1. The result of wavelength $L$ calculation

Wave number  $k$  can be changed into wavelength with a simple relation, i.e.  $L = \frac{2\pi}{k}$ . The result of wave length calculation for various wave periods in a number of water depths is shown on Fig.1, where the calculation was done at bottom slope  $\frac{\partial h}{\partial x} = -0.005$ , weighting coefficient  $\gamma = 2.483$ , and wave amplitude  $A = 0.6$  m.

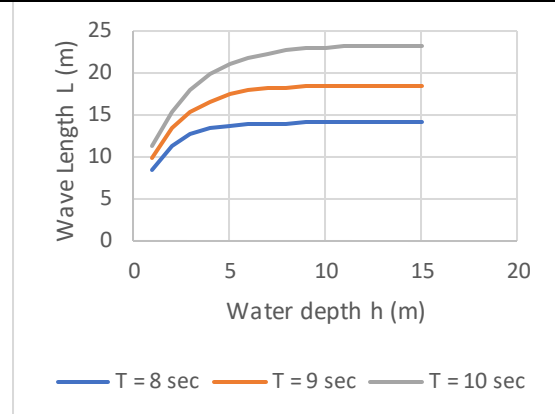


Fig.1: Graph of wave length against water depth

Fig.1 shows that wave with wave period of 8 sec, wavelength constant starts at water depth 9.0 m or deep water  $h_0 = 8$  m with  $L_0 = 14.0$  m, whereas at wave period 9 sec.,  $h_0 = 10.0$  m with  $L_0 = 18.4$  m, at wave period 10 sec.  $h_0 = 12$  m, with  $L_0 = 23.2$  m. The wave length is quite realistic, quite in line with what exists in the nature.

At the equations for calculating wave number, there are wave amplitude as the variables, hence the wave length that was produced will be determined by the size of the wave amplitude  $A$ . Fig.2 shows graph of wavelength against water depth using wave with wave period of 8 sec., wave amplitude  $A$ , 0.60 m, 0.80 m and 1.0 m. It is shown that the bigger the wave amplitude, the shorter the wave length. The calculation was done using  $\frac{\partial h}{\partial x} = -0.005$ , weighting coefficient  $\gamma = 2.483$ .

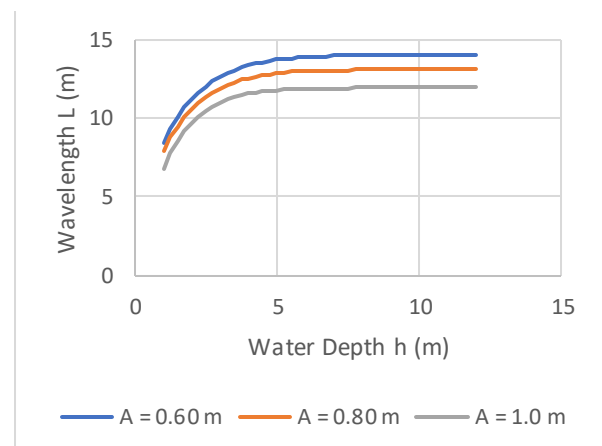


Fig.2: The influence of wave amplitude on wave length

With the presence of the influence of wave amplitude on wave length, the calculation of wave length change from a water depth to a shallower water depth should have been calculated along with shoaling analysis. There is also the influence of bottom slope on wave length which will be discussed on the next research due to space limitation.

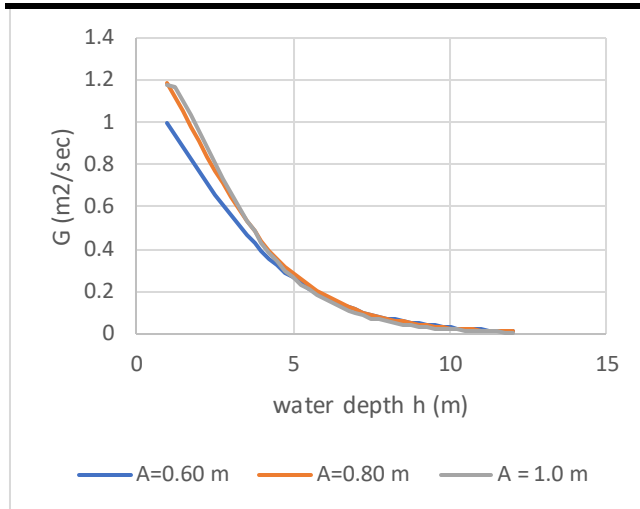


Fig.3: The value of wave constant  $G$

Fig.3 shows the value of wave constant  $G$  for wave with wave period of 8 sec., where the influences of wave amplitude and water depth are visible. At the deep water, the influence of wave amplitude is not that big where the value of  $G$  at the three wave amplitude values is almost the same. The difference is visible at shallow water, where the bigger the wave amplitude, the bigger the value of  $G$  but at big wave amplitude, i.e. 0.80 m and 1.0 m where the difference is very small. Considering that basically particle velocity is a function of wave constant  $G$ , then if wave constant  $G$  is eliminated with wave amplitude, then a very accurate relation between wave constant  $G$  and wave amplitude is needed in order to obtain an accurate particle velocity.

#### IV. SHOALING AND BREAKING MODEL

In the journey to shallower water, there are 3 (three) wave parameters with changing values, i.e. wave number  $k$ , wave constant  $G$  and wave amplitude  $A$ , where in this case wave amplitude  $A$  has been absorbed as variable of a wave. Therefore, at shoaling and breaking analysis there are 3 (three) unknowns, i.e.  $\frac{\partial k}{\partial x}$ ,  $\frac{\partial G}{\partial x}$  and  $\frac{\partial A}{\partial x}$  where the changes occurred as a result of changes in water depth  $h$ . There is an unchanged wave characteristic, i.e.

$$\beta(\eta_0) = \beta\left(\frac{A_0}{2}\right) = \alpha_0 e^{k_0(h_0 + \frac{A_0}{2})} + e^{-k_0(h_0 + \frac{A_0}{2})}$$

and

$$\beta_1(\eta_0) = \beta_1\left(\frac{A_0}{2}\right) = \alpha_0 e^{k_0(h_0 + \frac{A_0}{2})} - e^{-k_0(h_0 + \frac{A_0}{2})}$$

Index 0 shows the value at deep water. The conservation of the two characteristics is the consequence of wave number conservation law, where  $\frac{\partial \beta(\eta_0)}{\partial x} = 0$  and  $\frac{\partial \beta_1(\eta_0)}{\partial x} = 0$ .

As a governing equation of changing equation  $\frac{\partial k}{\partial x}$ ,  $\frac{\partial G}{\partial x}$  and  $\frac{\partial A}{\partial x}$  are wave number conservation equation, i.e. (10)

$$\left(h + \frac{A}{2}\right) \frac{\partial k}{\partial x} + \frac{k}{2} \frac{\partial A}{\partial x} + k \frac{\partial h}{\partial x} = 0 \quad \text{.....(10)}$$

The next equation is KFSBC equation in the form of an equation for wave amplitude  $A$  (25),

$$A = \frac{Gk}{\gamma\sigma} \left( \beta_1\left(\frac{A}{2}\right) - (1 + \mu)\beta\left(\frac{A}{2}\right) \frac{kA}{2} \right) \quad \text{.....(25)}$$

Defined,

$$f = \left( \beta_1\left(\frac{A_0}{2}\right) - (1 + \mu)\beta\left(\frac{A_0}{2}\right) \frac{kA}{2} \right) \quad \text{.....(30)}$$

$$A = \frac{Gk}{\gamma\sigma} f \quad \text{.....(31)}$$

Wave amplitude equation (31) was differentiated against horizontal- $x$  axis

$$\gamma\sigma \frac{\partial A}{\partial x} = \left( \frac{\partial G}{\partial x} k + G \frac{\partial k}{\partial x} \right) f \quad \text{.....(32)}$$

Then  $\frac{\partial A}{\partial x}$  is substituted to (10)

$$\left(h + \frac{A}{2}\right) \frac{\partial k}{\partial x} + k \frac{\partial h}{\partial x} + \frac{k}{2\gamma\sigma} \left( \frac{\partial G}{\partial x} k + G \frac{\partial k}{\partial x} \right) f = 0 \quad \text{.....(33)}$$

Hence (33) meet KFSBC. For (27) to meet energy conservation equation, then  $\frac{\partial G}{\partial x}$  is substituted to (16),  $\frac{\partial G}{\partial x} = -\mu k G$ , to obtain changing equation of wave number  $\frac{\partial k}{\partial x}$  that meets wave number conservation equations, KFSBC and energy conservation.

$$\left( \left(h + \frac{A}{2}\right) + \frac{Gkf}{2\gamma\sigma} \right) \frac{\partial k}{\partial x} = -k \frac{\partial h}{\partial x} + \frac{\mu G k^3 f}{2\gamma\sigma} \quad \text{.....(34)}$$

At (34), there is variable  $\mu$  which is a function of  $\frac{\partial k}{\partial x}$ .

Therefore, the calculation of  $\frac{\partial k}{\partial x}$  with (34) was done using iteration, i.e. the first step,  $\frac{\partial k}{\partial x}$  was calculated with (8),  $\frac{\partial k}{\partial x} = -\frac{k}{h} \frac{\partial h}{\partial x}$ , the value of  $\mu$  was calculated with (18). Then,  $\frac{\partial k}{\partial x}$  was calculated with (34) and  $\mu$  was recalculated with (18). This step is repeated over and over again until a stable value of  $\frac{\partial k}{\partial x}$  where generally 5-6 iterations have obtained stable value of  $\frac{\partial k}{\partial x}$ . After a stable value of  $\frac{\partial k}{\partial x}$  was obtained,  $\frac{\partial G}{\partial x}$  was calculated with (16) and  $\frac{\partial A}{\partial x}$  with (10). Then, the value of variable at the point  $x = x + \delta x$  was calculated at the depth of  $h_{x+\delta x} = h_x + \delta x \frac{\partial h}{\partial x}$  where  $\frac{\partial h}{\partial x}$  is negative, using Taylor series.

$$k_{x+\delta x} = k_x + \delta x \frac{\partial k}{\partial x}$$

$$G_{x+\delta x} = G_x + \delta x \frac{\partial G}{\partial x}$$

$$A_{x+\delta x} = A_x + \delta x \frac{\partial A}{\partial x}$$

The calculation was done from the deep water until coastal water, until breaker point and afterward

#### 4.1. Breaking Characteristics

At the equation for the wave amplitude, i.e. (30) and (31), there is breaking condition when  $f = 0$ ,

$$\left(\beta_1 \left(\frac{A_0}{2}\right) - (1 + \mu)\beta \left(\frac{A_0}{2}\right) \frac{kA}{2}\right) = 0 \dots (35)$$

$\beta_1 \left(\frac{A_0}{2}\right)$  and  $\beta \left(\frac{A_0}{2}\right)$  are values at deep water, where  $\beta_1 \left(\frac{A_0}{2}\right) = \beta \left(\frac{A_0}{2}\right)$ , so that if (35) is divided by  $\beta \left(\frac{A_0}{2}\right)$  the result is

$$\left(1 - (1 + \mu) \frac{kA}{2}\right) = 0$$

$$\frac{k_b A_b}{2} = \frac{1}{(1 + \mu)} \dots (36)$$

In (36), there is the influence of bottom slope on the breaking parameter that was absorbed at the value of  $\mu$ . The formulation process has stated that wave is sinusoidal, i.e. in (22). (23) and (24), then relation  $A_b = \frac{H_b}{2}$  applies, therefore breaker length index is obtained.

$$\frac{H_b}{L_b} = \frac{2}{(1 + \mu)\pi} \dots (37)$$

In (37), there is the influence of bottom slope on breaker height, whereas the previous section has shown that there is the influence of bottom slope on wavelength. So, in general, bottom slope will have an influence on the breaking wave. However, more detail discussion will be done in the next research.

#### 4.2. The result of Shoaling-Breaking Model

As an example, a model was executed for wave with wave period  $T = 8$  sec., deep water wave height is  $H_0 = 2.32$  m., bottom slope  $\frac{dh}{dx} = -0.005$ , weighting coefficient  $\gamma = 2.483$ . As seen in Fig.4, breaking occurs at breaker depth  $h_b = 3.66$  m, with breaker height  $H_b = 2.95$  m.

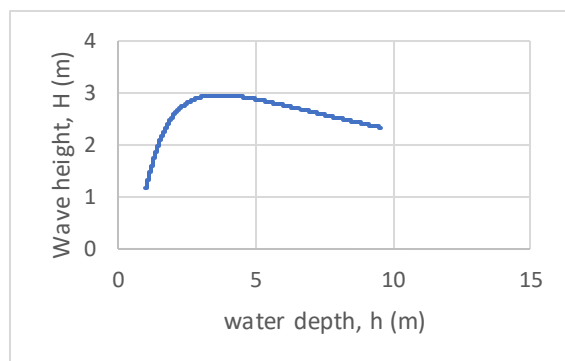


Fig. 4: Shoaling-breaking, wave period  $T = 8$  sec.,  $H_0 = 2.30$  m.

The comparison with breaker height index equations is presented on Table (1). As comparator, the average values of 5 empirical breaker index equations that were obtained from laboratories experiments were used, i.e. Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992) and Rattana

Pitikonand Shibayama (2000). Whereas comparator for breaker depth, equation from SPM (1984) was used.

Table.1: Comparison of the result of the model with empirical equation

| T (sec) | $H_0$ (m) | $H_b$ (m) |       | $h_b$ (m) |       |
|---------|-----------|-----------|-------|-----------|-------|
|         |           | Model     | BHI   | Model     | SPM   |
| 8       | 2,32      | 2,95      | 2,71  | 3,66      | 3,38  |
| 9       | 2,94      | 3,73      | 3,43  | 4,63      | 4,28  |
| 10      | 3,63      | 4,6       | 4,23  | 5,71      | 5,28  |
| 11      | 4,39      | 5,57      | 5,12  | 6,91      | 6,39  |
| 12      | 5,22      | 6,63      | 6,09  | 8,22      | 7,61  |
| 13      | 6,13      | 7,78      | 7,15  | 9,65      | 8,93  |
| 14      | 7,11      | 9,02      | 8,29  | 11,19     | 10,36 |
| 15      | 8,16      | 10,36     | 9,52  | 12,85     | 11,89 |
| 16      | 9,29      | 11,79     | 10,83 | 14,62     | 13,53 |

Note: BHI: average from 5 (five) Breaker Height Index equation

The comparison between the result of the model with empirical breaker height equation is shown on Table (1). As deep water wave height  $H_0$ ,  $0.9 \times H_{0-max}$  was used in every wave period, where  $H_{0-max} = \frac{g}{\gamma^2 \sigma^2}$  m (Hutahaean (2019b)), whereas deep water depth  $h_0 = \frac{1.8\pi}{k_0}$ , bottom slope  $\frac{dh}{dx} = -0.005$ , weighting coefficient  $\gamma = 2.483$ . Breaker height  $H_b$  from the model is bigger than breaker height from BHI, with an increasing pattern of differences with the increase in wave period. Similar differences pattern occur at breaker depth  $h_b$ , but with not so big differences. For more clear information see Fig. 5 for breaker height comparison and Fig. 6. for breaker depth comparison.

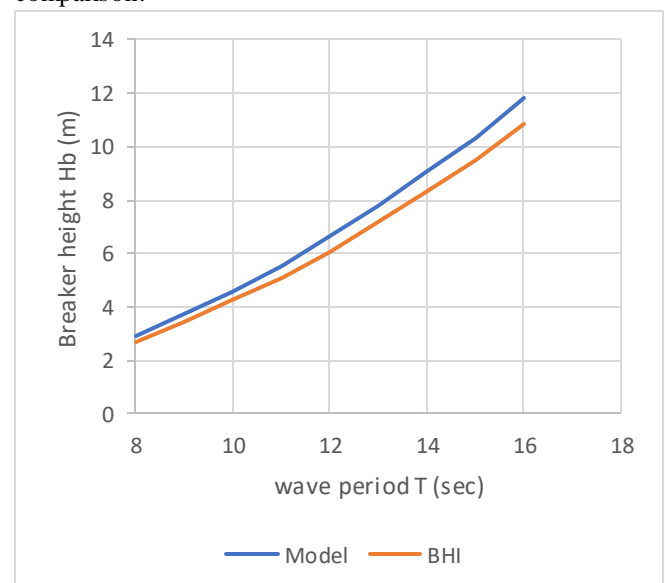


Fig.5: Comparison of breaker height



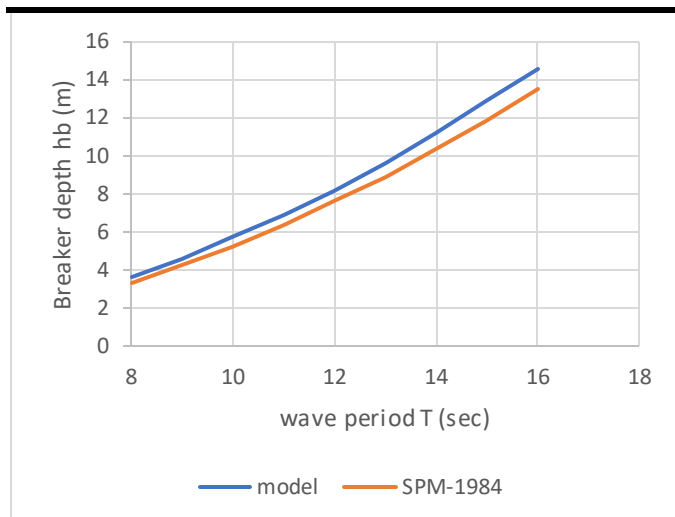


Fig.6: Comparison of breaker depth

Table.2: Comparison of breaker height index  $\frac{H_b}{h_b}$ 

| T<br>(sec) | $\frac{H_b}{h_b}$ |     | Model        |                   |
|------------|-------------------|-----|--------------|-------------------|
|            | Model             | BHI | $L_b$<br>(m) | $\frac{H_b}{L_b}$ |
| 8          | 0,81              | 0,8 | 4,6          | 0,64              |
| 9          | 0,81              | 0,8 | 5,82         | 0,64              |
| 10         | 0,81              | 0,8 | 7,19         | 0,64              |
| 11         | 0,81              | 0,8 | 8,7          | 0,64              |
| 12         | 0,81              | 0,8 | 10,35        | 0,64              |
| 13         | 0,81              | 0,8 | 12,15        | 0,64              |
| 14         | 0,81              | 0,8 | 14,09        | 0,64              |
| 15         | 0,81              | 0,8 | 16,17        | 0,64              |
| 16         | 0,81              | 0,8 | 18,4         | 0,64              |

$\frac{H_b}{h_b}$  comparison is shown in Table (2), which shows that the result of the model is quite close with the result of empirical equation. Breaker steepness cannot be compared since there were wave-length differences, where breaker length model  $L_b$  is quite short, quite in accordance with what exist in the nature. Breaker steepness  $\frac{H_b}{L_b}$  is quite in accordance with analytical equation (37), i.e.  $\frac{H_b}{L_b} \approx \frac{2}{\pi}$ .

## V. CONCLUSION

Wave dynamic calculation using wave constant  $G$  can be done easily and provide a quite good calculation result. The wave constant execution enables energy conservation equation execution which is a relation between wave constant changes with wave number changes.

The execution of sinusoidal wave assumption, simplify the correlation of calculation result in the form of wave amplitude with wave height, where generally the

information needed is wave height. In addition, the execution of sinusoidal wave assumption in this research found no difficulty in the use of even big wave amplitude.

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