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# Study on Wave Type at Water Wave Surface Equation Obtained from Kinematic Free Surface Boundary Condition (KFSBC)

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***Abstract*—*In this research, water wave surface equation was formulated by integrating kinematic free surface boundary condition against time. Then, a study was conducted on the type of wave produced by the water wave surface equation.*
***Keywords*— Integration of Kinematic Free Surface Boundary Condition, wave profile.**

## I. INTRODUCTION

Water wave surface equation of Airy water wave theory was formulated by working on small amplitude wave assumption (Dean (1991)) that produces sinusoidal wave which can be defined that wave height is two times the amplitude. Water wave surface formulation was done by working on Bernoulli equation on water wave surface with small amplitude assumption, so the water wave surface coincides with still water level. Then, KFSBC was done to formulate dispersion equation. Therefore, Bernoulli equation along with KFSBC can produce sinusoidal wave type.

In this research, water wave surface equation was obtained by integrating KFSBC against time without working on small amplitude wave assumption. In that water wave surface equation there are two wave constants that must be known their values, i.e. wave constant $G$and wave number $k$ that are found in the velocity potential solution of Laplace equation. The equation to calculate those two variables was formulated using KFSBC and surface momentum equation.

The characteristic of the produced water wave surface was studied using Wilson criteria (1963). Based on the criteria there are 4 (four) types of wave beginning with wave with the smallest amplitude, i.e. Airy's waves, Stoke's waves, Cnoidal waves and Solitary waves. In the deep water, the water wave surface equation that was obtained, produces 3 (three) types of the first wave, depending on the wave amplitude as the input.

## II. THE FORMULATION OF WATER WAVE SURFACE EQUATION

Water wave surface equation will be formulated using velocity potential equation and KFSBC. Velocity potential equation of Dean (1991) is,

$\Phi(x,z,t) = G\cos kx \cosh k(h+z) \sin\sigma t$ ......(1)

$x$is horizontal axis, $z$is vertical axis where $z=0$at the surface of still water level, $t$ time, $G$ wave constant, $k$ wave number, $\sigma = \frac{2\pi}{T}$, angular frequency, $T$ wave period and $h$ still water depth. The equation was obtained by completing Laplace equation with variable separation method, where $\cosh k(h+z)$is just a $z$function, so that $\frac{\partial \cosh k(h+z)}{\partial t} = \sinh k(h+z)\frac{\partial k(h+z)}{\partial t} = 0$, thereby

$\frac{\partial k(h+z)}{\partial t} = 0$ ........(2)

For all $z$ value. For$z=\eta$, where$\eta=\eta(x,t)$is the elevation of water surface with respect to still water level,

$\frac{\partial k(h+\eta)}{\partial t} = 0$ ........(3)

Equations (2) and (3) are called wave number conservation equation against time-$t$.

With the velocity potential, the equations of particle velocity horizontal-$x$direction and vertical-$z$ direction can be obtained, sequentially as follows

$u = -\frac{\partial\Phi}{\partial x} = Gk\sin kx\cosh k(h+z)\sin\sigma t$ ....(4)

$w = -\frac{\partial\Phi}{\partial z} = -Gk\cos kx\sinh k(h+z)\sin\sigma t$ ...(5)

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Particle velocity at water wave surface was obtained by substituting $z$ with $\eta = \eta(x,t)$, where $\eta$ is the elevation of water wave surface against still water level.

$u_\eta = Gksinkxcoshk(h+\eta)sin\sigma t$ ....(6)

$w_\eta = -Gkcoskxsinhk(h+\eta)sin\sigma t$ .....(7)

Water wave surface equation was formulated by integrating KFSBC, $\gamma \frac{\partial \eta}{\partial t} = w_\eta - u_\eta \frac{\partial \eta}{\partial x}$ against time-$t$, . Substitute (6) and (7),

$$\gamma \frac{\partial \eta}{\partial t} = -Gksinhk(h+\eta)coskxsin\sigma t$$

$-Gkcoshk(h+\eta)sinkxsin\sigma t \frac{\partial \eta}{\partial x}$ ......(8)

$\gamma$ is weighting coefficient at weighted total acceleration equation with a value around 2.202-3.0. This research uses $\gamma = 2.483$ (Hutahaean (2019a-b)).

To make the writing easier, the following equations were defined

$\beta_1(\eta) = sinhk(h+\eta)$........(9)

$\beta(\eta) = coshk(h+\eta)$ ......(10)

Substitute (9) and (10) to (8) and by multiplying the right side with $\frac{\sigma}{\sigma}$, then (8) becomes,

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\sigma coskxsin\sigma t$$

$-\frac{Gk}{\gamma\sigma}\beta(\eta)\sigma sinkxsin\sigma t \frac{\partial \eta}{\partial x}$.......(11)

(11) was integrated against time-$t$ by bearing in mind (3), i.e. at $z = \eta$, $\frac{\partial k(h+\eta)}{\partial t} = 0$ applies, which means that $\beta_1(\eta)$ and $\beta(\eta)$ have constant values against time $t$, thus the integration of the first term right side of the equation was completed by integrating $sin\sigma t$,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)\, coskxcos\sigma t$$

$$-\frac{Gk}{\gamma\sigma}\beta(\eta)\,\sigma sinkx \int sin\sigma t \frac{\partial \eta}{\partial x} dt$$

The integration of the second term right side of the equation will be completed using partial integration method, as follows

Assume a function of $f = cos\sigma t \frac{\partial \eta}{\partial x}$. This $cos\sigma t$ function was used so when it was differentiated against time $t$, $sin\sigma t \frac{\partial \eta}{\partial x}$ will be formed, i.e.

$$\frac{\partial f}{\partial t} = -\sigma sin\sigma t \frac{\partial \eta}{\partial x} + cos\sigma t \frac{\partial^2 \eta}{\partial t \partial x}$$

This differential equation was multiplied with $dt$ and integrated against time $t$,

$$f = -\sigma \int sin\sigma t \frac{\partial \eta}{\partial x} dt + \int cos\sigma t \frac{\partial^2 \eta}{\partial t \partial x} dt$$

Substitute $f$ and the first term right side was moved to the left and $f$ was moved to the right and both equations were divided by $\sigma$,

$$\int sin\sigma t \frac{\partial \eta}{\partial x} dt = -\frac{1}{\sigma} cos\sigma t \frac{\partial \eta}{\partial x} + \frac{1}{\sigma}\int cos\sigma t \frac{\partial^2 \eta}{\partial t \partial x} dt$$

The integration of the second term right side of the equation can be completed the same way, but with an assumption that $\frac{\partial^3 \eta}{\partial t^2 \partial x}$ is a very small number, the integration can be completed by integrating just the $cos\sigma t$ element.

$$\int sin\sigma t \frac{\partial \eta}{\partial x} dt = -\frac{1}{\sigma} cos\sigma t \frac{\partial \eta}{\partial x} + \frac{1}{\sigma^2} sin\sigma t \frac{\partial^2 \eta}{\partial t \partial x}$$

Substitute the result of integration,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)coskxcos\sigma t - \frac{Gk}{\gamma\sigma}\beta(\eta)sinkxsin\sigma t$$

$$\left(-cos\sigma t \frac{\partial \eta}{\partial x} + \frac{1}{\sigma} sin\sigma t \frac{\partial^2 \eta}{\partial t \partial x}\right)$$

Working on an assumption that $\frac{1}{\sigma} sin\sigma t \frac{\partial^2 \eta}{\partial t \partial x}$ is a very small number and can be ignored,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)\, coskxcos\sigma t$$

$$+\frac{Gk}{\gamma\sigma}\beta(\eta) sinkxsin\sigma t cos\sigma t \frac{\partial \eta}{\partial x}$$

The equation was differentiated against horizontal-$x$ axis.

$$\frac{\partial \eta}{\partial x} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta) ksinkxcos\sigma t$$

$$+\frac{Gk}{\gamma\sigma}\beta(\eta)\, kcoskxsin\sigma t cos\sigma t \frac{\partial \eta}{\partial x}$$

$$+\frac{Gk}{\gamma\sigma}\beta(\eta)\, sinkxsin\sigma t cos\sigma t \frac{\partial^2 \eta}{\partial x^2}$$

or

$\frac{\partial \eta}{\partial x} = \left(-\frac{Gk}{\gamma\sigma}\beta_1(\eta) ksinkx + \frac{Gk}{\gamma\sigma}\beta(\eta) kcoskxsin\sigma t \frac{\partial \eta}{\partial x} + \frac{Gk}{\gamma\sigma}\beta(\eta) sinkxsin\sigma t \frac{\partial^2 \eta}{\partial x^2}\right) cos\sigma t$......(12)

In accordance with the provision at the velocity potential equation where there is $t$ function only, $x$ function only and $z$ function only, then at the water wave surface using variable from velocity potential equation, water surface equation also has variable that is $t$ function only an $x$ function only, so it can be stated that the general form of (12) is,

$$\frac{\partial \eta}{\partial x} = f(x)cos\sigma t$$

In this equation $f(x)$ is just a function of $x$. $f(x)$ in (12) is,

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$$f(x) = \left(-\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx + \frac{G\,k}{\gamma\sigma}\beta(\eta)\,kcoskxcos\sigma tf(x) + \frac{Gk}{\gamma\sigma}\beta(\eta)sinkxcos\sigma t\frac{\partial f}{\partial x}\right)$$

Bearing in mind (3) $\beta_1(\eta)$and$\beta(\eta)$should be constant numbers against time $t$and against horizontal $x$ axis. In the second and third terms right side of the equation there is a function of time $t$, i.e.$cos\sigma t$ , then the term shouldn't be there. Therefore$f(x)$is,

$$f(x) = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)ksinkx$$

Thereby

$$\frac{\partial\eta}{\partial x} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\,ksinkxcos\sigma t$$

Substitute this equation to (11),

$$\frac{\partial\eta}{\partial t} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta)\sigma coskxsin\sigma t + \left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta)\beta_1(\eta)\sigma ksin^2kxsin\sigma tcos\sigma t$$

is integrated against time $t$. Integration will be completed by bearing in mind wave number conservation equation (3), as in the previous section, thus integration was done by integrating only sinusoidal element and bearing in mind that $sin\sigma tcos\sigma t = \frac{1}{2}sin2\sigma t$ ,

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta)\,coskxcos\sigma t - \frac{1}{4}\left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta)\beta_1(\eta)\,ksin^2kxcos2\sigma t$$

..(13)

To determine the values of $\beta_1(\eta)$, $\beta(\eta)$, an approach was done that water surface equation is sinusoidal, i.e.

$\eta_0(x,t) = Acoskxcos\sigma t$ .......(14)

Therefore, the approach of hyperbolic function is $\beta_1(\eta) = \beta_1(\eta_0)$, $\beta(\eta) = \beta(\eta_0)$.

$$\eta(x,t) = \frac{Gk}{\gamma\sigma}\beta_1(\eta_0)coskxcos\sigma t - \frac{1}{4}\left(\frac{Gk}{\gamma\sigma}\right)^2\beta(\eta_0)\beta_1(\eta_0)ksin^2kxcos2\sigma t$$

..(15)

## III. THE CALCULATION OF $G$AND$k$

To calculate $G$and$k$, two equations are needed, i.e. KFSBC and surface momentum equation.

By working on (14) on (11),

$$\frac{\partial\eta_0}{\partial t} = -\frac{Gk}{\gamma\sigma}\beta_1(\eta_0)\sigma coskxsin\sigma t - \frac{Gk}{\gamma\sigma}\beta(\eta_0)\sigma sinkxsin\sigma t\frac{\partial\eta_0}{\partial x}$$

At the characteristic point, i.e. at $coskx = sinkx = cos\sigma t = sin\sigma t$, the equation becomes,

$A = \frac{Gk}{\gamma\sigma}\beta_1\left(\frac{A}{2}\right) - \frac{Gk}{\gamma\sigma}\beta\left(\frac{A}{2}\right)\frac{kA}{2}$ ......(16)

Surface momentum equation where convective acceleration term is ignored is,

$\gamma\frac{\partial u_\eta}{\partial t} = -g\frac{\partial\eta_0}{\partial x}$ ........(17)

At the characteristic point, where $\frac{\partial u_\eta}{\partial t}$was obtained from (6) equation $\gamma\sigma G\beta\left(\frac{A}{2}\right) = gA$was obtained

or,

$G = \frac{gA}{\gamma\sigma\beta\left(\frac{A}{2}\right)}$ ......(18)

Substitute (17) to (15),

$\gamma^2\sigma^2 = gktanhk\left(h + \frac{A}{2}\right) - \frac{gk^2A}{2}$ ...........(19)

was obtained. In the deep water, $tanhk\left(h + \frac{A}{2}\right) = 1$, then dispersion equation in the deep water is,

$\gamma^2\sigma^2 = gk - \frac{gk^2A}{2}$.......... (20)

This equation is quadratic equation of wave number $k$where$k$can be calculated using simple method, i.e. finding the square root. Therefore, the calculation of $G$ and $k$was done by calculating $k$with (20) and then $G$was calculated with (18).

As an illustration of the result of the calculation, wave lenght at$L$was calculated where$L = \frac{2\pi}{k}$ , for wave with wave period of 8 sec., wave amplitude 0.6 m, 0.8 m and 1.0 m. The result of calculation, shown on Fig. 1shows that there is an impact of wave amplitude on wavelength, i.e. the bigger the wave amplitude, the shorter the wavelength.

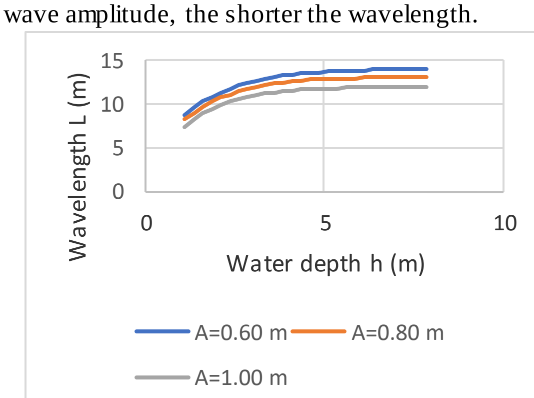


*Fig.1. The impact of wave amplitude A on wavelength* $L = \frac{2\pi}{k}$

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In (20), there is a maximum value of wave amplitude, i.e. when determinant value $D = g^2 - 4\left(\frac{gA}{2}\right)(\gamma^2\sigma^2) = 0$. In this condition wave amplitude $A = A_{max}$ , with the value,

$A_{max} = \frac{g}{2\gamma^2\sigma^2}$ ............(21)

At wave amplitude maximum where determinant $= 0$ , wave number becomes

$k_0 = \frac{1}{A_{max}}$ ............(22)

In the deep water $\tanh hk_0\left(h_0 + \frac{A_0}{2}\right) = \tanh k_0 h_0\left(1 + \frac{A_0}{2h_0}\right) \approx \tanh k_0 h_0 = 1$ . Consider $\frac{A_0}{2h_0} \ll 1$.

where $\tanh(1.65\pi) = 0.999937$ . Then deep water depth $h_0$

$h_0 = \frac{1.65\pi}{k_0} = 1.65\,\pi A_{max}$ ...........(23)

The use of coefficient 1.65 in (23) is the result of breaker depth calibration in shoaling-breaking model against breaker-depth from SPM(1984). The model is not discussed here.

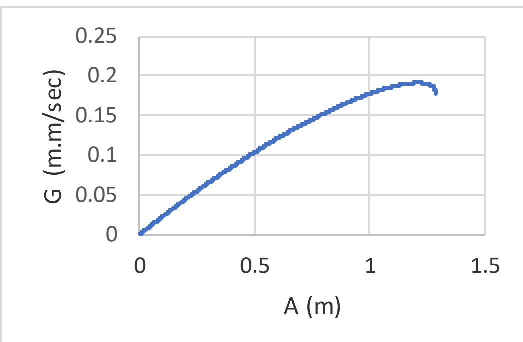


*Fig.2. Graph of the value of G as a function of wave amplitude A*

Fig.2 shows graph of value $G$ as the function of wave amplitude, for wave with wave period of 8 sec in the deep water. It shows that the value of $G$ grows bigger as the wave amplitude grows bigger, but there is a wave amplitude value where $G$ reaches maximum value, i.e. the value of $G$ decreases as wave amplitude grows bigger, until it has negative value at wave amplitude equals to $A_{max}$. There is a similar phenomenon for other wave period, where $G_{max}$ was achieved at wave amplitude of 0.91 $A_{max}$. Therefore, for a wave period, the wave amplitude value that should be used is

$A_{max} = \frac{0.91\,g}{2\gamma^2\sigma^2}$ ............(24)

In (22) and (23) $A_{max}$ was used in (24). Bearing in mind (3), the calculation of wave number $k$ for a wave moving from a certain depth to another depth cannot be calculated with (19), it should be with wave number conservation law. The result shown on Fig.1. should be read as a wave with wave constant values $G$ and determined wave amplitude. Therefore, Fig.1. is not the value of wave amplitude $k$ moving from deep water to shallow water.

## IV. TYPE OF WAVE PRODUCED BY WATER WAVE SURFACE EQUATION

### 4.1. General Shape

The general shape of water wave surface produced by(15) is shown on fig.3. The shape of water wave surface is not symmetrical between wave crest and wave trough where $\eta_{max} > |\eta_{min}|$, $\eta_{max}$ is the elevation of wave crest above still water level (line of elevation $z = 0$) , whereas $\eta_{min}$ is the elevation of wave trough below line $z = 0$. Wave profile like this is called cnoidal wave profile (Wilson(1963)). With wave height $H = \eta_{max} - \eta_{min}$.

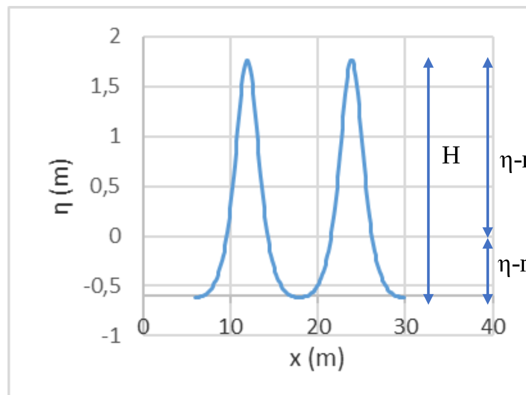


*Fig.3. General shape of water wave surface.*

### 4.2. Wilson Criteria (1963)

Wilson (1963) classified wave type based on the value of $\frac{\eta_{max}}{H}$ . Airy waves or sinusoidal waves types have a value of $\frac{\eta_{max}}{H} < 0.505$. This shows that the shape of wave is symmetrical, where $\eta_{max} \approx |\eta_{min}|$. Type Stoke's wave type is still quite close with the sinusoidal character. Furthermore, cnoidal waves type is not symmetrical at all where there is a form of wave with the wave trough that almost coincides with the line $z = 0$ i.e., where $\frac{\eta_{max}}{H} \approx 1.0$. This type of wave is called solitary waves type.

*Table.1: Wilson criteria (1963)*

| Wave Type | $\frac{\eta_{max}}{H}$ |
|---|---|
| Airy waves | $< 0.505$ |
| Stoke's waves | $< 635$ |

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| | |
|---|---|
| Cnoidal waves | $0.635 < \frac{\eta_{max}}{H} < 1$ |
| Solitary waves | $= 1$ |

4.3. Sinusoidal Type (Airy Waves Type)

Table (2) shows the result of the calculation with a very small wave amplitude where the value of $\frac{\eta_{max}}{H} = 0.503$ was achieved, which means that the wave profile is not really symmetrical, wave crest is still bigger than wave trough, but it is very close with the symmetry, where the value of $\frac{H}{A} = 2$ was achieved, or $H = 2A$. Therefore, it can be determined that as a criteria Airy wave type or sinusoidal wave type is $H = 2A$. The example of sinusoidal wave profile is on Fig.4., wherewave crest with wave trough is symmetrical, i.e. $\eta_{max} = |\eta_{min}|$.

*Table.2: Wave amplitude for sinusoidal wave, at deep water.*

| $T$ (sec.) | $A$ (m) | $\frac{\eta_{max}}{H}$ | $H$ (m) | $\frac{H}{A}$ |
|---|---|---|---|---|
| 8 | 0,017 | 0,503 | 0,035 | 2 |
| 9 | 0,02 | 0,503 | 0,041 | 2 |
| 10 | 0,023 | 0,503 | 0,047 | 2 |
| 11 | 0,027 | 0,503 | 0,053 | 2 |
| 12 | 0,03 | 0,503 | 0,06 | 2 |
| 13 | 0,033 | 0,502 | 0,067 | 2 |
| 14 | 0,037 | 0,502 | 0,073 | 2 |
| 15 | 0,04 | 0,502 | 0,081 | 2 |

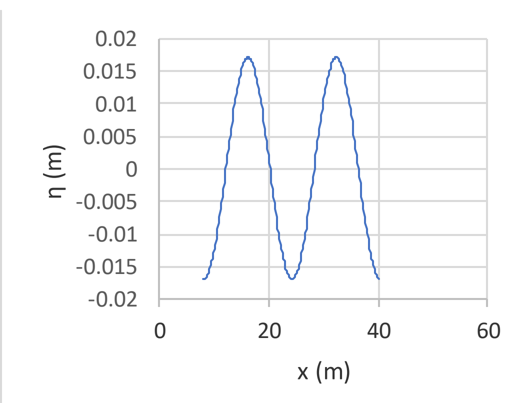


*Fig.4. Sinusoidal wave $T = 8$ sec., $A = 0.017$ m, at deep water.*

Fig.4. is the shape of wave surface sinusoidal for wave period 8 sec., which shows that wave crest and wave trough are symmetrical, i.e., $\eta_{max} = |\eta_{min}|$.

4.4. Cnoidal Type

Table (3) and Table (4) show the result of the calculation of wave profile characteristic with wave amplitude $0.523A_{max}$ and $A_{max}$.

With wave amplitude of $0.523A_{max}$, $\frac{\eta_{max}}{H} = 0.635$ was obtained, which, according to Wilson criteria is the maximum limit of Stoke's wave type. The value of $\frac{H}{A} = 2.099$, is still quite close with 2.

*Table.3: Wave profile at wave amplitude $A = 0.523A_{max}$*

| $T$ (sec.) | $A$ (m) | $\frac{\eta_{max}}{H}$ | $H$ (m) | $\frac{H}{A_{max}}$ |
|---|---|---|---|---|
| 7 | 0,47 | 0,635 | 0,986 | 2,099 |
| 8 | 0,614 | 0,635 | 1,288 | 2,099 |
| 9 | 0,777 | 0,635 | 1,631 | 2,099 |
| 10 | 0,959 | 0,635 | 2,013 | 2,099 |
| 11 | 1,161 | 0,635 | 2,436 | 2,099 |
| 12 | 1,381 | 0,635 | 2,899 | 2,099 |
| 13 | 1,621 | 0,635 | 3,402 | 2,099 |
| 14 | 1,88 | 0,635 | 3,946 | 2,099 |
| 15 | 2,158 | 0,635 | 4,529 | 2,099 |

With wave amplitude of $A_{max}$, $\frac{\eta_{max}}{H} = 0.803$ was obtained, with the value of $\frac{H}{A} = 2.726 > 2$ with a deviation of 36.3 % of 2.

*Table.4: Wave profile at wave amplitude $A = A_{max}$*

| $T$ (sec.) | $A$ (m) | $\frac{\eta_{max}}{H}$ | $H$ (m) | $\frac{H}{A}$ |
|---|---|---|---|---|
| 7 | 0,899 | 0,803 | 2,449 | 2,726 |
| 8 | 1,174 | 0,803 | 3,199 | 2,726 |
| 9 | 1,485 | 0,803 | 4,049 | 2,726 |
| 10 | 1,834 | 0,803 | 4,998 | 2,726 |
| 11 | 2,219 | 0,803 | 6,048 | 2,726 |
| 12 | 2,641 | 0,803 | 7,198 | 2,726 |
| 13 | 3,099 | 0,803 | 8,447 | 2,726 |
| 14 | 3,594 | 0,803 | 9,797 | 2,726 |
| 15 | 4,126 | 0,803 | 11,246 | 2,726 |

The profile of wave with wave period 8 sec., wave amplitude $A = 1.174$ m or $A = A_{max}$., can be seen on

Fig.5. The cnoidal profile that was formed is very clear and almost perfect.

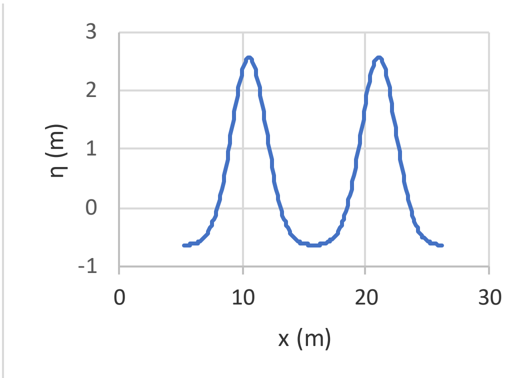


*Fig.5. Cnoidal wave* $T = 8\ sec., A = 1.174\ m$, *at deep water.*

## V. CONCLUSION

In the deep water, there are three types of wave produced by kinematic free surface boundary condition, i.e.Airy waves, Stoke's waves dan cnoidal waves. At a very small wave amplitude, Airy waves type was produced with symmetrical wave profile between wave crest and wave trough and wave height has a value twice of wave amplitude value. By enlarging wave amplitude, Stoke's wave type will be obtained, where the wave height is still close to twice the wave amplitude. Cnoidal wavetype was obtained by enlarging wave amplitude more than the wave amplitude at Stoke's waves, where at cnoidal waves the value of wave height is bigger than twice the wave amplitude.

The second conclusion is a calculation with an assumption that wave height is twice wave amplitude for a large wave amplitude is inaccurate

As has been stated, the research was done at deep water. Therefore, the next research should be on wave profile at shallow water, particularly at breaker point. This is important to interpret the result of shoaling breaker analysis with wave amplitude variable. It is also to examine wave height produced by a model using wave height as its variable by defining wave height that is twice wave amplitude

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