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The impact of single-disk and multi-disk theories on wind turbine design

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Abstract *— Aerodynamics is an experimental science, and outstanding design results are achieved through trial and error. This article primarily discusses the bottlenecks encountered in innovative wind power development, pointing out the shortcomings of the Betz limit as applied to new wind power theories and the long-standing misconceptions caused by the conclusions of the "Betz limit" in guiding aerodynamic design for novel wind turbines. It argues that the Betz limit is applicable only under the conditions of traditional single-disc horizontal-axis wind turbines and is not suitable for vertical-axis wind turbines. Instead, vertical-axis wind turbines should be explained using a new framework—the multi-disc theory—along with practical examples of its application.*

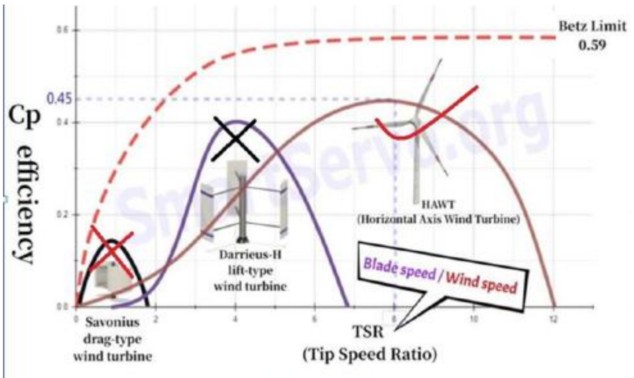
Foreword：

In aerodynamics, different shapes exhibit different aerodynamic characteristics. Traditional horizontal axis wind turbines (HAWT) designed using Blade Element Momentum (BEM) theory(1878) and the Betz limit law(1921) share highly consistent structural forms and deliver nearly identical performance, indicating that the design tools and methodologies for conventional HAWTs have become standardized.

By contrast, the VAWT designed using the same Blade Element Momentum theory and Betz limit principles display widely varying configurations and performance levels often with significant differences. This demonstrates that the design tools and methods developed for traditional HAWTs cannot be directly applied to VAWTs, and that innovative design tools and methodologies are therefore required.

Because the rotor of a HAWT moves within a two-dimensional plane, as it rotates and the blades pass through different azimuthal positions, the aerodynamic characteristics at the same point on a blade remain unchanged. In other words, the aerodynamic behavior at a given spatial position on the blade is independent of the blade's azimuth angle during rotation.

If the turbine is well designed, according to the Betz limit theory, the maximum theoretical energy conversion efficiency at the blade tip can reach 59.3% [1]. However, the curve described in the Betz limit does not actually represent the overall efficiency characteristics of a horizontal-axis wind turbine; rather, it describes the performance of a single blade rotating about a central axis. Expressed mathematically, the Betz limit law can be interpreted as follows: **the efficiency of a blade is a function of its length from root to tip, and the maximum efficiency occurs when the tip-speed ratio reaches approximately 5–8 times the wind speed**.



Betz Limit Law

Under the same airflow conditions, a two-bladed wind turbine generates slightly more power than a single-bladed HAWT; however, the efficiency of each individual blade decreases. Similarly, a traditional three-bladed turbine produces slightly more power than a two-bladed turbine, yet the efficiency of each blade is lower than that of a two-bladed configuration. In other words, increasing the number of blades leads to a gradual increase in total power output at the same wind condition, but reduces the efficiency of each individual blade.

Most traditional HAWTs adopt a three-blade configuration not because it delivers the highest aerodynamic efficiency, but because it achieves the lowest cost of electricity generation.

In addition to traditional horizontal-axis wind power technology, from the beginning of this century to around 2015, various types of vertical-axis wind turbines with diverse configurations were developed. These included lift-type designs, drag-type designs, and even hybrid of lift-type and drag-type designs combining both lift and drag. Some vertical turbines continuously altered their external shapes, with different configurations representing different aerodynamic characteristics. However, influenced by the design philosophy of traditional HAWT, as well as misconceptions stemming from the Betz limit law and blade element momentum theory particularly regarding tip speed ratio and solidity, most VAWT designs, with very few exceptions, were unsuccessful.

They not only exhibited very low power generation efficiency, but also lacked aerodynamic over-speed protection. As a result, they faced a dilemma: they could not generate electricity at low wind speeds, and risked structural failure at high wind speeds. Consequently, they were unable to achieve commercialization comparable to HAWT.

I. THE RELATIONSHIP OF WIND ENERGY BETWEEN AVERAGE WIND SPEED AND CONSTANT WIND SPEED.

The wind energy is a function of the cube of wind speed. In practice, wind speed is expressed as an average value. The energy density under average wind speed conditions is much greater than that under constant wind speed conditions[4]. Under extreme conditions, the wind energy corresponding to an average wind speed can be up to four times that of a constant wind speed.

For example, assuming standard atmospheric pressure, at a constant wind speed of 5m/s, the wind energy per square meter over two hours is 153.125Wh. However, under an average wind speed of 5m/s, the wind energy over two hours would be far greater than 153.125Wh. Suppose that within the two-hour period corresponding to the 5m/s average wind speed, the wind speed is 10m/s for one hour and 0 m/s for the other hour. In that case, the wind energy under the 5m/s average wind speed would be 612.5Wh/hour, four times the energy under a constant wind speed of 5m/s. The reason is that wind energy and wind speed are not linearly related.

Therefore, actual wind energy is related to wind frequency distribution and follows a Rayleigh distribution. In natural environments, the wind energy corresponding to an average wind speed is typically about 1.5 times that under a constant wind speed of the same value. This is the most important factor that cannot be represented by a simple wind speed–power curve when describing wind turbine performance.

II. MAIN CHARACTERISTICS OF A SINGLE-DISC ROTOR HAWT

It is well known that traditional HAWT technology is already quite mature. However, in recent years, apart from the continuous increase in single-unit turbine capacity, there has been little technological innovation.

According to the Betz limit law, the aerodynamic efficiency of a blade in a traditional HAWT is a function of blade length. The efficiency is highest near the blade tip. With good design, the theoretical aerodynamic efficiency at the tip can reach about 50%. However, as one moves closer to the blade root, the aerodynamic efficiency decreases, and near the rotational center of the rotor it is almost zero.

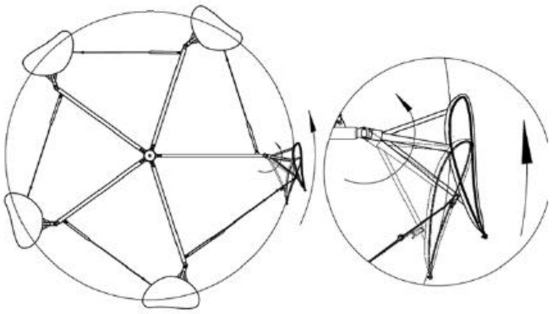
When mechanical transmission efficiency, generator efficiency, and wind misalignment losses are also taken into account, the average wind-to-electric conversion efficiency of horizontal-axis wind turbines calculated based on the incoming wind speed is generally around 20%.

The leveled cost of electricity (LCOE) can be expressed as: (Total investment and maintenance cost of the wind turbine)/(Total electricity generated over the turbine's lifetime).

III. BASIC CHARACTERISTICS OF VAWT

Early lift-type VAWT had relatively fixed blade angles, only changing the blade angle using centrifugal force in over-speed protection wind speed ranges. The aerodynamic efficiency of these vertical wind turbines did not increase with changes in blade angle; instead, it reduced aerodynamic efficiency to achieve overspeed protection when the wind speed exceeded the rated wind speed. Because these vertical wind turbines used elastic components for angle control, their diameter could not be made large, and they were typically limited to 10 kilowatt power level vertical wind turbines.

Below fig is a VAWT structure using centrifugal force for overspeed protection



VAWT structure using centrifugal force for overspeed protection

The core technology of this type VAWT consists of six key factors.

3.1 First is airfoil selection. A non-symmetrical low-speed airfoil must be chosen, rather than the symmetrical airfoils commonly recommended in textbooks, such as NACA0012, NACA0015, or NACA0018[5]. Aerodynamic theory originates from the aviation industry, where speeds of (200–300)km/h are still considered low speed; therefore, the airfoil requirements differ from HAWT.

3.2 Second, the concave side of the blade centerline should face outward, rather than the commonly assumed convex side facing outward.

3.3 Third, an appropriate blade initial angle must be set, usually with an optimal range between 4° to 8°.

3.4 Fourth, the ratio between the width of a single blade and the rotor diameter must be considered. The optimal ratio lies between 0.16 and 0.165. However, this ratio is not the same as the solidity concept used in horizontal-axis wind turbines and should not be simply interpreted as equivalent to horizontal turbine solidity.

For VAWT with fixed blade pitch, the optimal tip speed ratio is between 1.4 and 1.5, rather than the 3–5 range often cited as optimal for Darrieus turbines under Betz-limit-based theory, nor the mistaken conclusion that efficiency becomes negligible below 3. It is precisely this misconception that has led to the wide variety of lift-type vertical turbine configurations.

3.5 Fifth element is the number of blades. Because a vertical rotor with fixed blade pitch produces opposite torque when blades are in the upwind and downwind positions, practical verification shows that a five-blade rotor achieves the highest aerodynamic efficiency for this type of vertical-axis wind turbine.

3.6 Sixth is over-speed protection structure instead of a load resistor.

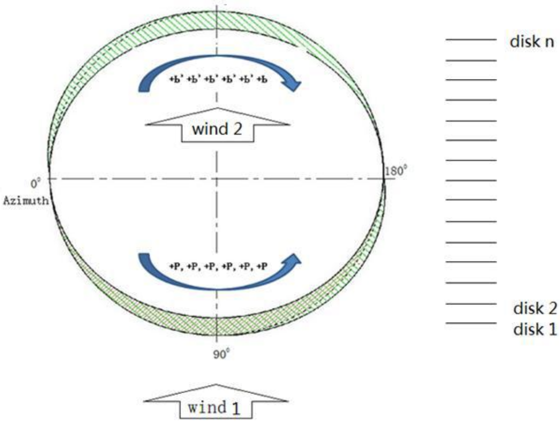
Below fig is a VAWT structure with relatively fixed blade angle

VAWT with relatively fixed blade angle

IV. CHARACTERISTICS OF VAWT WITH ATTACK ANGLE REGULATION STRUCTURE.

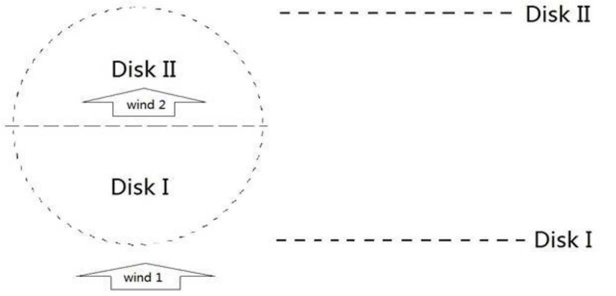
In a VAWT with a relatively fixed blade angle, as the rotor rotates and the blades are at different positions on the circumference, the angle of blade continuously changes with the blade position, it means, **the attack angle of blade always changes.**

Consequently, the magnitude and direction of the torque also change with the blade attack angle. Wind tunnel experiments have verified that the torque is high at some positions, low at others, positive at some positions, and negative at others. The final output torque of the rotor is the resultant torque of these torques, resulting in relatively low aerodynamic efficiency for the vertical wind turbine. The diagram below assumes that the ratio of rotor height to rotor diameter is less than 5%. Wind 1, flowing through the upwind blades, reaches wind 2 at the downwind blades under the continuous push of subsequent winds. Wind 2 is the same as wind 1. At this point, the aerodynamic characteristics of the blades are different at each wind section position (disk 1, disk 2…disk n), and can be considered as countless disks. Below fig shows a torque variation law of blades at different position.



Torque variation of blades at different position of VAWT

Simply put, the blades on the upwind side and the blades on the downwind side have the same absolute value of average torque, but in opposite directions. Therefore, they can be simplified to two disks with the blades on the upwind and downwind sides.



VAWT simplified of multiple disks

By adjusting the blade angle in real time based on the blade's position, rotor speed, wind direction, wind speed, and rotor power, the magnitude and direction of the torque at different blade positions can be adjusted, greatly improving the efficiency of vertical wind turbines. The mechanism for real-time blade angle adjustment uses a blade pivot at a certain position on the blade chord line, allowing the blade to rotate around this pivot. Control is achieved through a servo mechanism, it calls "active real-time attack angle regulation" technology.

This technology of active real-time attack regulation was verified through wind tunnel testing in 2014. In the experiment, the cross-sectional area of the wind tunnel test section was 10 square meters, the rotor height was 1 meter, and the rotor diameter was 1.36 meters. Using a magneto-resistive meter as a load, a servo mechanism was used to individually control the real-time change of the angle of each blade. The optimal angle was set for each blade at different positions, thus forming a family of angles. At a wind tunnel speed of 2 m/s, a stable torque of 1 Nm and a rotational speed of 44 rpm were measured. Calculated using traditional methods (ignoring blockage), the wind energy conversion efficiency reached 68%, exceeding the Bates limit of 59.3%. However, this does not mean the Bates limit is incorrect. Rather, the Bates limit describes a single-disc rotor moving in two-dimensional space, while a vertical rotor moves in three-dimensional space as a multi-disc rotor. When the three-dimensional vertical rotor is unfolded by its diameter, it is equivalent to two rotors, thus verifying the correctness of the multi-disc theory. Below fig is a wind tunnel test of VAWT with multiple disks.

Wind tunnel test of VAWT with multiple disks structure

Using the technology of active real-time attack regulation, even when the wind turbine is operating at overpower, it can maintain a constant power output through real-time blade angle changes. Even with vertical wind turbines employing "active real-time variable angle of attack" technology, improper angle settings at different blade positions can result in power output differences of several times, or even tens of times.

Calculations show that the blade tip speed ratio of this experimental setup is 1.6, which is far lower than the erroneous conclusion in the Bates limit that the optimal tip speed ratio for vertical wind turbines is between 3 and 5, and that anything below 3 is inefficient. It is also far lower than the 5-8 tip speed ratio for HAWT. This demonstrates that the description of vertical wind turbine characteristics in the Bates limit is flawed, regardless of whether the blade angle is relatively fixed or varies with the blade position. The reason for this is that the Bates limit only applies to traditional single-disk HAWT and not to multi-disk VAWT. This is because horizontal axis wind turbines rotate in two-dimensional space, while vertical axis wind turbines rotate in three-dimensional space, resulting in different aerodynamic characteristics and conditions. In vertical axis wind turbines, changes in blade position are equivalent to changes in the angle of attack, which has a significant impact on aerodynamic characteristics. HAWT, however, do not exhibit the characteristics of attack angle.

Unlike the "variable pitch" technology of HAWT, the attack angle is a vector, while the pitch is not. Variable attack angle and variable pitch are fundamentally different. This is their most important difference

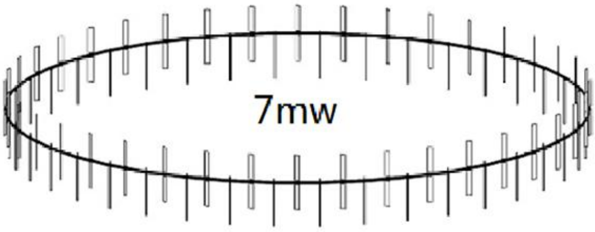
V. THE TECHNOLOGY OF ACTIVE REAL-TIME ATTACK ANGLE REGULATION HAS BEEN APPLIED TO VAWT DESIGN

Even when the blades are positioned at the optimal angle in every VAWT using "active real-time attack angle regulation" technology, which maximizes the wind energy conversion efficiency, the torque generated by the blades varies greatly depending on their position. The torque generated by the blades in the first quadrant is much greater than that in the fourth quadrant, and similarly, the torque generated by the blades in the second quadrant is much greater than that in the third quadrant. This is because when the upwind wind blows past the front half of the wind turbine and reaches the interior of the vertical wind turbine, becoming similar to the downwind in a HAWT, the wind speed decreases, and the work done by the blades in the rear half of the rear wind turbine is much lower than the work done by the blades in the front half of the rear wind turbine.

Wind energy is a function of the cube of wind speed. If the wind speed downwind inside a vertical wind turbine can be restored to the incoming wind speed, the wind energy utilization rate of the vertical wind turbine will be significantly improved from 68%. Due to the characteristics of wind, when wind blows over an object, it will gradually return to its original speed under the action of subsequent wind; we simply call this the incoming wind speed. Let's assume that the diameter of the wind turbine is infinitely large and the blade height is low enough. In this way, the wind will gradually return to the incoming wind speed after passing over the upwind blades, so that the downwind blades can also be affected by the same wind as the upwind blades.

To ensure that the downwind wind speed inside the rotor can recover to the incoming wind speed, the rotor diameter is designed based on the blade length. Setting the rotor diameter to blade height ratio to be above 10:1 ensures that the wind speed recovers to the incoming wind speed by the time it reaches the lower blades, significantly improving the wind energy conversion efficiency beyond 68%.

The three most expensive components of a VAWT are the generator, the rotor support structure, and the blades. Reducing the cost of these three components while achieving high wind energy conversion efficiency can drastically reduce the cost of wind power generation. This is the basic design philosophy of modular & combined turbines. Below fig is its structure.



Structure of a modular & combined wind turbine

VI. CONCLUSIONS

Electricity is a special commodity. The first principle of the electricity industry is the cost of power generation per kilowatt-hour. The cost of power generation is the most important economic indicator for power plants and is the lifeblood of power generation companies. In the future, wind power technology, which does not rely on various subsidies and has low generation costs that can compete with thermal and hydropower, is the future development direction of the wind power industry.

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